A Supplement to "the Double Dividend from Carbon Regulations in Japan"

Shiro Takeda*

Department of Economics, Kanto Gakuen University
200 Fujiakucho, Ota city, Gunma, 373–8515, Japan.

e-mail: `<zbc08106@park.zero.ad.jp>`

January, 2006

`$Id: double_dividend_supplement.tex,v 1.7 2006/03/09 09:42:06 st Exp $`

Abstract

This is a supplementary paper to the paper "the Double Dividend from Carbon Regulations in Japan". In this paper, we describe the complete model structure, parameterizations, data construction, and GAMS code for the simulation.

Contents

*corresponding author, email: `<zbc08106@park.zero.ad.jp>`.

1 Description of the model

1.1 Overview of the model

The model is based on a multisector dynamic general equilibrium model used in Böhringer et al. (1997), Rutherford et al. (2002), and Rutherford and Light (2002). It consists of the following six parts: production side, households, the government, Armington sectors, investment good production sector, and the rest of the world. We assume that there are 27 sectors and goods, and that all markets are perfectly competitive. Using intermediate inputs and primary factors, production sectors create and sell goods and pay factor income to households.

To represent the demand side, we assume an infinitely lived representative household.[1] The household supplies primary factors to production sectors and enjoys consumption. In our model, it is assumed that the household owns the capital stock and earns rental revenues by lending it to production sectors. Moreover, the household is assumed to purchase investment goods so as to accumulate the capital stock.

The model is an open economy, and the country trade goods with the rest of the world (ROW). Like other CGE analyses, we use the Armington assumption. The Armington assumption implies that domestically produced goods and imported goods are imperfect substitutes. Domestic goods and imported goods are aggregated through a CES function. The government obtains revenues through various taxes and spend them for transfer to the household and government expenditure. As taxes on production, we consider capital tax, labor tax, and indirect tax on production. and as taxes on households, we consider capital income tax, labor income tax, and consumption tax.

In the followings, we present the model description. First, we present the model as an infinite horizon model, and afterward modify it to a finite horizon model in which the simulation is conducted. Let s denote period index and i denote sector and good index. Moreover, we define the following sets:

- I $\cdots$ A set of all goods and sectors (see Table 1)
- $\mathrm{ES} = \{\mathrm{COC}, \mathrm{SLA}, \mathrm{COK}, \mathrm{CRU}, \mathrm{PET}, \mathrm{NAT}, \mathrm{GAS}, \mathrm{LIM}\}$ $\cdots$ Emission sources.
- $\mathrm{EC} = \{\mathrm{COC}, \mathrm{SLA}, \mathrm{CRU}, \mathrm{PET}, \mathrm{NAT}, \mathrm{GAS}\}$
- $\mathrm{CL} = \{\mathrm{COK}, \mathrm{LIM}\}$.
- $\mathrm{ELE} = \{\mathrm{ELE}\}$ $\cdots$ Electricity.
- $\mathrm{ENE} = \mathrm{EC} \cup \mathrm{ELE}$ $\cdots$ Energy goods
- $\mathrm{NENE} = I \setminus \mathrm{ENE}$ $\cdots$ Non-energy goods
- $\mathrm{EN} = \mathrm{ES} \cup \mathrm{ELE}$ $\cdots$ Energy goods + COL + LIM
- $\mathrm{NEN} = I \setminus \mathrm{EN}$ $\cdots$ Non-energy goods – COL – LIM

In Figure 1, the flows of goods, primary factors, taxes, and emissions are graphically depicted.

[1] In the simulation, we use a finite horizon model. However, we assume an infinite horizon to describe the model structure.

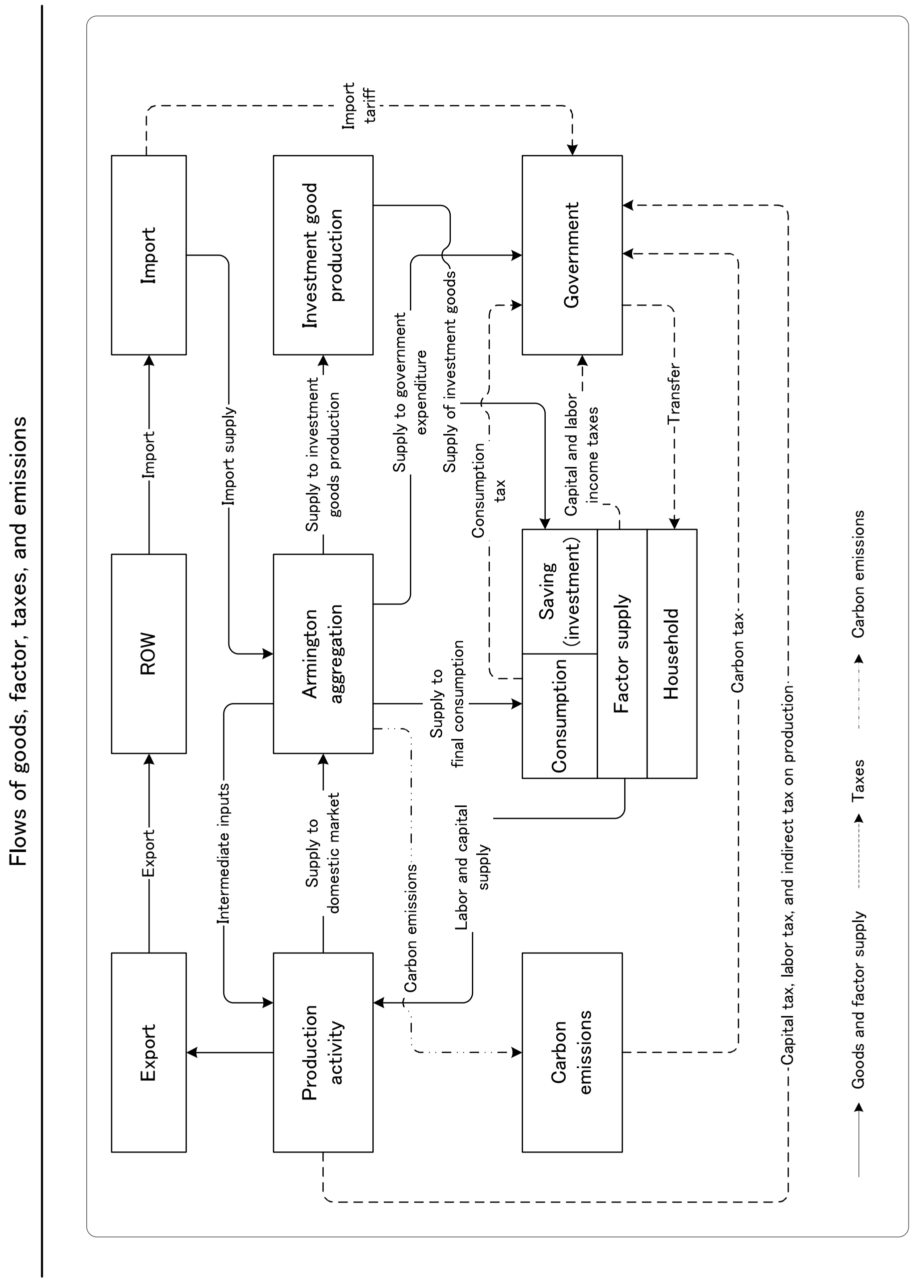

Figure 1: Flow of goods, factors, and taxes.

Table 1: Sector identifiers (27 sectors)

Identifier	Sector description
AGR	Agriculture, forestry and fishery
LIM	Limestone (Materials for ceramics)
COC	Coking coal
SLA	Steam coal, lignite and anthracite
CRU	Crude petroleum
NAT	Natural gas
OMI	Other minings
FOO	Foods
TET	Textile products
PPP	Pulp, paper and wooden products
CHM	Chemical products
PET	Petroleum refinery products
OPP	Other petroleum products (naphtha and others)
COK	Coke (Coal products)
CSC	Ceramic, stone and clay products
IAM	Iron and metal
MAC	Machinery
OIP	Other industrial products
CON	Construction
ELE	Electricity
GAS	Gas supply
SWW	Steam, and hot water supply, water supply and waste disposal services
COM	Commerce
RES	Real estate
TCB	Transport, communication and broadcasting
PUB	Public administration
SER	Services

1.2 Production side

Using intermediate inputs and primary factors (labor and capital stock), each sector produces a good under CRS technology so as to maximize profits. All markets are perfectly competitive and all producers act as price takers. We assume that capital stock is owned not by industries but by households and that industries borrow capital stock from households and pay rental price. It follows that investment is made not by industries but by households. It is also assumed that each industry is imposed capital and labor taxes on employment of labor and capital respectively. Moreover, each industry pays ad valorem tax on his output (indirect tax on production).

1.2.1 Production function

First, let us explain the input side of the production. In the production function, inputs are categorized into the following four types:[2]

- Non-energy intermediate inputs
- Energy intermediate inputs for combustion purpose
- Energy intermediate inputs for non-combustion purpose

[2] More precisely, inputs are classified into the following six types: (1) non-energy intermediate inputs except LIM and COL, (2) energy intermediate inputs for combustion purpose except ELE, (3) ELE, (4) COL and LIM for combustion purpose, (5) emission sources for non-combustion purpose, and (6) primary factors. In the explanation in the text, we simplify the argument a little.

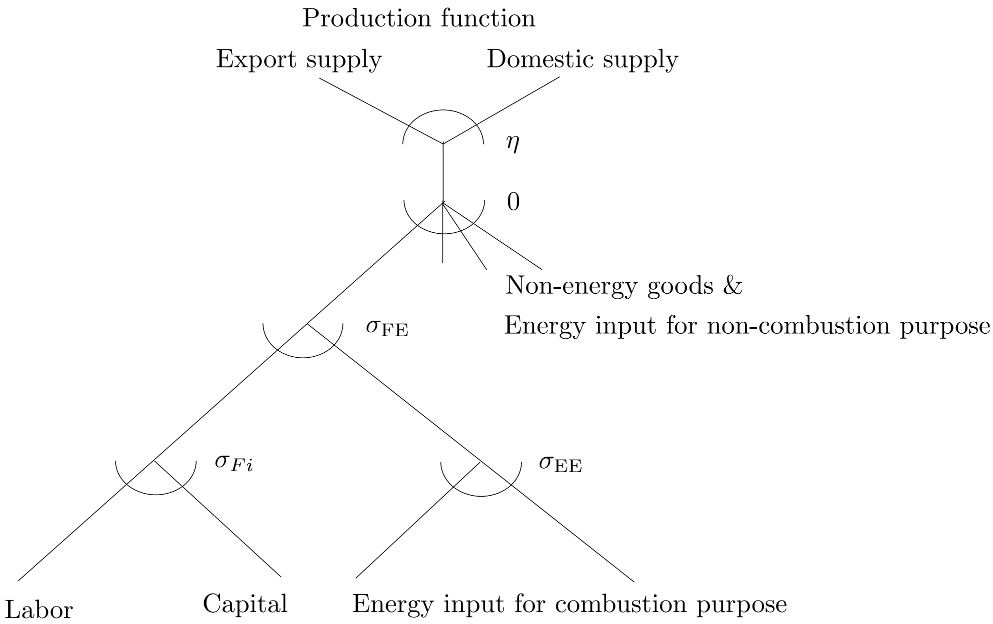

Figure 2: Production function

- Primary factors (labor and capital)

Energy goods indicate COC (coking coal), SLA (steam coal, lignite, and anthracite), CRU (crude oil), NAT (natural gas), PET (petroleum refinery products), GAS (gas supply), and ELE (electricity), and non-energy goods are all other goods. Note that although LIM (limestone) and COK (coke) are emission sources, they are not treated as energy goods, and that ELE (electricity) is classified as an energy good, but it is not classified as an emission source because its direct use and consumption do not emit carbon. Moreover, we distinguish energy inputs for combustion purpose and those for non-combustion purpose and exclude the latter in calculating carbon emissions. Energy inputs for non-combustion purpose are treated in the same way as ordinary non-energy inputs because most of them are used as materials for secondary energy sectors.

The production function is assumed to have nested CES structure represented by Figure 2. The numerical values and sigmas in the figure represent elasticities of substitution between inputs. The input structure is explained as follows. First, in the third stage, labor and capital are aggregated into a primary factor composite through a CES function with elasticity ($\sigma_{F,i}$). Let Q^F_{si} denote the amount of primary factor composite, and let L^D_{si} and K^D_{si} denote labor and capital inputs respectively. Then, this relation is expressed as[3]

$$Q^F_{si} = f^F_i(L^D_{si}, K^D_{si}) = \left[\alpha^{\mathrm{FL}}_i (L^D_{si})^{\frac{\sigma_{F,i}-1}{\sigma_{F,i}}} + (1-\alpha^{\mathrm{FL}}_i)(K^D_{si})^{\frac{\sigma_{F,i}-1}{\sigma_{F,i}}}\right]^{\frac{\sigma_{F,i}}{\sigma_{F,i}-1}} \tag{1}$$

Note that elasticity of substitution between labor and capital $\sigma_{F,i}$ differs between sectors.

[3]Strictly speaking, the function $f^F_i(\cdot)$ should be written as

$$Q^F_{si} = \left[\alpha^{\mathrm{FL}}_i (L^D_{si})^{\frac{\sigma_{F,i}-1}{\sigma_{F,i}}} + \alpha^{\mathrm{FK}}_i (K^D_{si})^{\frac{\sigma_{F,i}-1}{\sigma_{F,i}}}\right]^{\frac{\sigma_{F,i}}{\sigma_{F,i}-1}}$$

because we omit the scale factor. However, for the notational simplification, we use notation in (1). The same argument is applied to notations in the subsequent functions (e.g. $f^{\mathrm{FE}}_i(\cdot)$).

Similarly, energy goods for combustion purpose are aggregated into an energy composite through a CES function with elasticity σ_{EE}. This relation is represented by the following equation.

$$Q_{si}^{\text{EE}} = f_i^{\text{EE}}(\{Q_{sji}^E\}) = \left[\sum_{j \in \text{ENE}} \alpha_{ji}^E (Q_{sji}^E)^{\frac{\sigma_{\text{EE}}-1}{\sigma_{\text{EE}}}} \right]^{\frac{\sigma_{\text{EE}}}{\sigma_{\text{EE}}-1}}$$

where Q_{si}^{EE} is the amount of energy composite and Q_{sji}^E is sector i's input of energy good j.

Next, in the second stage, primary factor composite and energy composite are aggregated into primary factor-energy composite through a CES function with elasticity σ_{EC}. Let Q_{si}^{FE} denote the amount of primary factor-energy composite, then this relation is expressed by the following equation.

$$Q_{si}^{\text{FE}} = f_i^{\text{FE}}(Q_{si}^F, Q_{si}^{\text{EE}}) = \left[\alpha_i^F (Q_{si}^F)^{\frac{\sigma_{\text{FE}}-1}{\sigma_{\text{FE}}}} + (1-\alpha_i^F)(Q_{si}^{\text{EE}})^{\frac{\sigma_{\text{FE}}-1}{\sigma_{\text{FE}}}} \right]^{\frac{\sigma_{\text{FE}}}{\sigma_{\text{FE}}-1}}$$

Finally, in the first stage, output of sector i (Q_{si}) is determined with fixed coefficient aggregation of primary factor-energy composite (Q_{si}^{FE}), non-energy intermediate inputs (Q_{sji}^{NEN}), emission sources for non-combustion purpose (Q_{sji}^{NC}), and COL and LIM inputs for combustion purpose (Q_{sji}^{CL}):

$$\begin{aligned} Q_{si} &= f_i^Q(\{Q_{sji}^{\text{NEN}}\}, \{Q_{sji}^{\text{NC}}\}, \{Q_{sji}^{\text{CL}}\}, \{Q_{si}^{\text{FE}}\}) \\ &= \min \left[\left\{ \frac{Q_{sji}^{\text{NEN}}}{\bar{a}_{ji}^{\text{NEN}}} \right\}_{j \in \text{NEN}}, \left\{ \frac{Q_{sji}^{\text{NC}}}{\bar{a}_{ji}^{\text{NC}}} \right\}_{j \in \text{ES}}, \left\{ \frac{Q_{sji}^{\text{CL}}}{\bar{a}_{ji}^{\text{CL}}} \right\}_{j \in \text{CL}}, \frac{Q_{si}^{\text{FE}}}{\bar{a}_i^{\text{FE}}} \right] \end{aligned}$$

where $\bar{a}$ denote fixed input coefficient for each input.

When the production function is represented by multi-stage CES function, we can consider input choice in each stage separately. Based on this property of CES functions, we define price indices for inputs. Again, let us start with the third stage. Producers who try to maximize their profits choose labor and capital to minimize costs. From this cost minimizing behavior, we can define the price index for primary factor composite as follows.

$$p_{si}^F \equiv \min_{L,K} \left[\tilde{p}_{si}^L L + \tilde{r}_{si}^K K | f_i^F(L,K) = 1 \right]$$

where $\tilde{p}_{si}^L \equiv (1+t_i^L)p_s^L$ and $\tilde{r}_{si}^K \equiv (1+t_i^K)r_s^K$ are producer wage rate and rental price respectively, and t_i^L and t_i^K are labor tax rate and capital tax rate respectively. Price index for primary factor composite is the minimum cost for achieving one unit of primary factor composite. From the specification of (1), p_{si}^F is expressed as

$$p_{si}^F = \left[(\alpha_i^{\text{FL}})^{\sigma_{F,i}} (\tilde{p}_{si}^L)^{1-\sigma_{F,i}} + (1-\alpha_i^{\text{FL}})^{\sigma_{F,i}} (\tilde{r}_{si}^K)^{1-\sigma_{F,i}} \right]^{\frac{1}{1-\sigma_{F,i}}}$$

Similarly, we can define the price index for energy composite as follows.

$$\begin{aligned} p_{si}^{\text{EE}} &\equiv \min_{\{Q_j^E\}} \left[\sum_{j \in \text{ENE}} \tilde{p}_{sj}^E Q_j^E | f_i^{\text{EE}}(\{Q_j^E\}) = 1 \right] \\ &= \left[\sum_j (\alpha_{ji}^E)^{\sigma_{\text{EE}}} (\tilde{p}_{sj}^E)^{1-\sigma_{\text{EE}}} \right]^{\frac{1}{1-\sigma_{\text{EE}}}} \end{aligned}$$

where $\tilde{p}_{sj}^E = p_{sj}^E$ ($j \in \text{EC}$) and $\tilde{p}_{s,\text{ELE}}^E = p_{s,\text{ELE}}^A$. p_{sj}^E is the price of emission sources for combustion purpose and $p_{s,\text{ELE}}^A$ is the price of electricity. This price index represents minimum cost for achieving one unit of energy composite.

$$\left.\begin{array}{r} \left.\begin{array}{rcl} p^L_s, r^K_s & \longrightarrow & p^F_{si} \\ p^E_{sj}\ (j \in \mathrm{EC}), p^A_{s,\mathrm{ELE}} & \longrightarrow & p^{\mathrm{EE}}_{si} \end{array}\right\} \longrightarrow \qquad p^{\mathrm{FE}}_{si} \\ \\ p^E_{sj}\ (j \in \mathrm{CL}) \\ \\ p^A_{sj} \end{array}\right\} \longrightarrow c^Q_{si}$$

Figure 3: Price indices and unit cost

The same argument is applicable to the second stage decision. Since profit-maximizing producers choose inputs of primary factor composite and energy composite so as to minimize costs, the price index for primary factor-energy composite is defined as follows.

$$\begin{aligned} p^{\mathrm{FE}}_{si} &\equiv \min_{\{Q^F, Q^{\mathrm{EE}}\}} \left[p^F_{si} Q^F + p^{\mathrm{EE}}_{si} Q^{\mathrm{EE}} | f^{\mathrm{FE}}_i(Q^F, Q^{\mathrm{EE}}) = 1\right] \\ &= \left[(\alpha^F_i)^{\sigma_{\mathrm{FE}}} (p^F_{si})^{1-\sigma_{\mathrm{FE}}} + (1-\alpha^F_i)^{\sigma_{\mathrm{FE}}} (p^{\mathrm{EE}}_{si})^{1-\sigma_{\mathrm{FE}}}\right]^{\frac{1}{1-\sigma_{\mathrm{FE}}}} \end{aligned}$$

Finally, in the top level of the production function, output is fixed coefficient aggregation of primary factor-energy composite, non-energy intermediate inputs, energy inputs for non-combustion purpose, and COL and LIM inputs for combustion purpose. From the Leontief technology, unit cost of production is represented by linear function of input prices.

$$c^Q_{si} = \sum_{j \in \mathrm{NEN}} p^A_{sj} \bar{a}^{\mathrm{NEN}}_{ji} + \sum_{j \in \mathrm{ES}} p^A_{sj} \bar{a}^{\mathrm{NC}}_{ji} + \sum_{j \in \mathrm{CL}} p^E_{sj} \bar{a}^{\mathrm{CL}}_{ji} + p^{\mathrm{FE}}_{si} \bar{a}^{\mathrm{FE}}_{si}$$

where p^A_{sj} denote the price of intermediate input j. The above relations between prices are summarized in Figure 3.

Next, let us consider the output side of the production. Following Böhringer et al. (1997), Rutherford et al. (2002), and Rutherford and Light (2002), we assume that goods produced for domestic use and goods produced for export are differentiated, and that they are allocated through a CET (constant elasticity of transformation) function. Thus, domestic supply D_{si}, export supply X_{si}, and output Q_{si} have the following relation.

$$Q_{si} = f^O_i(X_{si}, D_{si}) = \left[\alpha^X_i (X_{si})^{\frac{1+\eta}{\eta}} + (1-\alpha^X_i)(D_{si})^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}$$

where η denotes constant elasticity of transformation. Given the price of domestic good p^D_{si} and the price of export good p^X_{si}, profit-maximizing producers allocate output to domestic and export markets so as to maximize their revenues. Thus, we can define the price index of output as follows.

$$p^Q_{si} = \max_{X,D} \left[p^X_{si} X + p^D_{si} D | f^O_i(X, D) = 1\right]$$

This price index p^Q_{si} indicates maximum revenues derived from one unit of output. From (2), p^Q_{si} is expressed as

$$p^Q_{si} = \left[(\alpha^X_i)^{-\eta} (p^X_{si})^{1+\eta} + (1-\alpha^X_i)^{-\eta} (p^D_{si})^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

Since we assume that the ad valorem tax t^Q_i is imposed on output, unit revenue for producers is given by $(1-t^Q_i)p^Q_{si}$.

From the above arguments, we can express zero profit condition for sector i as follows.

$$(1-t^Q_i)p^Q_{si} = c^Q_{si}$$

1.2.2 Demand and supply

In this section, demands for production inputs are derived. Since we have already defined unit cost function and price indices, we can easily derive demands for inputs by using Shephard's lemma. For example, one can obtain demand for labor by deriving demand for primary factor-energy composite, demand for primary factor composite, and demand for labor in turn and then combining these threes.

First, by Shephard's lemma, demand for primary factor-energy composite per unit of output is

$$\frac{\partial c^Q_{si}}{\partial p^{\mathrm{FE}}_{si}} = \bar{a}^{\mathrm{FE}}_i$$

Similarly, demand for primary factor composite per unit of primary factor-energy composite is given by

$$\frac{\partial p^{\mathrm{FE}}_{si}}{\partial p^F_{si}} = \left[\frac{\alpha^F_i p^{\mathrm{FE}}_{si}}{p^F_{si}}\right]^{\sigma_{\mathrm{FE}}}$$

Finally, demand for labor per unit of primary factor composite is

$$\frac{\partial p^F_{si}}{\partial \tilde{p}^L_{si}} = \left[\frac{\alpha^{\mathrm{FL}}_i p^F_{si}}{\tilde{p}^L_{si}}\right]^{\sigma_{F,i}}$$

Combining three demand functions above, we can derive total labor demand of sector i (L^D_{si}) as follows.

$$L^D_{si} = \frac{\partial c^Q_{si}}{\partial \tilde{p}^L_{si}} Q_{si} = \frac{\partial c^Q_{si}}{\partial p^{\mathrm{FE}}_{si}} \frac{\partial p^{\mathrm{FE}}_{si}}{\partial p^F_{si}} \frac{\partial p^F_{si}}{\partial \tilde{p}^L_{si}} Q_{si} = \left[\frac{\alpha^{\mathrm{FL}}_i p^F_{si}}{\tilde{p}^L_{si}}\right]^{\sigma_{F,i}} \left[\frac{\alpha^F_i p^{\mathrm{FE}}_{si}}{p^F_{si}}\right]^{\sigma_{\mathrm{FE}}} \bar{a}^{\mathrm{FE}}_i Q_{si}$$

By applying the same procedure, we can derive total demand for capital of sector i (K^D_{si}).

$$K^D_{si} = \frac{\partial c^Q_{si}}{\partial \tilde{r}^K_{si}} Q_{si} = \frac{\partial c^Q_{si}}{\partial p^{\mathrm{FE}}_{si}} \frac{\partial p^{\mathrm{FE}}_{si}}{\partial p^F_{si}} \frac{\partial p^F_{si}}{\partial \tilde{r}^K_{si}} Q_{si} = \left[\frac{(1-\alpha^{\mathrm{FL}}_i) p^F_{si}}{\tilde{r}^K_{si}}\right]^{\sigma_{F,i}} \left[\frac{\alpha^F_i p^{\mathrm{FE}}_{si}}{p^F_{si}}\right]^{\sigma_{\mathrm{FE}}} \bar{a}^{\mathrm{FE}}_i Q_{si}$$

Next, let us derive demand for energy goods for combustion purpose. The procedure is the same as in deriving demands for primary factor. That is, derive demand for primary factor-energy composite, demand for energy composite, and demand for each energy good in turn and then combining these threes. Sector i's demand for energy good j for combustion purpose (E^D_{sji}) is given by the following equation.

$$E^D_{sji} = \left[\frac{\alpha^E_{ji} p^{\mathrm{FE}}_{si}}{p^E_{sj}}\right]^{\sigma_{\mathrm{EE}}} \left[\frac{(1-\alpha^F_i) p^{\mathrm{FE}}_{si}}{p^{\mathrm{EE}}_{si}}\right]^{\sigma_{\mathrm{FE}}} \bar{a}^{\mathrm{FE}}_i Q_{si}$$

As to electricity, the price is $p^A_{s,\mathrm{ELE}}$ (not p^E_{si}). So, demand for electricity is given by

$$E^D_{sji} = \left[\frac{\alpha^E_{ji} p^{\mathrm{FE}}_{si}}{p^A_{sj}}\right]^{\sigma_{\mathrm{EE}}} \left[\frac{(1-\alpha^F_i) p^{\mathrm{FE}}_{si}}{p^{\mathrm{EE}}_{si}}\right]^{\sigma_{\mathrm{FE}}} \bar{a}^{\mathrm{FE}}_i Q_{si}, \qquad j = \mathrm{ELE}$$

Other intermediate inputs are entered into the production function through Leontief technology. So, unit demands for them are given by constant input coefficients. Demand for non-energy intermediate inputs, demand for COK and LIM for combustion purpose, and demand for energy goods for non-combustion purpose are respectively as follows.

$$\begin{aligned} &\bar{a}^{\mathrm{NEN}}_{ji} Q_{si} && j \in \mathrm{NEN} \\ &\bar{a}^{\mathrm{CL}}_{ji} Q_{si} && j \in \mathrm{CL} \\ &\bar{a}^{\mathrm{NC}}_{ji} Q_{si} && j \in \mathrm{ES} \end{aligned}$$

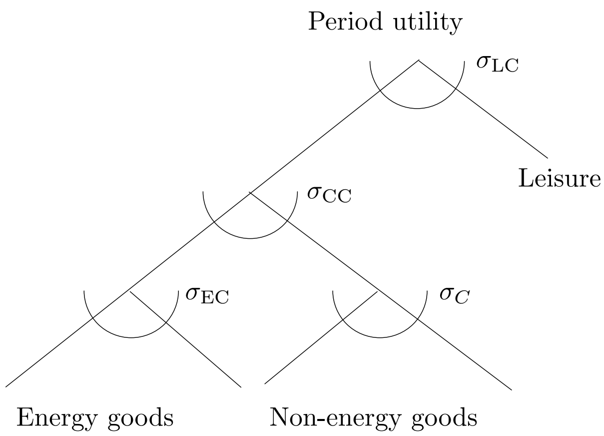

Figure 4: Period utility function

Next, consider supply functions. As demand function, we can derive supply function with Shephard's lemma, that is, differentiating the price index of output (unit revenue function) with supply prices. For example, export supply is given by

$$X_{si}^S = \frac{\partial p_{si}^Q}{\partial p_{si}^X} Q_{si} = \left[\frac{p_{si}^X}{\alpha_i^X p_{si}^Q} \right]^{\eta} Q_{si}$$

Similarly, domestic supply is

$$D_{si}^S = \frac{\partial p_{si}^Q}{\partial p_{si}^D} Q_{si} = \left[\frac{p_{si}^D}{(1-\alpha_i^X) p_{si}^Q} \right]^{\eta} Q_{si}$$

1.3 Household

In this section, we explain household behavior. To represent household behavior, we assume an infinitely lived representative household. The household utility in each period (period utility) depends on consumption and leisure, and then lifetime utility is determined by period utility in all periods. The household acts as not only a consumer but also a supplier of labor. Moreover, we assume that capital stock is owned by the household and is lent to producers. Thus, the household earns rental revenue as well as labor income. To accumulate the capital stock, the household purchases investment goods with his savings. Consumption, labor supply, and savings are determined through optimizing behavior of the household.

1.3.1 Utility function

Lifetime utility U is a CES function of period utility W_s: [4]

$$U = U(\{W_s\}_s) = \left[\sum_{s=t}^{\infty} \alpha_s^W (W_s)^{\frac{\sigma_U - 1}{\sigma_U}} \right]^{\frac{\sigma_U}{\sigma_U - 1}} \tag{2}$$

where α_s^W is a parameter for discounting utility and t is the initial period.

[4] In ordinary dynamic models、the following lifetime utility function is usually used.

$$U = \sum_{s=t}^{\infty} \Delta^{t-s} \frac{\sigma}{\sigma - 1} \left[W_s^{\frac{\sigma_U - 1}{\sigma_U}} - 1 \right]$$

where Δ is subjective discount factor and other notations are the same as (1). Since this utility function is a monotonic transformation of (2), utility maximizing choices derived from two utility functions coincide.

Period utility is a nested CES function which depends on leisure, energy consumption goods, and non-energy consumption goods (see Figure 4). The structure of period utility is as follows. First, in the third stage, energy consumption goods are aggregated into energy composite through a CES function with elasticity σ_{EC}.

$$C_s^{\mathrm{ENE}} = C^{\mathrm{ENE}}(\{C_{si}\}_{i\in\mathrm{ENE}}) = \left[\sum_{i\in\mathrm{ENE}} (\alpha_i^{\mathrm{EC}})(C_{si})^{\frac{\sigma_{\mathrm{EC}}-1}{\sigma_{\mathrm{EC}}}}\right]^{\frac{\sigma_{\mathrm{EC}}}{\sigma_{\mathrm{EC}}-1}} \tag{3}$$

Similarly, non-energy consumption goods are aggregated into non-energy composite through a CES function with elasticity σ_C.

$$C_s^{\mathrm{NENE}} = C^{\mathrm{NENE}}(\{C_{si}\}_{i\in\mathrm{NENE}}) = \left[\sum_{i\in\mathrm{NENE}} (\alpha_i^{C})(C_{si})^{\frac{\sigma_C-1}{\sigma_C}}\right]^{\frac{\sigma_C}{\sigma_C-1}}$$

Next, in the second stage, energy composite and non-energy composite are aggregated into aggregate consumption through a CES function with elasticity σ_{CC}.

$$\bar{C}_s = \bar{C}(C_s^{\mathrm{NENE}}, C_s^{\mathrm{ENE}}) = \left[\alpha^{\mathrm{NENE}}(C_s^{\mathrm{NENE}})^{\frac{\sigma_{\mathrm{CC}}-1}{\sigma_{\mathrm{CC}}}} + (1-\alpha^{\mathrm{NENE}})(C_s^{\mathrm{ENE}})^{\frac{\sigma_{\mathrm{CC}}-1}{\sigma_{\mathrm{CC}}}}\right]^{\frac{\sigma_{\mathrm{CC}}}{\sigma_{\mathrm{CC}}-1}}$$

Finally, in the first stage, aggregate consumption $\bar{C}_s$ and leisure LE_s determine the level of period utility W_s through a CES function with elasticity σ_{LC}.

$$W_s = W(LE_s, \bar{C}_s) = \left[\alpha^{\mathrm{LE}}(LE_s)^{\frac{\sigma_{\mathrm{LC}}-1}{\sigma_{\mathrm{LC}}}} + (1-\alpha^{\mathrm{LE}})(\bar{C}_s)^{\frac{\sigma_{\mathrm{LC}}-1}{\sigma_{\mathrm{LC}}}}\right]^{\frac{\sigma_{\mathrm{LC}}}{\sigma_{\mathrm{LC}}-1}} \tag{4}$$

1.3.2 Expenditure minimizing behavior

From the duality, utility maximizing behavior is captured in terms of expenditure minimizing behavior. Below, we see the household behavior in terms of expenditure minimizing behavior. As presented in the previous paragraphs, utility function is a multi-stage CES function. In this case, we can understand optimizing behaviors in each stage separately. That is, we can divide utility maximizing behavior into the following four choices: (1) choice between period utility in different periods, (2) choice between leisure and consumption, (3) choice between energy composite and non-energy composite, and (4) choice between consumption goods. Using this property, we consider the household behavior in each stage separately.

First, let us consider the third stage. The relation between energy composite C_s^{ENE} and energy consumption goods C_{si} is given by (3). Under this relation, utility-maximizing household chooses the consumption bundle that minimizes expenditure. From this expenditure minimization, we can define the price index for energy composite as follows.

$$p_s^{\mathrm{EC}} \equiv \min_{\{C_i\}_{i\in\mathrm{ENE}}} \left[\sum_{i\in\mathrm{ENE}} \tilde{p}_{si}^C C_i \mid C^{\mathrm{ENE}}(\{C_i\}) = 1\right]$$

where $\tilde{p}_{si}^C = (1-s_i^C)p_{si}^E$ and $\tilde{p}_{s,\mathrm{ELE}}^C = (1-s_{\mathrm{ELE}}^C)p_{s,\mathrm{ELE}}^A$. s_i^C is the rate of consumption subsidy and $\tilde{p}_{si}^C$ is household price of energy good i. p_s^{EC} indicates the minimum expenditure required to obtain one unit of energy composite, given $\tilde{p}_{si}^C$. From (3), p_s^{EC} is expressed as

$$p_s^{\mathrm{EC}} = \left[\sum_{i\in\mathrm{ENE}} (\alpha_i^{\mathrm{EC}})^{\sigma_{\mathrm{EC}}}(\tilde{p}_{si}^C)^{1-\sigma_{\mathrm{EC}}}\right]^{\frac{1}{1-\sigma_{\mathrm{EC}}}}$$

Similarly, we define the price index for non-energy composite.

$$\begin{aligned} p_s^C &\equiv \min_{\{C_i\}_{i\in\mathrm{NENE}}} \left[\sum_{i\in\mathrm{NENE}} \tilde{p}_{si}^C C_i \mid C^{\mathrm{NENE}}(\{C_i\}) = 1\right] \\ &= \left[\sum_{i\in\mathrm{NENE}} (\alpha_i^C)^{\sigma_C}(\tilde{p}_{si}^C)^{1-\sigma_C}\right]^{\frac{1}{1-\sigma_C}} \end{aligned}$$

Table 2: Price indices

$$\left.\begin{array}{l} \left.\begin{array}{l} \left.\begin{array}{lcl} \tilde{p}^C_{si}\ (i \in \text{ENE}) & \longrightarrow & p^{\text{EC}}_s \\ \tilde{p}^C_{si}\ (i \in \text{NENE}) & \longrightarrow & p^C_s \end{array}\right\} \longrightarrow p^{\text{CC}}_s \\ \\ \qquad\qquad\qquad\qquad\qquad\quad p^{\text{LE}}_s \end{array}\right\} \longrightarrow R_{t,s} c^W_s = p^W_s \\ \\ \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad p^W_{s'} \end{array}\right\} \longrightarrow p^U$$

where $\tilde{p}^C_{si} = (1 - s^C_i) p^A_{si}$ is the consumer price of non-energy good i.

In the second stage, the price of composite consumption is defined as follows.

$$\begin{aligned} p^{\text{CC}}_s &\equiv \min_{\{C^{\text{NENE}}, C^{\text{ENE}}\}} \left[p^C_s C^{\text{NENE}} + p^{\text{EC}}_s C^{\text{ENE}} \mid \bar{C}(C^{\text{NENE}}, C^{\text{ENE}}) = 1 \right] \\ &= \left[(\alpha^{\text{NENE}})^{\sigma_{\text{CC}}} (p^C_s)^{1-\sigma_{\text{CC}}} + (1 - \alpha^{\text{NENE}})^{\sigma_{\text{CC}}} (p^{\text{EC}}_s)^{1-\sigma_{\text{CC}}} \right]^{\frac{1}{1-\sigma_{\text{EC}}}} \end{aligned}$$

Next, we consider choice between leisure and consumption in the first stage. To obtain one unit of period utility, the household chooses combination of leisure and consumption that minimizes expenditure. Thus, we can defined unit cost for period utility as follows.

$$\begin{aligned} c^W_s &\equiv \min_{LE_s, \bar{C}_s} \left[p^{\text{LE}}_s LE_s + \tilde{p}^{\text{CC}}_s \bar{C}_s \mid W(LE_s, \bar{C}_s) = 1 \right] \\ &= \left[(\alpha^{\text{LE}})^{\sigma_{\text{LC}}} (p^{\text{LE}}_s)^{1-\sigma_{\text{LC}}} + (1 - \alpha^{\text{LE}})^{\sigma_{\text{LC}}} (\tilde{p}^{\text{CC}}_s)^{1-\sigma_{\text{LC}}} \right]^{\frac{1}{1-\sigma_{\text{LC}}}} \end{aligned}$$

where $p^{\text{LE}}_s = (1 - t^I) p^L_s$ is the household price of leisure, t^I is the labor income tax rate, $\tilde{p}^{\text{CC}}_s = (1 + t^C) p^{\text{CC}}_s$ is the household price of consumption, and t^C is the consumption tax rate.

Once period utility in each period is determined, it determines lifetime utility. We can define the price index for lifetime utility as follows.

$$\begin{aligned} p^U &\equiv \min_{W_s} \left[\sum_{s=t}^{\infty} p^W_s W_s \mid U(\{W_s\}_s) = 1 \right] \\ &= \left[\sum_{s=t}^{\infty} (\alpha^W_s)^{\sigma_U} (p^W_s)^{1-\sigma_U} \right]^{\frac{1}{1-\sigma_U}} \end{aligned}$$

where p^W_s is the present price of period utility. p^W_s is represented as $p^W_s = R_{t,s} c^W_s$ by defining the market discount factor $R_{t,s}$ as follows:

$$R_{t,s} = \begin{cases} 1 & s = t \\ \prod_{l=t}^{s} (1 + r_s)^{-1} & s > t \end{cases}$$

where r_s denotes the interest rate for loans between periods $s - 1$ and s. Note that all prices and values except p^W_s are current prices and values, but p^W_s represents present price.

The relation between different price indices defined above are represented in Table 2. Using price indices, lifetime utility that the household achieves is represented by the following relation.

$$U = Y^H / p^U \tag{5}$$

where Y^H is lifetime income that will be defined later.

1.3.3 Compensated demand

In this section, demands for consumption and leisure are derived. Since we have so far defined price indices (expenditure functions), we consider compensated demand functions (Hicksian demand functions). Compensated demand functions can be derived by using Shephard's lemma. The approach here is the same as the one in deriving demands for production inputs.

First, by Shephard's lemma, unit demand for period utility is

$$\frac{\partial p^U}{\partial p_s^W} = \left[\frac{\alpha_s^W p^U}{p_s^W} \right]^{\sigma_U} \tag{6}$$

Similarly, unit demand for leisure is

$$\frac{\partial c_s^W}{\partial p_s^{\mathrm{LE}}} = \left[\frac{\alpha^{\mathrm{LE}} c_s^W}{p_s^{\mathrm{LE}}} \right]^{\sigma_{\mathrm{LC}}} \tag{7}$$

Combining (6) and (7), compensated demand for leisure is derived as follows.

$$LE_s = \left[\frac{\alpha^{\mathrm{LE}} c_s^W}{p_s^{\mathrm{LE}}} \right]^{\sigma_{\mathrm{LC}}} \left[\frac{\alpha_s^W p^U}{p_s^W} \right]^{\sigma_U} U \tag{8}$$

Next, let us consider consumption demand. Consumption demand is derived in two stages: (1) deriving demand for aggregate consumption, (2) deriving demand for each consumption good.

First, unit demand for aggregate consumption is

$$\frac{\partial p_s^W}{\partial \tilde{p}_s^{\mathrm{CC}}} = \left[\frac{(1 - \alpha^{\mathrm{LE}}) c_s^W}{\tilde{p}_s^{\mathrm{CC}}} \right]^{\sigma_{\mathrm{LC}}}$$

Combining this and (6), demand for aggregate consumption is given by

$$\bar{C}_s^D = \left[\frac{(1 - \alpha^{\mathrm{LE}}) c_s^W}{\tilde{p}_s^{\mathrm{CC}}} \right]^{\sigma_{\mathrm{LC}}} \left[\frac{\alpha_s^W p^U}{p_s^W} \right]^{\sigma_U} U$$

Next, we consider demand for each non-energy consumption good. Demand for non-energy consumption good $i \in \mathrm{NENE}$ is given by

$$C_{si}^D = \left[\frac{\alpha_i^C p_s^C}{\tilde{p}_i^C} \right]^{\sigma_C} \left[\frac{\alpha^{\mathrm{NENE}} p_s^{\mathrm{CC}}}{p_s^C} \right]^{\sigma_{\mathrm{CC}}} \bar{C}_s^D \qquad i \in \mathrm{NENE}$$

Similarly, demand for energy consumption good $i \in \mathrm{ENE}$ is

$$C_{si}^D = \left[\frac{\alpha_i^{\mathrm{EC}} p_s^{\mathrm{EC}}}{\tilde{p}_i^C} \right]^{\sigma_{\mathrm{EC}}} \left[\frac{(1 - \alpha^{\mathrm{NENE}}) p_s^{\mathrm{CC}}}{p_s^{\mathrm{EC}}} \right]^{\sigma_{\mathrm{CC}}} \bar{C}_s^D \qquad i \in \mathrm{ENE}$$

Finally, let us derive labor supply. Let $\bar{L}_s$ denote the total time available for leisure and work. Then, from (8), labor supply is given by

$$L_s^S = \bar{L}_s - LE_s$$

1.3.4 Lifetime income

To derive household lifetime income, it is necessary to consider flow budget constraint. The household derives income from four sources. First, he receives labor income. The actual labor income that the household receives is equal to $p_s^{\mathrm{LE}} L_s^S$, i.e., labor supply multiplied by household wage rate. However, since we assume that the household makes expenditure on leisure, it is necessary to use $p_s^{\mathrm{LE}} \bar{L}_s$ as labor income.

Second, the household supplies capital stock to industries and receives rental revenue (capital income). We assume that a tax (capital income tax) is imposed on this rental revenue. Let r_s^{KE}

denote after-tax rental price and t^A denote the rate of capital income tax. Then, $r_s^{\mathrm{KE}} = (1-t^A)r_s^K$ holds. The household capital income is given by $r_s^{\mathrm{KE}} K_s$.

In addition to factor income, the household receives transfer from the government, which is denoted as TRN_s. Moreover, the household receives return from asset other than capital. Let NA_s^H denote the value of household asset in the end of period $s-1$. Then return from this asset is represented by $r_s \mathrm{NA}_s^H$. In the reminder of the paper, "asset" refers to asset other than capital stock.

Summing up all the revenues above, household income in period s is expressed as

$$r_s^{\mathrm{KE}} K_s + r_s \mathrm{NA}_s^H + \mathrm{TRN}_s + p_s^{\mathrm{LE}} \bar{L}_s$$

Subtracting expenditure on consumption and leisure (EXP_s) from this amount, we can get household savings.[5] These savings are spent on investment INV_s and purchase of new asset $\mathrm{NA}_{s+1}^H - \mathrm{NA}_s^H$. Thus, the flow budget constraint of the household in period s is given by

$$\mathrm{INV}_s + \mathrm{NA}_{s+1}^H - \mathrm{NA}_s^H = r_s^{\mathrm{KE}} K_s + r_s \mathrm{NA}_s^H + \Omega_s \tag{9}$$

where $\Omega_s = p_s^{\mathrm{LE}} \bar{L}_s + \mathrm{TRN}_s - \mathrm{EXP}_s^H$.

Let p_s^I denote the price of investment good and I_s denote the amount of gross investment. Then, INV_s in (9) is equal to $p_s^I I_s$. We assume that there is adjustment cost for investment and that the following relation holds between net investment J_s and gross investment I_s:

$$I_s = J_s \left[1 + \Phi \left(\frac{J_s}{K_s} \right) \right] \tag{10}$$

where $J_s \Phi_s$ indicates adjustment cost. Below, we specify Φ_s as

$$\Phi \left(\frac{J_s}{K_s} \right) = \frac{\phi}{2} \frac{J_s}{K_s}$$

Since ϕ is a parameter determining adjustment cost, we call it *adjustment cost parameter.*

Inserting (10) into (9), we can get the following equation.

$$p_s^I J_s [1 + \Omega_s] + \mathrm{NA}_{s+1}^H - \mathrm{NA}_s^H = r_s^{\mathrm{KE}} K_s + r_s \mathrm{NA}_s^H + \Omega_s \tag{11}$$

Under the budget constraint of (11) and the constraint $K_{s+t} = (1-\delta)K_s + J_s$, the household chooses capital stock K_s, net investment J_s, and asset NA_s^H so as to maximize lifetime utility.[6]

$$\begin{aligned}
\max \quad & U(\{W_s\}) \\
\text{s.t.} \quad & p_s^I J_s [1 + \Omega_s] + \mathrm{NA}_{s+1}^H - (1 + r_s)\mathrm{NA}_s^H - r_s^{\mathrm{KE}} K_s - \Omega_s = 0 \\
& K_{s+1} = (1-\delta)K_s + J_s \\
& \text{Given } K_t \text{ and } \mathrm{NA}_t^H.
\end{aligned}$$

The Lagrangian function for this problem is given by[7]

$$\begin{aligned}
\mathcal{L} &= U(\{W_s\}) \\
&- \sum_{s=t}^{\infty} \lambda_s R_{t,s} \left[p_s^I J_s (1 + \Omega_s) + \mathrm{NA}_{s+1}^H - (1 + r_s)\mathrm{NA}_s^H - r_s^{\mathrm{KE}} K_s - \Omega_s \right] \\
&- \sum_{s=t}^{\infty} \mu_s R_{t,s} \left[K_{s+1} - (1-\delta)K_s - J_s \right]
\end{aligned}$$

[5] EXP_s is equal to $p_s^W W_s / R_{t,s}$.

[6] The household also chooses the level of W_s, but the decision of W_s has been considered in the previous sections. Thus we omit it here.

[7] We multiply Lagrange multipliers by discount factor so that they represent current value.

The first order conditions for NA_s^H, J_s, and K_s are given by

$$\frac{\partial \mathcal{L}}{\partial \mathrm{NA}_s^H} = 0: \quad \lambda_s = \lambda_{s-1} \qquad s > t \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial J_s} = 0: \quad \frac{\mu_s}{\lambda_s} = p_s^I \left[1 + \Phi_s + \Phi'_s \frac{J_s}{K_s}\right] \qquad s = t, t+1, \cdots \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial K_s} = 0: \quad r_s^K + \frac{\mu_s}{\lambda_s}(1-\delta) + p_s^I \Phi'_s \left[\frac{J_s}{K_s}\right]^2 = (1+r_s)\frac{\lambda_{s-1}}{\lambda_s}\frac{\mu_{s-1}}{\mu_s} \qquad s > t \tag{14}$$

As will be shown below, μ_s/λ_s represents the shadow price of capital. Thus, we set $p_s^K \equiv \mu_s/\lambda_s$. Moreover, we define p_s^{KA} as follows.

$$p_s^{\mathrm{KA}} \equiv -p_s^I \Phi'_s \left[\frac{J_s}{K_s}\right]^2 \tag{15}$$

This p_s^{KA} represents the reduction in adjustment cost resulted from one unit increase in K_s. We call this p_s^{KA} *adjustment premium.*

Using above notations, (13) and (14) become

$$p_s^K = p_s^I \left[1 + \Phi_s + \Phi'_s \frac{J_s}{K_s}\right] \qquad s = t, t+1, \cdots \tag{16}$$

$$r_s^K + p_s^K(1-\delta) + p_s^{\mathrm{KA}} = (1+r_s)p_{s-1}^K \qquad s = t+1, t+2, \cdots \tag{17}$$

Let us explain what these two conditions mean. First, (16) is interpreted as follows. Suppose that one unit of investment is made in period s. For this, it is necessary to purchase one unit of investment good, which costs p_s^I (the first term of the the RHS). In addition to this, it requires adjustment cost of $p_s^I \Phi_s$ (the second term). Moreover, the additional cost of $p_s^I \Phi'_s J_s/K_s$ arises because increase in net investment raises adjustment cost per unit $p_s^I \Phi_s$ (the third term). To sum up, the RHS of (16) represents marginal cost of net investment in period s. This is the amount of payment required to achieve one unit of capital stock. Thus, it is interpreted as the shadow price of capital.

Next, let us consider (17). The RHS of (17) represents marginal cost (or opportunity cost) of capital stock in period s. To acquire one unit of capital stock in period s, it costs p_{s-1}^K. In addition to this, one passes up return which could have been achieved if he invested it on another asset. Thus, the total cost of achieving one unit of capital in period s is $(1+r_s)p_{s-1}^K$.[8]

On the other hand, the LHS of (17) represents marginal revenue of capital in period s. The first term is rental revenue. One unit of capital stock in period s generates rental revenue of r_s^{KE}. The second term is revenue from selling capital. One unit of capital stock in the end of period $s-1$ becomes $1-\delta$ units of capital in the end of period s and can be sold for p_s^K. The third term is reduction in adjustment cost. Increase in capital stock reduces adjustment cost per unit of investment, thus the cost of investment is reduced. As already explained, this reduction is represented by adjustment premium p^{KA}. The sum of three terms is additional revenue from increase in one unit of capital stock.

As the above argument shows, (17) implies that marginal revenue from capital stock is equalized to its marginal cost.[9] This condition is naturally derived from the optimizing behavior of the household. Note that (17) holds only for $s > t$ because K_t is given exogenously for the household.[10]

[8] Actually, one can acquire capital stock only through new investment. However, if it is traded in a market, the shadow price becomes transaction price.

[9] Rewriting (17), we can interpret it in another way.

$$r_s = \frac{r_s^{\mathrm{KE}}}{p_{s-1}^K} - \frac{p_s^K}{p_{s-1}^K}\delta + \frac{p_s^K - p_{s-1}^K}{p_{s-1}^K} + \frac{p_s^{\mathrm{KA}}}{p_{s-1}^K}$$

The LHS of this equation represents the rate of return when one yen is invested to assets other than capital. On the other hand, the RHS which is the sum of (1) the rate of return from rental revenue, (2) the rate of loss resulting from depreciation, (3) the rate of capital gain, and (4) the rate of return from reduction in adjustment cost, represents the rate of return from capital stock. Thus, (17) represents no arbitrage condition between capital stock and other assets.

[10] However, if the economy is in the steady state at the initial period, (17) holds for $s = t$

By the definition of Φ_s, (16) and (17) become

$$p_s^K = p_s^I \left[1 + \phi \frac{J_s}{K_s}\right] \qquad s = t, t+1, \cdots \tag{18}$$

$$r_s^K + p_s^K(1-\delta) + p_s^{\mathrm{KA}} = (1+r_s)p_{s-1}^K \qquad s = t+1, t+2, \cdots \tag{19}$$

Using these optimal conditions for investment and capital stock, we can derive lifetime budget constraint from flow budget constraint. Flow budget constraint is given by the following equation.

$$p_s^I J_s \left[1 + \frac{\phi}{2}\frac{J_s}{K_s}\right] + \mathrm{NA}_{s+1}^H - \mathrm{NA}_s^H = r_s^{\mathrm{KE}} K_s + r_s \mathrm{NA}_s^H + \Omega_s$$

Adding $p_s^I \phi (J_s)^2/(2K_s)$ to both sides, we have

$$p_s^I J_s \left[1 + \phi \frac{J_s}{K_s}\right] + \mathrm{NA}_{s+1}^H = r_s^{\mathrm{KE}} K_s + p_s^I \frac{\phi}{2}\frac{J_s^2}{K_s} + (1+r_s)\mathrm{NA}_s^H + \Omega_s$$

By (18), this equation reduces to

$$p_s^K J_s + \mathrm{NA}_{s+1}^H = r_s^{\mathrm{KE}} K_s + p_s^I \frac{\phi}{2}\frac{J_s^2}{K_s} + (1+r_s)\mathrm{NA}_s^H + \Omega_s$$

Moreover, from $J_s = K_{s+1} - (1-\delta)K_s$ and the definition of p_s^{KA}, we have

$$p_s^K K_{s+1} + \mathrm{NA}_{s+1}^H = \left[r_s^{\mathrm{KE}} + p_s^K(1-\delta) + p_s^{\mathrm{KA}}\right] K_s + (1+r_s)\mathrm{NA}_s^H + \Omega_s \tag{20}$$

Since (19) holds for $s > t$, (20) becomes

$$p_s^K K_{s+1} + \mathrm{NA}_{s+1}^H = (1+r_s)\left[p_{s-1}^K K_s + \mathrm{NA}_s^H\right] + \Omega_s \qquad s > t \tag{21}$$

Applying forward iteration to this, we have

$$p_t^K K_{t+1} + \mathrm{NA}_{t+1}^H = R_{t,T}\left[p_T^K K_{T+1} + \mathrm{NA}_{T+1}^H\right] - \sum_{s=t+1}^{T} R_{t,s}\Omega_s \tag{22}$$

As to period t, (20) holds as is.

$$p_t^K K_{t+1} + \mathrm{NA}_{t+1}^H = \left[r_t^{\mathrm{KE}} + p_t^K(1-\delta) + p_t^{\mathrm{KA}}\right] K_t + (1+r_t)\mathrm{NA}_t^H + \Omega_t \tag{23}$$

From (22) and (23), lifetime budget constraint is derived as follows.

$$\begin{aligned}\left[r_t^K + p_t^K(1-\delta) + p_t^{\mathrm{KA}}\right] K_t + &(1+r_t)\mathrm{NA}_t^H \\ &- R_{t,T}\left[p_T^K K_{T+1} + \mathrm{NA}_{T+1}^H\right] + \sum_{s=t}^{T} R_{t,s}\Omega_t = 0\end{aligned}$$

Extending this to infinite horizon and assuming the following no Ponzi game condition:

$$\lim_{T\to\infty} R_{t,T}\left[p_T^K K_{T+1} + \mathrm{NA}_{T+1}^H\right] = 0$$

the lifetime budget constraint can be derived as follows.

$$\begin{aligned}\sum_{s=t}^{\infty} R_{t,s}\mathrm{EXP}_s = &\left[r_t^K + p_t^K(1-\delta) + p_t^{\mathrm{KA}}\right] K_t \\ &+ (1+r_t)\mathrm{NA}_t^H + \sum_{s=t}^{\infty} R_{t,s}\left[p_s^{\mathrm{LE}}\bar{L}_s + \mathrm{TRN}_s\right]\end{aligned} \tag{24}$$

The RHS of this equation is equal to Y^H in (5).

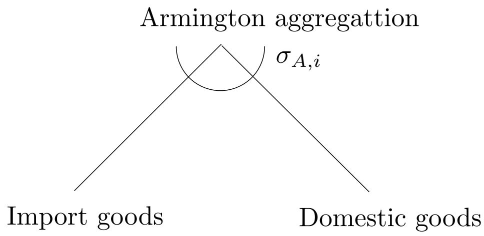

Figure 5: Armington Aggregation

1.4 Other activities

1.4.1 Armington aggregation

Like other CGE analyses, we use the Armington assumption (Armington, 1969). The Armington assumption implies that domestically produced goods and imported goods are imperfect substitutes. Domestic goods and imported goods are aggregated through a CES function in Figure 5. A good aggregated from domestic and import goods is called *Armington good* and its quantity is represented by A_{si}.

$$A_{si} = A_i(D_{si}, M_{si}) = \left[\alpha_i^{\mathrm{AD}}(D_{si})^{\frac{\sigma_{A,i}-1}{\sigma_{A,i}}} + (1-\alpha_i^{\mathrm{AD}})(M_{si})^{\frac{\sigma_{A,i}-1}{\sigma_{A,i}}}\right]^{\frac{\sigma_{A,i}}{\sigma_{A,i}-1}}$$

The produced Armington good is used for intermediate inputs, final consumption, investment, and government expenditure (Figure 1).

It is assumed that domestic and import goods are chosen so as to minimize cost. Thus, we can define the price index for Armington good.

$$\begin{aligned} p_{si}^A &\equiv \min\ \left[p_{si}^D D + \tilde{p}_{si}^M M | A_i(D, M) = 1\right] \\ &= \left[\alpha_i^{\mathrm{AD}}(p_{si}^D)^{1-\sigma^A} + (1-\alpha_i^{\mathrm{AD}})(\tilde{p}_{si}^M)^{1-\sigma^A}\right]^{\frac{1}{1-\sigma^A}} \end{aligned}$$

where $\tilde{p}_{si}^M = (1+t_i^M)p_{si}^M$ is the domestic price of import good. From this price index, we can derive demands for domestic good D_{si}^{AD} and import good M_{si}.

$$\begin{aligned} D_{si}^{\mathrm{AD}} &= \frac{\partial p_{si}^A}{\partial p_{si}^D} A_{si} = \left[\frac{\alpha^{\mathrm{AD}} p_{si}^A}{p_{si}^D}\right]^{\sigma^A} A_{si} \\ M_{si} &= \frac{\partial p_{si}^A}{\partial p_{si}^M} A_{si} = \left[\frac{(1-\alpha^{\mathrm{AD}}) p_{si}^A}{p_{si}^M}\right]^{\sigma^A} A_{si} \end{aligned}$$

1.4.2 Investment goods

It is assumed that an investment good is a fixed coefficient composite of 27 Armington goods. Let I_s denote the amount of investment good, A_{si}^I denote Armington good i used for investment, and a_i^I denote the amount of Armington good i required for producing one unit of investment good. Then, the following equation holds.

$$I_s = \min_i \left[\frac{A_{si}^I}{a_i^I}\right]$$

From this, the price index of investment good is derived as follows

$$p_s^I = \sum_i p_{si}^A a_i^I$$

1.4.3 Trade

We assume that Japan is a small country and the world prices of goods are given constant. Let p_s^{ROW} denote the world price of good i and p^{FX} denote exchange rate (yen per dollar). Then, the price of import good i in yen is given by

$$p_{si}^M = p^{\mathrm{FX}} p_{si}^{\mathrm{ROW}}$$

On the other hand, the price of export good satisfies the following relation.

$$p_{si}^{\mathrm{ROW}} = p_{si}^X / p^{\mathrm{FX}}$$

Without loss of generality, we normalize the world prices of all goods to unity.

$$p_{si}^{\mathrm{ROW}} = 1$$

Then, the prices of import and export goods are expressed as

$$p_{si}^M = p^{\mathrm{FX}} \qquad p_{si}^X = p^{\mathrm{FX}}$$

These equations represent relation between import, export and foreign exchange. For example, suppose that one unit of good s is exported in period s. This brings about revenue of p_{si}^X. However, since it is equal to p^{FX}, one unit of export is equivalent to one unit of foreign exchange. Similarly, suppose that one unit of good s is imported in period s. For this, it costs p_{si}^M, which is equal to p^{FX}. So, one unit of import is also equivalent to one unit of foreign exchange. As the above arguments show, export and import have one-to-one relation with foreign exchange. As will be explained later, exchange rate is determined so that current account is balanced intertemporally.[11]

1.5 Government

The government obtains revenue by collecting taxes, which funds government expenditure and subsidies. The taxes on production are capital tax, labor tax, and indirect tax on production, and taxes on households are labor income tax, capital income tax and consumption tax.

1.5.1 Expenditure of the government

The expenditure of government includes "government expenditure" and "transfer to households". Government expenditure indicates final consumption and investment by government. Transfer to households is transfer such as social security contributions. The amount of transfer is expressed as TRN_s.

Government expenditure is spent on a good, which is a fixed coefficient composite of 27 Armington goods. Let a_i^G denote the amount of Armington good i required to acquire one unit of government expenditure. Then, the price index of government expenditure is given as follows.

$$p_s^G = \sum_i p_{si}^A a_i^G$$

The value of government expenditure is equal to $p_s^G G_s$ where G_s is real government expenditure.

[11] Foreign exchange rate here has little to do with actual foreign exchange rate. It is the price for an hypothetical good called foreign exchange and plays the similar role to ordinary good prices and is determined so as to balance current account intertemporally (that is, to clear market for foreign exchange).

1.5.2 Government income

Government income is derived from taxes. Government net income in period s is given by

$$\begin{aligned} M_s^G = & \sum_i t_i^L p_s^L L_{si}^D + \sum_i t_i^K r_s^K K_{si}^D + \sum_i t_i^Q p_{si}^Q Q_{si} \\ & + t^I p_s^L L_s^S + t^A r_s^K K_s + t^C p_s^{\mathrm{CC}} \bar{C}_s \\ & + \sum_i t_i^M p_{si}^M M_{si} - \sum_{i \in \mathrm{NENE}} s_i^C p_{si}^A C_{si}^D - \sum_{i \in \mathrm{ENE}} s_i^C p_{si}^E C_{si}^D \end{aligned}$$

where the first line indicates revenue from labor tax, capital tax, and indirect taxes on production, and the second line indicates revenue from labor income tax, capital income tax, and consumption tax, and the final line indicates revenue from import tariff and (less) subsidies.

1.5.3 Government intertemporal budget constraint

We assume that government budget is balanced intertemporally. This means that government budget at each point in time can be negative or positive. To derive intertemporal budget constraint, we need to consider flow budget constraint. Government flow budget constraint which excludes asset income is given by

$$\mathrm{GS}_s = M_s^G - p_s^G G_s - \mathrm{TRN}_s^G$$

Adding asset income $r_s \mathrm{NA}_s^G$ to this, we get government flow budget constraint.

$$\mathrm{NA}_{s+1}^G - \mathrm{NA}_s^G = \mathrm{GS}_s + r_s \mathrm{NA}_s^G$$

Applying forward iteration to this equation, we have

$$(1 + r_t)\mathrm{NA}_t^G = R_{t,T} \mathrm{NA}_{T+1}^G - \sum_{s=t}^{T} R_{t,s} GS_s$$

Intertemporally balanced government budget means that government net asset after infinite periods is equal to net asset at the initial period:

$$(1 + r_t)\mathrm{NA}_t^G = \lim_{T \to \infty} R_{t,T} \mathrm{NA}_{T+1}^G$$

This implies that the following equation must hold.

$$\sum_{s=t}^{\infty} R_{t,s} \left[p_s^G G_s + \mathrm{TRN}_s \right] = \sum_{s=t}^{\infty} M_s^G$$

1.6 Carbon emissions and carbon tax

1.6.1 Price of emission sources for combustion purpose

Thus far, the price of emission sources for combustion purpose has been expressed as p_{si}^E. To use emission sources for combustion purpose, it is necessary to pay carbon tax. Let γ_i denote the amount of emission per unit of emission source $i \in \mathrm{EC}$ and t_s^{CE} denote the level of carbon tax. Then, we can express the price of emission sources for combustion purpose as follows.

$$p_{si}^E = p_{si}^A + \gamma_i t_s^{\mathrm{CE}}$$

This means that the price of emission sources for combustion purpose is the sum of the price of Armington good and payment to carbon tax.[12]

[12] Although we assume the common γ_i for final consumption and intermediate demand of all sectors, it is more desirable to assume different values for different uses. The assumption of the common γ_i is made because to assume different values for different uses increases the number of endogenous variables significantly and thus makes computation difficult.

1.6.2 The amount of carbon emissions

Let A_{si}^{EC} denote Armington good i used for combustion purpose. It is given by

$$A_{si}^{\mathrm{EC}} = \sum_j \bar{a}_{ij}^{\mathrm{CL}} Q_{sj} + C_{si}^D \qquad i \in \mathrm{CL}$$

$$A_{si}^{\mathrm{EC}} = \sum_j E_{sij}^D + C_{si}^D \qquad i \in \mathrm{EC}$$

Summing emissions from all emission sources, we have total carbon emissions.

$$\mathrm{CE}_s^D = \sum_{i \in \mathrm{ES}} \gamma_i A_{si}^{\mathrm{EC}} \tag{25}$$

1.7 Market equilibrium

In this section, equilibrium conditions for good and factor markets are presented. Basically, the LHS represents supply and the RHS represents demand.

1.7.1 Markets for domestic goods

Domestic goods are supplied by production sectors and demanded by Armington aggregation.

$$D_{si} = D_{si}^{\mathrm{AD}}$$

1.7.2 Markets for Armington goods

Armington goods are supplied by Armington aggregation activity and demanded for final consumption, intermediate inputs, investment, and government expenditure. Below, we distinguish market clearing conditions according to the following four types of goods: (1) non-energy goods (exc. LIM and COK), (2) electricity, (3) LIM and COK, and (4) emission sources (exc. LIM and COK).

Market clearing condition for non-energy $i \in$ NENE (exc. LIM and COK):

$$A_{si} = C_{si}^D + \sum_j \bar{a}_{ij}^Q Q_{sj} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad i \in \mathrm{NENE}$$

Market clearing condition for LIM and COK:

$$A_{si} = C_{si}^D + \sum_j \bar{a}_{ij}^{\mathrm{CL}} Q_{sj} + \sum_j \bar{a}_{ij}^{\mathrm{NC}} Q_{sj} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad i \in \mathrm{CL}$$

Market clearing condition for electricity:

$$A_{si} = C_{si}^D + \sum_j E_{sij} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad i \in \mathrm{ELE}$$

Market clearing condition for emission sources $i \in$ EC (exc. LIM and COK):

$$A_{si} = C_{si}^D + \sum_j E_{sij} + \sum_j \bar{a}_{ij}^{\mathrm{NC}} Q_{sj} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad i \in \mathrm{EC}$$

1.7.3 Labor market

Labor is supplied by the household and demanded by production sectors.

$$L_s^S = \sum_i L_{si}^D$$

1.7.4 Market for renting capital

Renting capital is supplied by the household and demanded by production sectors.

$$K_s = \sum_i K_{si}^D$$

1.7.5 Current account (market for foreign exchange)

Our model assumes that current account is balanced intertemporally. Current account in period s is given by

$$\mathrm{CAS}_s = \mathrm{NA}_{s+1}^F - \mathrm{NA}_s^F = \mathrm{TS}_s + r_s^{\mathrm{ROW}} \mathrm{NA}_s^F$$

where $\mathrm{TS}_s \equiv \sum_i p_{si}^W [X_{si} - M_{si}]$ is trade surplus and $r_s^{\mathrm{ROW}} \mathrm{NA}_s^F$ is return from foreign assets. Note that the interest rate applied to foreign assets is the world interest rate r_s^{ROW}.

Applying the forward iteration to the above equation, we have

$$(1 + r_t^{\mathrm{ROW}}) \mathrm{NA}_t^F = R_{t,T}^{\mathrm{ROW}} \mathrm{NA}_{T+1}^F - \sum_{s=t}^{T} R_{t,s}^{\mathrm{ROW}} \mathrm{TS}_s \tag{26}$$

Furthermore, setting $T \to \infty$, we have

$$(1 + r_t^{\mathrm{ROW}}) \mathrm{NA}_t^F = \lim_{T\to\infty} R_{t,T}^{\mathrm{ROW}} \mathrm{NA}_{T+1}^F - \sum_{s=t}^{\infty} R_{t,s}^{\mathrm{ROW}} \mathrm{TS}_s$$

The assumption of intertemporally balanced current account means

$$\lim_{T\to\infty} R_{t,T} \mathrm{NA}_{T+1}^F = (1 + r_t) \mathrm{NA}_t^F$$

This implies that the following condition must hold.

$$\sum_{s=t}^{\infty} R_{t,s}^{\mathrm{ROW}} \mathrm{TS}_s = 0 \tag{27}$$

From the arguments in section 1.4.3, (27) is written as follows

$$\sum_{s=t}^{\infty} \sum_i R_s^{\mathrm{ROW}} X_{si} = \sum_{s=t}^{\infty} \sum_i R_s^{\mathrm{ROW}} M_{si} \tag{28}$$

We can interpret this as equilibrium condition for foreign exchange. As explained in Section 1.4.3, export and import have one-to-one relation with foreign exchange. Namely, one unit of X_{si} brings one unit of foreign exchange, and one unit of foreign exchange brings one unit of M_{si}. From this relation, we can interpret that the LHS of (28) represents supply of foreign exchange and the RHS of (28) represents demand for foreign exchange. Foreign exchange p^{FX} is adjusted so that market for foreign exchange is balanced.

1.7.6 Carbon tax

Total carbon emissions CE_s^D is given by (25). The government determines the level of carbon tax t_s^{CE} so that CE_s^D is equal to the exogenous target level of carbon emissions CE_s^S:

$$\mathrm{CE}_s^S = \mathrm{CE}_s^D$$

2 Simulation

In this section, we explain the approaches and assumptions for simulation.

Table 3: Value of elasticity parameters

Notation	Description	Value
η	Elasticity of transformation between domestic supply and export supply	4
σ_{FE}	Elasticity of substitution between primary factor composite and energy composite	0.5
σ_{Fi}	Elasticity of substitution between capital and labor in sector i	Table 5
σ_{EE}	Elasticity of substitution between energy intermediate inputs	0.5
σ_{Ai}	Armington elasticity of good i	Table 4
σ_{CC}	Elasticity of substitution between energy composite and non-energy composite in utility function	0.3
σ_{EC}	Elasticity of substitution between energy consumption goods	2
σ_{C}	Elasticity of substitution between non-energy consumption goods	1
σ_{U}	Intertemporal elasticity	0.5

Table 4: The values of Armington elasticity ($\sigma_{A,i}$)

Goods	Value
AGR, FOO, TET	2.2
OMI, LIM, COC, SLA, CRU, NAT, IAM, MAC, OIP, ELE, GAS, SWW	2.8
PPP	1.8
CHM, PET, OPP, COK, CSC, CON, COM, RES, TCB, CAB, PUB, SER	1.9

Source: GTAP version 5 data.

Table 5: The values of elasticity of substitution between capital and labor ($\sigma_{F,i}$)

Sector	Value
AGR, FOO	0.237
OMI, LIM, COC, SLA, CRU, NAT	0.2
TET, PPP, CHM, PET, OPP, COK, CSC, IAM, MAC, OIP	1.26
ELE, GAS, SWW, RES, TCB, PUB, CON, SER,	1.4
COM	1.68

Source: GTAP version 5 data.

2.1 Elasticity of substitution

Table 3 presents exogenously determined elasticity of substitution. Actually, these values should be based on empirical estimates for Japan, but reasonable estimates for these parameters are not available. So, we determine them by using values in Böhringer et al. (1997), Rutherford et al. (2002), and Rutherford and Light (2002). Similarly, since reasonable estimates for elasticity of substitution of capital-labor and Armington elasticity for Japan are not available, we use values in GTAP data (Table 4 and Table 5).

2.2 Static calibration

Parameters with the static nature are calibrated from the benchmark data. These parameters include:

[1] Share parameters in CES functions (α_i^F、α_i^X, α_i^{FL} etc.)

[2] Elasticity of substitution between leisure and consumption in utility function (σ_{LC})

2.2.1 Share parameters in CES functions

CES functions include parameters (α), which are called share parameters. We calibrate these share parameters in the same way as other CGE analysis do.[13]

2.2.2 Elasticity of substitution between leisure and consumption in the utility function

Generally, we determine elasticities of substitution exogenously. However, we calibrate the elasticity of substitution between leisure and consumption by providing a value of the wage elasticity of labor supply (ε_L) exogenously. Following Bessho et al. (2003), we assume a benchmark value of the labor supply elasticity of 0.19. Calibration is done in the following way.

Let LE^U denote uncompensated demand for leisure. From (4), LE^U is given by

$$LE^U = \left[\frac{\alpha^{\mathrm{LE}}}{p^{\mathrm{LE}}}\right]^{\sigma_{\mathrm{LC}}} \frac{X}{(\alpha^{\mathrm{LE}})^{\sigma_{\mathrm{LC}}}(p^{\mathrm{LE}})^{1-\sigma_{\mathrm{LC}}} + (1-\alpha^{\mathrm{LE}})^{\sigma_{\mathrm{LC}}}(p^{\mathrm{CC}})^{1-\sigma_{\mathrm{LC}}}}$$

where $X \equiv p^{\mathrm{LE}}\bar{L} + r^{\mathrm{KE}}K + r\mathrm{NA}^H + \mathrm{TRN} - \mathrm{SAVE}^H$. All variables in the above equation represent benchmark values and thus time index is omitted.

From this leisure demand, we can derive wage elasticity of leisure demand ($\varepsilon_{\mathrm{LE}}$) as follows:

$$\varepsilon_{\mathrm{LE}} \equiv \frac{\ln LE^U}{\ln p^{\mathrm{LE}}} = -\left[1 - \frac{p^{\mathrm{LE}}LE}{X}\right]\sigma_{\mathrm{LC}} + \frac{p^{\mathrm{LE}}L}{X}$$

Rearranging this, the relation between σ_{LC} and ε_{LE} is derived

$$\sigma_{\mathrm{LC}} = \frac{1}{1 - p^{\mathrm{LE}}LE/X}\left[-\varepsilon_{\mathrm{LE}} + \frac{p^{\mathrm{LE}}L}{X}\right] \tag{29}$$

On the other hand, since labor supply is $L = \bar{L} - LE$, wage elasticity of labor supply is

$$\varepsilon_L \equiv \frac{\ln L}{\ln p^{\mathrm{LE}}} = -\varepsilon_{\mathrm{LE}}\frac{LE}{L} \tag{30}$$

From (29) and (30), we can derive the relation between σ_{LC} and ε_L.

$$\sigma_{\mathrm{LC}} = \frac{1}{1 - p^{\mathrm{LE}}LE/X}\left[\varepsilon_L\frac{L}{LE} + \frac{p^{\mathrm{LE}}L}{X}\right]$$

If the benchmark values of p^{LE}, LE, L, X are given and if wage elasticity of labor supply ε_L is determined, we can calibrate σ_{LC} from this equation.

2.3 Baseline equilibrium

The dynamic equilibrium without any emission regulations (the *baseline equilibrium*) is derived in the following way.

Step 1: First, we assume the steady state equilibrium with zero growth rate and calibrate parameters and variables.

Step 2: The values of total labor endowment, which is set to be constant in Step 1, are changed to desired levels.

Step 3: Finally, we calibrate technology growth rates so that rates of growth in GDP and carbon emissions derived from the model equilibrium coincide with exogenously given target growth rates. At the same time, we determine the path of government expenditure so that the benchmark level of government expenditure-GDP ratio is kept constant.

[13] See, for example, Rutherford (1998)、Shoven and Whalley (1992, p. 115)

2.3.1 Step 1

In Step 1, we assume that all exogenous variables are constant and that the economy is in steady state. Exogenous variables which are assumed to be constant are:

[1] Total labor endowment $\bar{L}_s$

[2] Government expenditure G_s

[3] Transfer from the government to the household $\frac{\mathrm{TRN}_s}{p_s^G}$

In addition to these variables, this paper assumes that the world interest rate is constant over time. From this assumption and the assumption that international lending and borrowing of money is allowed, the domestic interest rate is equalized to the constant world interest rate. In the followings, we do not distinguish the world interest rate and the domestic interest rate, and refer to them as r_s or r. This constant interest rate is calibrated later.

From the assumption of steady state, all quantity and price variables except p_s^W become constant over time in the baseline equilibrium. Since p_s^W presents present price (not current price) it declines at the constant rate.

$$p_{s+1}^W = \frac{p_s^W}{1+r}$$

From the assumptions above, the paths of period utility W_s and the price index p_s^W are determined. Then, we can calibrate the discount factor α_s^W in lifetime utility function (2) in the same way as calibrating other share parameters.

Next, we calibrate depreciation rate δ, benchmark value of rental price r_t^K, capital stock K_t, and interest rate r. To do this, we use the assumption of steady state. Calibration is done in the following way. In steady state, capital stock are constant and thus $K_{s+1} = K_s$ holds. From this, $K_{s+1} = (1-\delta)K_s + J_s$ can be written as

$$J_s/K_s = \delta \tag{31}$$

On the other hand, net investment J_s and gross investment I_s have the following relation:

$$I_s = J_s\left[1 + \frac{\phi}{2}\frac{J_s}{K_s}\right]$$

By (31), this can be written as

$$I_s = \delta K_s\left[1 + \frac{\phi}{2}\delta\right]$$

Moreover, since we normalize the benchmark price of investment good to unity (i.e. $p_t^I = 1$), gross investment I_t is equal to the benchmark value of investment V_t^I. Thus, we have

$$V_t^I = \delta K_s\left[1 + \frac{\phi}{2}\delta\right] \tag{32}$$

This is the first condition for the baseline equilibrium to be in steady state.

Next, consider rental price. For the benchmark year to be in steady state, interest rate r, rental price r_t^{KE}, depreciation rate δ and adjustment premium p_t^{KA} must satisfy (19). Moreover, since all prices variables become constant in steady state, we have $p_{t-1}^K = p_t^K$. Thus, (19) becomes

$$r_t^{\mathrm{KE}} + p_t^K(1-\delta) + p_t^{\mathrm{KA}} = (1+r_t)p_t^K \tag{33}$$

On the other hand, from (18), we have

$$p_t^K = p_t^I[1+\phi\delta] = 1 + \phi\delta$$

Substituting this into (33), we can derive the relation that rental price must satisfy.

$$r_t^{\mathrm{KE}} = [1 + \phi\delta](r + \delta) - p_t^{\mathrm{KA}} \tag{34}$$

From (15) and (31), the benchmark value of adjustment premium p_t^{KA} is given by

$$p_t^{\mathrm{KA}} = p_t^I \frac{\phi}{2}\left[\frac{J_t}{K_t}\right]^2 = \frac{\phi}{2}\delta^2 \tag{35}$$

Finally, rental price r_t^{KE}, capital stock K_t, and the value of payment to capital V_t^K must satisfy the following relation.

$$r_t^{\mathrm{KE}} K_t = V_t^K \tag{36}$$

(32) and (34)–(36) are the conditions for the baseline equilibrium to be in steady state.[14] Among the variables appeared in four conditions, V_t^K and V_t^I are determined by the benchmark data, and ϕ is determined exogenously. Given V_t^K, V_t^I, g, and ϕ, we calibrate r, δ, r_t^K, and K_t by solving the following constrained optimization problem.[15]

$$\begin{aligned} \min_{r,\delta,r_t^{\mathrm{KE}},K_t,p_t^{\mathrm{KA}}} \quad & \text{Loss} = \left[\frac{r-\bar{r}}{\bar{r}}\right]^2 + \left[\frac{\delta-\bar{\delta}}{\bar{\delta}}\right]^2 \\ \text{s.t.} \quad & V_t^I = \delta K_s\left[1 + \phi\delta/2\right] \\ & r_t^{\mathrm{KE}} = [1+\phi\delta](r+\delta) - p_t^{\mathrm{KA}} \\ & p_t^{\mathrm{KA}} = \phi\delta^2/2 \\ & r_t^{\mathrm{KE}} K_t = V_t^K \end{aligned} \tag{37}$$

where $\bar{r}$ and $\bar{\delta}$ are exogenously given target values for interest rate and depreciation rate respectively. (37) means that the problem's objective is to minimize the squared sum of the rates of discrepancy from target values. For the calibration, we assume $\bar{r} = 0.03$ and $\bar{\delta} = 0.07$ as target values. From this calibration, we derived $r = 0.032$, $\delta = 0.064$, $r_t^{\mathrm{KE}} = 0.098$, $K_t = 1561$, and $p_t^{\mathrm{KA}} = 0.001$.

2.3.2 Step 2

In Step 1, we assume that total labor endowment $\bar{L}_s$ is constant over time so as to assure the steady state. Here, we change the value of $\bar{L}_s$ to desired levels. The new path of total labor endowment is derived from the benchmark total labor endowment and projected labor growth rates. The value of projected labor growth rates from 1995 to 2050 are taken from Yashiro et al. (1997). Since the value of projected labor growth rates after 2051 are not available, we use projected growth rate of population (16–50 years) as a substitute for it. Projected growth rate of population is taken from NIPSSR (2002). Table 6 displays the rates of labor growth used in the simulation. From the table, we can see that total labor endowment will continue decline over time.

2.3.3 Step 3

The remaining exogenous variables are technology parameters and government expenditure. There are at least two approaches to determine these variables:

[1] To determine them exogenously independently of the model.

[2] To determine them by calibration within the model.

[14] More precisely, (36) must always be satisfied even if the economy is not in steady state.
[15] Similar calibration is conducted in Böhringer et al. (1997).

Table 6: Rate of growth of total labor endowment (%)

Year	Rate	Year	Rate	Year	Rate	Year	Rate
1995	−0.17	2020	−0.80	2045	−0.96	2070	−1.02
1996	−0.17	2021	−0.80	2046	−0.96	2071	−1.00
1997	−0.17	2022	−0.80	2047	−0.96	2072	−0.99
1998	−0.17	2023	−0.80	2048	−0.96	2073	−0.96
1999	−0.17	2024	−0.80	2049	−0.96	2074	−0.95
2000	−0.66	2025	−0.80	2050	−1.04	2075	−0.93
2001	−0.66	2026	−0.80	2051	−1.02	2076	−0.92
2002	−0.66	2027	−0.80	2052	−0.98	2077	−0.89
2003	−0.66	2028	−0.80	2053	−0.92	2078	−0.88
2004	−0.66	2029	−0.80	2054	−0.91	2079	−0.86
2005	−0.66	2030	−1.10	2055	−0.88	2080	−0.84
2006	−0.66	2031	−1.10	2056	−0.91	2081	−0.82
2007	−0.66	2032	−1.10	2057	−0.90	2082	−0.80
2008	−0.66	2033	−1.10	2058	−0.95	2083	−0.78
2009	−0.66	2034	−1.10	2059	−0.97	2084	−0.76
2010	−0.87	2035	−1.10	2060	−0.97	2085	−0.74
2011	−0.87	2036	−1.10	2061	−1.00	2086	−0.73
2012	−0.87	2037	−1.10	2062	−1.03	2087	−0.72
2013	−0.87	2038	−1.10	2063	−1.00	2088	−0.70
2014	−0.87	2039	−1.10	2064	−1.03	2089	−0.70
2015	−0.87	2040	−0.96	2065	−1.08	2090	−0.69
2016	−0.87	2041	−0.96	2066	−1.08	2091	−0.68
2017	−0.87	2042	−0.96	2067	−1.07	2092	−0.67
2018	−0.87	2043	−0.96	2068	−1.06	2093	−0.67
2019	−0.87	2044	−0.96	2069	−1.04	2094	−0.66

In Approach [1], we can determine the values of technology parameters and government expenditure freely without any restrictions. Moreover, once the values of technology parameters and government expenditure are determined, we can easily derive the baseline equilibrium. On the other hand, to use Approach [1], it is necessary to acquire estimates of technology parameters and government expenditure from other data. In addition, Approach [1] may cause the problem that values of variables deviate significantly from their realistic values. For example, GDP derived from Approach [1] may deviate significantly from the plausible estimate. On the other hand, in Approach [2], we need not prepare estimates of technology parameters and government expenditure, and can adjust paths of important variables so that they do not deviate from the realistic values. However, in Approach [2], the degree of freedom in values of exogenous variables is lost and calibration complicates the derivation of the baseline equilibrium.

As the above arguments indicate, both approaches have advantages and disadvantages. In this paper, we employ Approach [2] mainly because it is quite difficult to prepare reliable estimates of technology parameters and government expenditure.

As technology improvement, we consider two types: primary factor-augmented technology improvement and energy augmented technology improvement. From this, Q_{si}^{FE} is modified as follows:

$$Q_{si}^{\mathrm{FE}} = \left[\alpha_i^F \left(\beta_s^F Q_{si}^F\right)^{\frac{\sigma_{\mathrm{FE}}-1}{\sigma_{\mathrm{FE}}}} + (1-\alpha_i^F)\left(\beta_s^E Q_{si}^{\mathrm{EE}}\right)^{\frac{\sigma_{\mathrm{FE}}-1}{\sigma_{\mathrm{FE}}}}\right]^{\frac{\sigma_{\mathrm{FE}}}{\sigma_{\mathrm{FE}}-1}} \tag{38}$$

where β_s^F and β_s^E are technology parameters for primary factor and energy inputs respectively. The rise in β_s^F and β_s^E means technology growth. From this modification of Q_{si}^{FE}, the price index and demand functions corresponding to Q_{si}^{FE} are also modified. Note that technology parameter β_s^F and β_s^E are common in all sectors.

Next, we assume that rates of technology improvement are constant. So, the following relations hold:

$$\beta_s^F = (1+\xi^F)^{s-t}\beta_t^F \qquad \beta_s^E = (1+\xi^E)^{s-t}\beta_t^E \tag{39}$$

where ξ^F and ξ^E are rates of growth in β_t^F and β_t^E respectively. Since the values of β_t^F and β_t^E in the benchmark year are equal to unity, β_s^F and β_s^E become functions of ξ^F and ξ^E.

Variables calibrated below are two technology growth rates ξ^F and ξ^E, and government expenditure G_s. To calibrate these variables, it is necessary to provide conditions that they must satisfy. Here, we calibrate the variables by imposing the following conditions:

[1] ξ^F is calibrated so that GDP growth rate derived from the model is equal to the target level.

[2] ξ^E is calibrated so that carbon emission growth rate derived from the model is equal to the target level.

[3] G_s is calibrated so that the benchmark level of government expenditure-GDP ratio is kept constant over time.[16]

The above calibrations are conducted simultaneously. Target growth rates of GDP and carbon emissions are derived from projections by AIM/Trend model (AIM Project Team, 2002). Since AIM/Trend model provides projections of GDP and carbon emissions up to 2032. We calibrate ξ^F and ξ^E so that the growth rates of GDP and carbon emissions from 1995 to 2032 derived by our model are coincide with those derived by AIM/Trend model. On the other hand, in calibrating G_s, we make adjustment on lump-sum taxes so as to balance the government budget. In the simulation examining the double dividend hypothesis, technology growth rates and government expenditure calibrated here are assumed to be exogenously constant.

2.4 Finite horizon model

Although we have so far assumed that a representative household lives over an infinite horizon, it is necessary to determine a terminal period to solve the model numerically.

[16] That is, G_s is calibrated so that $p_s^G G_s/\mathrm{GDP}_s = p_t^G G_t/\mathrm{GDP}_t$ holds for any s where GDP_s is GDP in period s.

2.4.1 The restriction on terminal period

In the subsequent simulation, we set the terminal period at 2095; thus, the time period covered by the model is 1995 to 2095. There is one problem in setting the terminal period for a dynamic model. That is, if no special condition is imposed on the terminal adjustment, investment becomes very low as the terminal period approaches because capital stock existing after the terminal period is worthless. To avoid this problem, we adopt the approach used in Böhringer et al. (1997) and Lau et al. (2002) — that is, we impose the following restriction on investment in the terminal period:

$$\frac{J_T}{J_{T-1}} = \frac{W_T}{W_{T-1}} \tag{40}$$

where T is the terminal period. This restriction means that the growth rate of investment in the terminal period must be equal to the growth rate of period utility in that period. By imposing this restriction, the model yields a path of investment that is similar to one in an infinite horizon model. For more details on the terminal condition, see Lau et al. (2002).

2.4.2 Budget constraint in finite horizon model

In a finite horizon model, the household lifetime budget constraint (24) becomes:

$$\begin{aligned}\sum_{s=t}^{T} p_s^W W_s = \left[r_t^{\mathrm{KE}} + p_t^K(1-\delta) + p_t^{\mathrm{KA}}\right] K_t + \mathrm{NA}_t^H \\ -R_{t,T}\left[p_T^K K_{T+1} + \mathrm{NA}_{T+1}^H\right] + \sum_{s=t}^{T}\left[p_s^{\mathrm{LE}} \bar{L}_s + \mathrm{TRN}_s\right]\end{aligned}$$

where K_{T+1} and NA_{T+1}^H are capital stock and net asset in the post-terminal period. While no Ponzi game assures $R_{t,T}[p_T^K K_{T+1} + \mathrm{NA}_{T+1}^H]$ converges to zero in an infinite horizon model, it cannot be ignored in a finite horizon model. However, in the simulation, we assume $\mathrm{NA}_t^H = \mathrm{NA}_{T+1}^H = 0$, that is, the household does not have net asset both in the initial period and in the post-terminal period.

Similarly, government budget constraint and current account are given by[17]

$$\sum_{s=t}^{T} R_{t,s}\left[p_s^G G_s + \mathrm{TRN}_s\right] = \sum_{s=t}^{T} R_{t,s} M_s^G$$

$$\sum_{s=t}^{T}\sum_{i} p_s^{\mathrm{RF}} X_{si} = \sum_{s=t}^{T}\sum_{i} p_s^{\mathrm{RF}} M_{si}$$

In this section, we explain how the benchmark dataset for the simulation was created. The benchmark year in the simulation is 1995 and all the data refer to 1995. Most of the data are taken from "1995 Input-Output Table" (MCAG 1999, hereafter IO table).

3 Calculation of MEB

In this section, we explain the way for calculating MEB (marginal excess burden). There are two approaches for MEB calculation: (1) the differential approach, and (2) the balanced-budget approach (Ballard, 1990). Since the differential approach assumes that government expenditure is constant, it involves only the substitution effect. On the other hand, the balanced-budget approach assumes that

[17] For these conditions to be satisfied, the following relations must hold.

$$(1+r_t)\mathrm{NA}_t^G = R_{t,T}\mathrm{NA}_{T+1}^G$$
$$(1+r_t^{\mathrm{ROW}})\mathrm{NA}_t^F = R_{t,T}^{\mathrm{ROW}}\mathrm{NA}_{T+1}^F$$

In the simulation later, we assume $\mathrm{NA}_t^G = \mathrm{NA}_t^F = 0$. Thus, NA_{T+1}^G and NA_{T+1}^F are also zero.

government expenditure is changed. So, it involves changes in the amount of resources taken from the private sector. Generally speaking, the former is suited for comparing distortions of different taxes and the latter is suited for evaluating costs of a specific public project. Since our main purpose here is to compare the size of distortions caused by different taxes, we use the differential approach. MEB is calculated in the following procedure:

[1] First, tax rate is raised by one percent.

[2] Increase the amount of lump-sum transfer to households so as to keep government revenue constant.

[3] Then, MEB is calculated according to the following formula:

$$\text{MEB} = -100 \times \frac{\text{EV}}{changeinlump - sumtransfer} \tag{41}$$

where EV is equivalent variation caused by change in tax rate.

4 Data

4.1 Data of emission sources

We use energy dataset of 3EID (Embodied Energy and Emission Intensity Data for Japan Using Input-Output Table, Nansai et al. 2002) to calculate the volume of carbon emissions. Other assumptions and approach used in this paper are summarized as follows:

[1] In 3EID, petroleum-based energies such as gasoline, kerosene, light oil, heavy oil (A, B, and C), jet oil, and LPG are disaggregated. However, we aggregate these energies into single energy called PET (petroleum refinery products). We aggregate these energies by converting the physical quantity data to the calorific data and then summing up them.

[2] We do not classify ELE (electricity) as an ES because its direct use and consumption does not emit carbon. ESs in our data refer to the goods that emit carbon directly when used.

[3] Although OPP (other petroleum products) in Table 1 includes goods such as naphtha, which are regarded as sources of carbon emissions in 3EID, we do not include them among ESs. However, since emission shares from these goods are very low, the total volume of carbon emissions is not much different even if we do not include these goods to ESs.

[4] In 3EID, even if a good is classified as an ES, it is categorized depending on whether it is used for energy (combustion) or non-energy (non-combustion) purposes. Following 3EID, our data and model also distinguish between ES input for combustion purpose and that for non-combustion purpose.

[5] COK (coke) is usually classified as an energy good. However, since we treat it in the production function in the same way as other material inputs, we do not include it among energy goods. LIM (limestone) is also defined as a non-energy good: we treat it as a material.

[6] All ESs other than LIM are measured in calorific units, while LIM is measured in tonnes.

To derive carbon emissions, we multiply the volume of ES inputs by carbon emission coefficients. The emission coefficients are taken from 3EID, except for the coefficient for PET (see Table 7). In 3EID, petroleum based energies are disaggregated into gasoline, kerosene, light oil, heavy oil (A, B, and C), jet oil, and LPG, and different carbon coefficients are used for different energies. However, in our data, these energies are aggregated into one energy, PET, and there is no coefficient for PET in 3EID. Thus, we calculated the carbon coefficient for PET from carbon coefficients for petroleum-based energies.

$$\text{Carbon coefficient for PET} = \sum_i \theta_i \gamma_i$$

Table 7: Carbon coefficient*

ESs	Carbon coefficient	Unit
COC	1.045	$tC/10^8kcal$
SLA	1.015	$tC/10^8kcal$
COK	1.231	$tC/10^8kcal$
CRU	0.792	$tC/10^8kcal$
PET	0.783	$tC/10^8kcal$
NAT	0.585	$tC/10^8kcal$
GAS	0.597	$tC/10^8kcal$
LIM	0.12	tC/t

*All coefficients except for PET's are taken from 3EID.

where θ_i denotes the emission share of petroleum-based energy i, and γ_i denotes the carbon coefficient of petroleum-based energy i. That is, we derived the emission coefficient for PET by calculating the weighted average of petroleum-based energy's emission coefficients.

4.2 Tax data

We classify existing taxes in Japan to the following eight taxes: (1) labor income tax, (2) capital income tax, (3) consumption tax, (4) capital tax, (5) labor tax, (6) indirect tax on production, (7) import tariff, and (8) subsidies for consumption.

Let us explain these eight taxes. First, "labor income tax" indicates a tax on labor income of households. The data of this labor income tax is derived from Inagaki (2002). Similarly, "capital income tax" is a tax on capital income of households. In our model, capital income means household's income derived by renting capital stock to industries. The data of this capital income tax is also derived from MOF (1996). Third, "consumption tax" is a tax on consumption literally. The rate of consumption tax is assumed to be 3%, which is equal to the consumption tax rate at 1995. These three taxes are taxes imposed on households.

"Labor tax" and "capital tax" are taxes imposed on production side. Labor tax indicates social security contributions by employers. Although social security contributions by employers are not a tax in the strict sense, we regard these as a tax on production and refer to them as labor tax. The data of labor tax is derived from "social security contributions by employers" in IO table. On the other hand, capital tax indicates corporation tax and corporation inhabitant tax. Since our model assumes that these taxes are imposed on employment of the capital stock, we call them as capital tax. The data of capital tax is derived from MOF (1997) and NTA (1997).

"Indirect tax on production" indicates taxes represented by "indirect taxes minus current subsides" in IO table. Although "indirect taxes minus current subsides" include various taxes on industries, we treat these taxes as a single tax and call it as "indirect tax on production". This tax is incorporated into the model as ad-valorem tax on outputs. "Import tariff" is a tax on imports and derived from "custom duties" and "commodity taxes on imported goods" in IO table. Finally, "subsidies for consumption" indicates "individual consumption expenditure of government" in IO table. Individual consumption expenditure of government is government expenditure for households such as medical insurance. Since this plays the same role as consumption subsides, it is called "Subsidies for consumption" in this paper.

Among eight taxes and subsides above, taxes swapped for emission regulation in the simulation are the following five taxes: labor income tax, capital income tax, consumption tax, capital tax, labor tax.

4.3 Data construction

The benchmark dataset is created according to the following procedure:

[1] Sector aggregation.

[2] Adjustment for expenditure outside the household

[3] Value added data (factor income and taxation on production)

[4] Final demand data (final consumption, investment, government expenditure, and trade data)

[5] Household data (consumption, income, taxation on household, saving, and transfer)

[6] Other adjustment.

4.4 Sector aggregation

First, we aggregate the original 519×403 IO table to 35 sector table. 35 sectors after aggregation are displayed in Table 8. For the simulation, we further aggregate sectors to 27. However, adjustment of data is conducted in 35 sector table.

4.5 Expenditure outside the household

In IO table, the column and row representing "expenditure outside the household" are included in the value added and final demand sectors respectively. However, in the standard SNA, it is included in endogenous sectors. We aggregate "expenditure outside households" into SER (services) in the endogenous sectors so as to keep consistency with SNA data. To do this, we employ the approach in Kuroda et al. (1997, p.49).

4.6 Value added data

At this stage, value added data in each sector consist of the following six parts.

- Compensation of employees (CE)
- Contribution of employers to social insurance (CESI)
- Operating surplus (OS)
- Depreciation of fixed capital (DEP)
- Indirect taxes
- (Less) current subsidies

In our model, it is assumed that value added excluding indirect taxes and current subsidies is paid to primary factors (capital and labor). For this, we rearrange value added data to the following classification.

- Gross labor income
- Gross capital income
- Indirect taxes
- (Less) current subsidies

Table 8: Sector identifiers (35 sectors)

Identifiers	Sector description
AGR	Agriculture, forestry and fishery
LIM	Limestone (Materials for ceramics)
COC	Coking coal
SLA	Steam coal, lignite and anthracite
CRU	Crude petroleum
NAT	Natural gas
OMI	Other minings
FOO	Foods
TET	Textile products
PPP	Pulp, paper and wooden products
CHM	Chemical products
PET	Petroleum refinery products
OPP	Other petroleum products (naphtha and others)
COK	Coke (Coal products)
CSC	Ceramic, stone and clay products
IAS	Iron and steal
NFM	Non-ferrous metal
MET	Metal products
GMA	General machinery
EMA	Electrical machinery
TRE	Transportation equipment
PIN	Precision instruments
OIP	Other industrial products
CON	Construction
ELE	Electricity
GAS	Gas supply
SWW	Steam, and hot water supply, water supply and waste disposal services
COM	Commerce
FAI	Finance and insurance
RES	Real estate
TRN	Transport
CAB	Communication and broadcasting
PUB	Public administration
OPS	Other public services (private non-profit organizations)
SER	Services

where gross labor income and gross capital income are payments to labor and capital including taxes respectively. Note that payments to capital means rental cost of capital.

For this, we basically divide value added in the following way:

$$\begin{array}{lcl} \text{CE} + \text{CESI} & \longrightarrow & \text{gross labor income} \\ \text{OS} + \text{DEP} & \longrightarrow & \text{gross capital income} \end{array}$$

That is, CE and CESI are classified as payments to labor and OS and DEP are classified as payments to capital.

One problem of the above classification is that OS includes income for private incorporated enterprises. Since this income includes not only payments to capital but also payments to labor such as payments to domestic staff, it is not appropriate to regard all OS as payments to capital. To capture capital and labor income more precisely, it is necessary to include payments to labor in OS into labor income.

4.6.1 Payments to labor in OS

To create the data of payments to labor in OS from scratch, it is necessary to collect various data and thus it requires a lot of time and effort. To avoid this, we create the data of payments to labor in OS indirectly from other data. The data we use are KDB (Keio Economic Observatory Data Base; Kuroda et al. 1997), which includes the data of payments to labor in OS for 1992. We modify KDB data and create the data of payments to labor in OS for 1995.

4.6.2 Labor income

Gross labor income of each sector is given by

$$\text{Gross labor income} = \text{CE} + \text{CESI} + \text{payments to labor in OS}$$

Although CESI (contribution of employers to social insurance) is not a tax in a strict sense, it plays a similar role to that of a tax. Thus, we regard CESI as a tax and call it "labor tax". Gross labor income is written as

$$\text{Gross labor income} = \text{CE} + \text{labor tax} + \text{payments to labor in OS}$$

Moreover, we define net labor income as follows:

$$\text{Net labor income} = \text{gross labor income} - \text{labor tax}$$

This net labor income of each industry is paid to the household.

4.6.3 Capital income

Next, let us consider capital income. Gross capital income (capital income including taxes) is derived as follows.

$$\text{Gross capital income} = \text{OS} - \text{payments to labor in OS} + \text{DEP}$$

Note that DEP (depreciation of fixed capital) is included in capital income.

4.6.4 Capital tax

We assume that a tax is imposed on capital income and call it "capital tax". As actual taxes corresponding to this capital tax, we consider national corporation tax and local corporation inhabitant tax. We do not distinguish these two taxes and call them capital tax as a whole. The value of capital tax of each sector is derived as follows.

[1] First, we derive total value of corporation tax and corporation inhabitant tax from MOF (1997). The value is 16.8 trillion yen (see Table 9).

[2] On the other hand, corporation tax payments by industry are provided by NTA (1997).

[3] Combining these two data, we create capital tax data by industry.

4.6.5 Indirect tax on production

We have so far distinguished indirect taxes and current subsidies. Below, we only consider net indirect tax which is defined as indirect taxes minus current subsidies and call it "indirect tax on production". This tax is incorporated into the model as ad valorem tax on output.

Table 9: The value of corporation tax and corporation inhabitant tax in 1995

Tax	Value (trillion yen)
Corporation tax	13.7
Corporation inhabitant tax	3.1
Total	16.8

Source: MOF (1997).

4.7 Final demand

In this subsection, we explain final demand data. Final demand data are mainly derived from IO table.

- Government expenditure
- Investment
- Export and import
- Final consumption

Final demand data in Japanese IO table include negative values. There are two reasons for this: (1) decrease in stocks is recorded as negative value, and (2) Japanese IO table employs Stone approach to deal with wastes. Before deriving components of final demand, we delete negative elements in final demand.

4.7.1 Government expenditure

The data of government expenditure is derived as the sum of "collective consumption expenditure of government" and "gross domestic fixed capital formation (public)" in IO table.

$$\begin{aligned}\text{Government expenditure} = &\ \text{collective consumption expenditure of government}\\ &+ \text{gross domestic fixed capital formation (public)}\end{aligned}$$

Note that government expenditure includes not only final consumption but also investment (fixed capital formation).

4.7.2 Investment

The data of gross investment is derived as the sum of "gross domestic fixed capital formation (private)" and "increase in stocks" in IO table.

$$\begin{aligned}\text{Gross investment} = &\ \text{“gross domestic fixed capital formation (private)”}\\ &+ \text{“increase in stocks”}\end{aligned}$$

4.7.3 Export and import

The value of import are basically derived from IO table. However, when we remove negative elements in final demand, some adjustments are made for import. Thus, the value of import in our data slightly deviates from the original value. In IO table, there are two taxes on import: (1) "custom duties" and (2) "commodity taxes on imported goods". We aggregate these two taxes into one tax called "import tariff". The value of export is also taken from IO table. However, we make scale adjustments for export so that the benchmark value of trade surplus is zero.

4.7.4 Final consumption

Next, we consider household final consumption. We refer to the amount that the household actually *pays* as "household gross consumption expenditure" and to the amount that the household actually *consumes* as "household net consumption expenditure". The reason why two different values of consumption expenditure are used is that the amount of two expenditure differs because of consumption tax and subsidy. Two expenditures are derived as follows.

$$\begin{aligned}&\text{Household net consumption expenditures}\\&\qquad\qquad = \text{consumption expenditures (private)}\\&\qquad\qquad + \text{individual consumption expenditures of government}\\&\text{Household gross consumption expenditures}\\&\qquad\qquad = \text{consumption expenditures (private)} + \text{consumption tax}\end{aligned}$$

Note that consumption expenditure in IO table includes not only actual consumption but also imaginary consumption through imputation. Because of this, consumption expenditure in our data exceeds actual consumption by imputed consumption. For example, the value of imputed rent amounts to about 24 trillion yen in 1995.

Individual consumption expenditure of government is government expenditure for households such as medical insurance. Since this plays the same role as consumption subsides, it is called "consumption subsidy" in this paper.

Consumption tax is literally a tax on consumption. As to other taxes, we derive tax rates from given tax payment and tax base by the following relation:

$$\text{Tax rate} = \frac{\text{Tax payment}}{\text{Tax base}}$$

However, as to consumption tax, tax payment is derived from exogenous tax rate and tax base by the relation: tax payment = tax rate × tax base. Assuming that consumption tax rate in 1995 is 3%, we derived consumption tax payment of 8.2 trillion yen.

4.8 Household

We have already derived household consumption expenditure. Here, we derive other household data.

- Income
- Taxation on income
- Saving
- Transfer

4.8.1 Household income

The household supplies labor and capital to production sectors and achieves factor income. The household labor income is the sum of industry's net labor income and the household capital income is the sum of industry's net capital income.

4.8.2 Taxes on household

We assume that the different taxes are imposed on household labor income and capital income. We refer to tax on labor income as labor income tax and to tax on capital income as capital income tax. The values of labor and capital income taxes are derived from MOF (1996) and MOF (1997).

4.8.3 Savings and transfer

Household disposal income is given by

$$\begin{aligned}\text{Disposal income} &= \text{factor income} - \text{direct taxes}\\ &\quad + \text{transfer to household} - \text{transfer from household}\end{aligned}$$

where "factor income" is the sum of labor and capital income, "direct taxes" are the sum of labor and capital income taxes, "Transfer to household" is transfer from government to household, and "Transfer from household" is transfer from household to government. Below, we only consider net transfer defined as transfer to household minus transfer from household. Thus, the above relation becomes

$$\text{Disposal income} = \text{factor income} - \text{direct taxes} + \text{net transfer}$$

On the other hand, in the expenditure side, household disposal income is spent on consumption or savings. Thus, we have

$$\text{Disposal income} = \text{gross consumption expenditure} + \text{savings}$$

From the above two relations, we have

$$\begin{aligned}&\text{Gross consumption expenditure} + \text{savings}\\ &\quad = \text{factor income} - \text{direct taxes} + \text{net transfer}\end{aligned}$$

While factor income, direct taxes, and gross consumption expenditure in (42) are already determined, savings and net transfer are still undetermined. Thus, let us determine savings and net transfer below. First, savings must satisfy the following IS balance.

$$\text{Savings} = \text{gross investment} + \text{current account} - \text{fiscal surplus}$$

Investment has already been derived, and current account has been adjusted to zero. Here, we assume that fiscal surplus at the benchmark year is zero. Thus, savings is exactly equal to investment which is already derived.

$$\text{Savings} = \text{gross investment}$$

Given savings, the following relation determines net transfer as residual.

$$\begin{aligned}\text{Net transfer} &= \text{Gross consumption expenditure} + \text{savings}\\ &\quad -\text{Factor income} + \text{direct taxes}\end{aligned}$$

4.9 Other adjustments

4.9.1 Reaggregation of sectors

So far, we have made various adjustments for data of 35 sectors. Here, we aggregate 35 sectors to 27 sectors in Table 1. The simulation is conducted in 27 sector data.

4.9.2 Remove negative elements in intermediate input

As already explained, Japanese IO table deals with wastes and by-products with Stone approach and therefore includes negative elements. Here, we remove all negative elements in intermediate inputs. Negative elements in intermediate inputs are simply set to zero.

4.9.3 Remove small values

Since sectors are aggregated into 27 sectors, aggregated sectors have relatively large values of output and input. On the other hand, since energy goods and sectors are relatively disaggregated, their output and input take small values. A large difference between the largest and smallest values in the data causes problems for numerical computation. To avoid this, we remove small values in intermediate inputs.

4.9.4 Restoring data consistency

In the original IO table, the row and column sums are equal. However, as a result of various adjustments, intermediate inputs lost consistency with final demand, value added, and output. To construct a meaningful dataset, it is necessary to make some adjustments for intermediate inputs. To make these adjustments, we employ a quadratic programming approach. In this approach, the values in the data are adjusted to minimize a loss function subject to consistency conditions.

Let Y_i^C denote total value of intermediate input of sector i and Y_i^R denote total intermediate demand for good i. Y_i^C and Y_i^R are determined from data of final demand, value added, and output. Let A_{ij} denote new intermediate input which has consistency with other data. A_{ij} must satisfy the following conditions:

$$\sum_j A_{ij} = Y_i^R \qquad \forall i \tag{42}$$

$$\sum_j A_{ji} = Y_i^C \qquad \forall i \tag{43}$$

Next, let us define the following loss function.

$$\text{Loss} = \sum_{i,j} \left[\frac{A_{ij} - \bar{A}_{ij}}{\bar{A}_{ij}} \right]^2 \tag{44}$$

where $\bar{A}_{ij}$ is present intermediate input which has lost consistency with other data. New intermediate input A_{ij} are derived by minimizing (44) under constraint (42) and (43).

4.10 Data of emission sources

Quantity data on emission sources (hereafter, ESs) are taken from 3EID (Nansai et al., 2002). 3EID is the dataset that records the input and consumption of various materials including fossil fuels and limestone. The data in 3EID are based on the Japanese quantity IO table. However, since the quantity IO table has several flaws, 3EID modifies these data by using energy-input data from other sources. To apply 3EID to our data, we aggregate the 399 sectors in 3EID into 27 sectors.

4.11 Benchmark data

In this subsection, we review the benchmark dataset to identify its characteristics. In the simulation, we examine the effects of a policy that swaps emission regulations and pre-existing distortionary taxes. Therefore, the data on ESs and energy, and on pre-existing taxes, are particularly important. Although we assume 27 sectors for the simulation, we use the sector classification in Table 10 for presenting the dataset.

4.11.1 Energy data

Tables 11 and 12 show the input and consumption of ESs (except for LIM) by industry in the benchmark year. The rows indicate industries and end uses (FCON) that use ESs, and the columns indicate the ESs that are used. The difference between Tables 11 and 12 is that the former represents inputs for combustion purpose and the latter represents inputs for non-combustion purpose. As

Table 10: Aggregated sectors (this classification is used only for data representation).

Identifiers	Explanation	Sectors
AGR	Agriculture, forestry and fishery.	AGR
MIN	Minings.	OMI, LIM, COC, SLA, CRU, NAT
FOO	Foods.	FOO
TET	Textile products.	TET
PPP	Pulp, paper and wooden products.	PPP
CHM	Chemical products.	CHM
PAC	Petroleum refinery products and coal products.	PET, OPP, COK
CSC	Ceramic, stone and clay products.	CSC
IAM	Iron and metal.	IAM
MAO	Machinery and other industrial products.	GMA, OIP
CRE	Construction and real estate.	CON, RES
ELE	Electricity.	ELE
GSW	Steam, and hot water supply, water supply and waste disposal services.	GAS, SWW
COM	Commerce.	COM
TCB	Transport, communication and broadcasting.	TCB
SER	Services.	SER, PUB

Table 11 shows, the input of ESs is highly concentrated in a few sectors. The sectors that use ESs intensively are ELE (electricity), IAM (iron and metal), CHM (chemical industry), TCB (transport, communication and broadcasting), and CSC (ceramic, stone, and clay products). It is likely that these sectors will be affected significantly by emission regulations. Table 11 also shows that the only ESs used for final consumption are PET and GAS. Table 12 shows that primary energy sources such as COC, CRU, and NAT are rarely used for combustion purpose and most of them are used in secondary energy sectors — that is, PAC and GSW.

Table 13 shows the share of each ES in intermediate input and final consumption. Since PET is a composite good that aggregates various petroleum-based energy sources such as gasoline, light oil, kerosene, heavy oil, jet fuel, and LPG, its share in the total input is large (about 58%). Table 14 shows the share of payments to ESs in total cost. This table also shows that sectors such as ELE, IAM, CHM, CSC, and TCB use energy goods intensively.

Table 11: ES input and consumption for combustion purpose (10^{13}kcal)

	COC	SLA	CRU	NAT	PET	COK	GAS	Total
AGR					6.97			6.97
FOO					4.52		0.85	5.37
TET					1.60		0.17	1.77
PPP		1.00			5.11	0.02	0.21	6.34
CHM		1.70	2.56	0.58	7.48	0.14	0.54	13.00
CSC		6.48		0.03	3.78	0.27	0.33	10.89
IAM	4.95	1.68		0.16	4.12	28.01	0.94	39.86
ELE		28.24	14.65	43.33	23.30		0.01	109.53
COM					4.48		0.57	5.05
TCB					73.85		0.25	74.10
SER		0.33			18.70	0.04	5.42	24.49
MIN					0.24	0.01		0.25
PAC		1.41			2.91	0.02		4.34
MAO		0.01		0.02	5.19	0.19	1.25	6.66
CRE					5.09		0.56	5.65
GSW		0.04			2.81		0.50	3.35
INT	4.95	40.89	17.21	44.12	170.15	28.70	11.60	317.62
FCON					47.48		9.12	56.60
Total	4.95	40.89	17.21	44.12	217.63	28.70	20.72	374.22

LIM (limestone) is excluded.

Table 12: ES input and consumption for non-combustion purpose (10^{13}kcal)

	COC	CRU	NAT	PET
CHM				2.45
PAC	39.83	221.68		
GSW	0.55		14.54	2.70

Table 13: ES input and consumption (%)

	INT	FCON	Total
COC	1.6		1.3
SLA	12.9		10.9
CRU	5.4		4.6
NAT	13.9		11.8
PET	53.6	83.9	58.2
COK	9.0		7.7
GAS	3.7	16.1	5.5
Total	100	100	100

INT is emissions from intermediate input and FCON is emissions from final consumption.

Table 14: ES cost share in each sector (%)

	Cost share (%)
AGR	1.10
FOO	0.40
TET	0.40
PPP	0.70
CHM	0.80
CSC	4.00
IAM	1.40
ELE	8.90
COM	0.20
TCB	5.60
SER	0.30
MIN	0.80
PAC	2.10
MAO	0.10
CRE	0.30
GSW	0.90

Table 15: Carbon emissions (MtC)

	LIM	Coal	Petroleum	Gas	Total
AGR			5.46		5.46
FOO			3.54	0.51	4.05
TET			1.26	0.10	1.36
PPP	0.04	1.04	4.00	0.13	5.21
CHM		1.90	7.88	0.66	10.44
CSC	12.12	6.91	2.96	0.21	22.20
IAM	2.70	41.37	3.22	0.66	47.95
ELE		28.67	29.84	25.35	83.87
COM			3.51	0.34	3.85
TCB			57.82	0.15	57.97
SER	0.07	0.39	14.64	3.23	18.33
MIN		0.02	0.19		0.21
PAC		1.45	2.28		3.74
MAO		0.25	4.06	0.76	5.07
CRE			3.98	0.33	4.32
GSW		0.04	2.20	0.30	2.54
INT	14.93	82.06	146.83	32.73	276.55
FCON			37.17	5.44	42.61
Total	14.93	82.06	184.00	38.17	319.17

Coal includes COC, SLA, and COK, and petroleum includes PET and CRU, and gas includes GAS and NAT. INT is emissions from intermediate input and FCON is emissions from final consumption.

4.11.2 Emission data

Table 15 and Figure 6 represent carbon emissions by sector and ES. “Coal” is coal-based energy COC, SLA, and COK, “petroleum” is petroleum-based energy CRU and PET, and “gas” is gas-based energy NAT and GAS. As the table shows, the energy-intensive sectors tend to emit more carbon. Total carbon emissions in our dataset are 319MtC, which is slightly lower than the emissions in 3EID (320MtC).[18]

Table 16 and Figure 7 report emission shares by ES. Emission sources with high (low) calorific shares in Table 13 tend to have high (low) emission shares. However, the emission shares of emission sources with high (low) emission coefficients are larger (smaller) than their calorific shares.

Table 17 presents emission shares and carbon intensities by sector. Since energy inputs are concentrated in a few sectors, emissions are also concentrated in these sectors. In particular, emissions from CHM, CSC, IAM, ELE, and TCB account for about 70% of total emissions. In addition to these sectors, final consumption (FCON) also has a considerable share (about 13%). Carbon intensity is defined as the amount of carbon emissions (in tonnes) generated in producing goods and services valued at one million yen. As the table shows, energy-intensive sectors such as CHM, CSC, IAM, ELE, and TCB have high carbon intensities.

[18] The value of 320MtC derived from the original 3EID data only includes emissions from ESs in Table ??. The discrepancy in carbon emissions between our dataset and the 3EID data is due to the treatment of petroleum-based energy. In our dataset, the petroleum-based energy sources are aggregated into a single good PET, while they are disaggregated in 3EID.

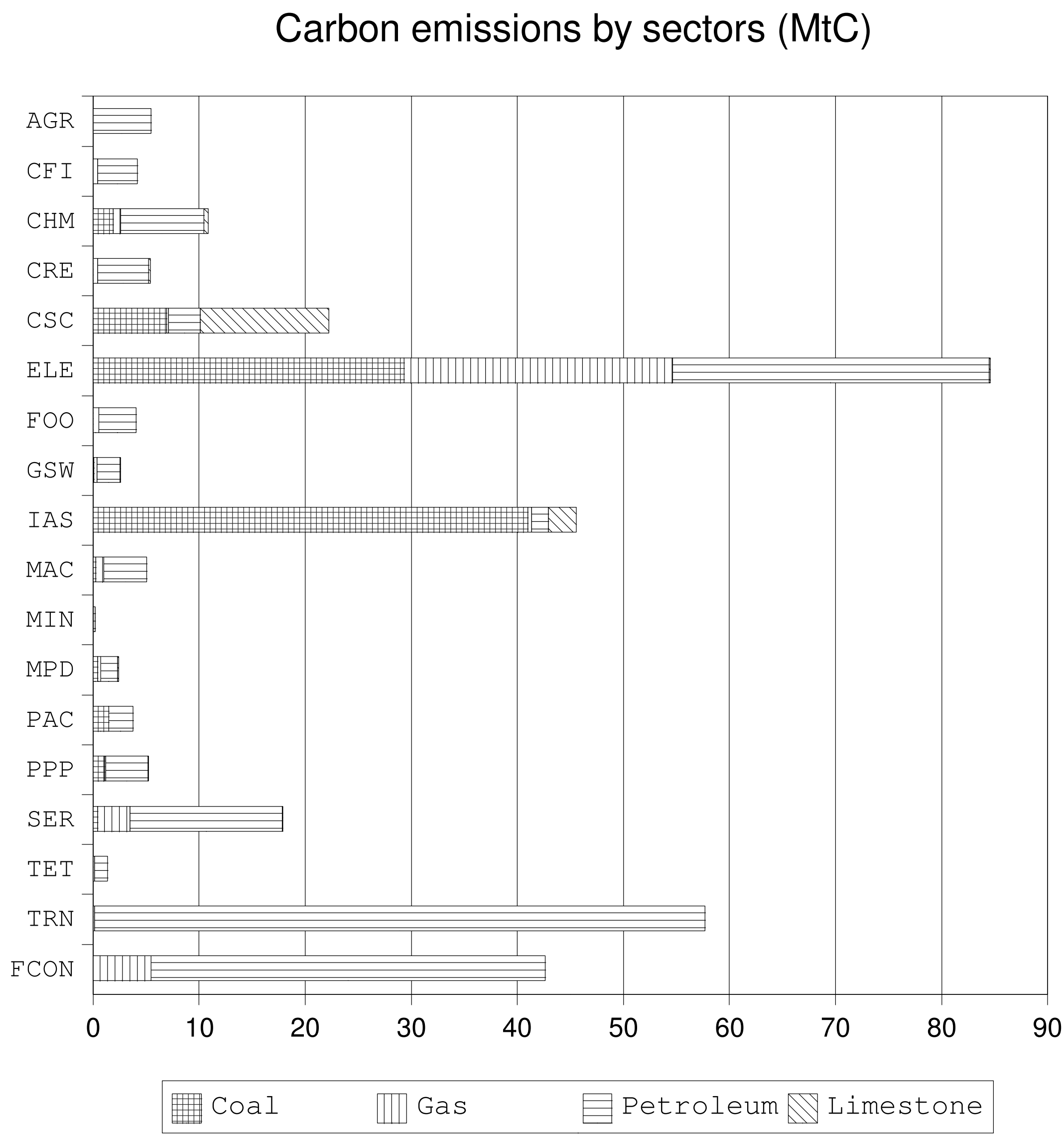

Figure 6: Carbon emissions (MtC)

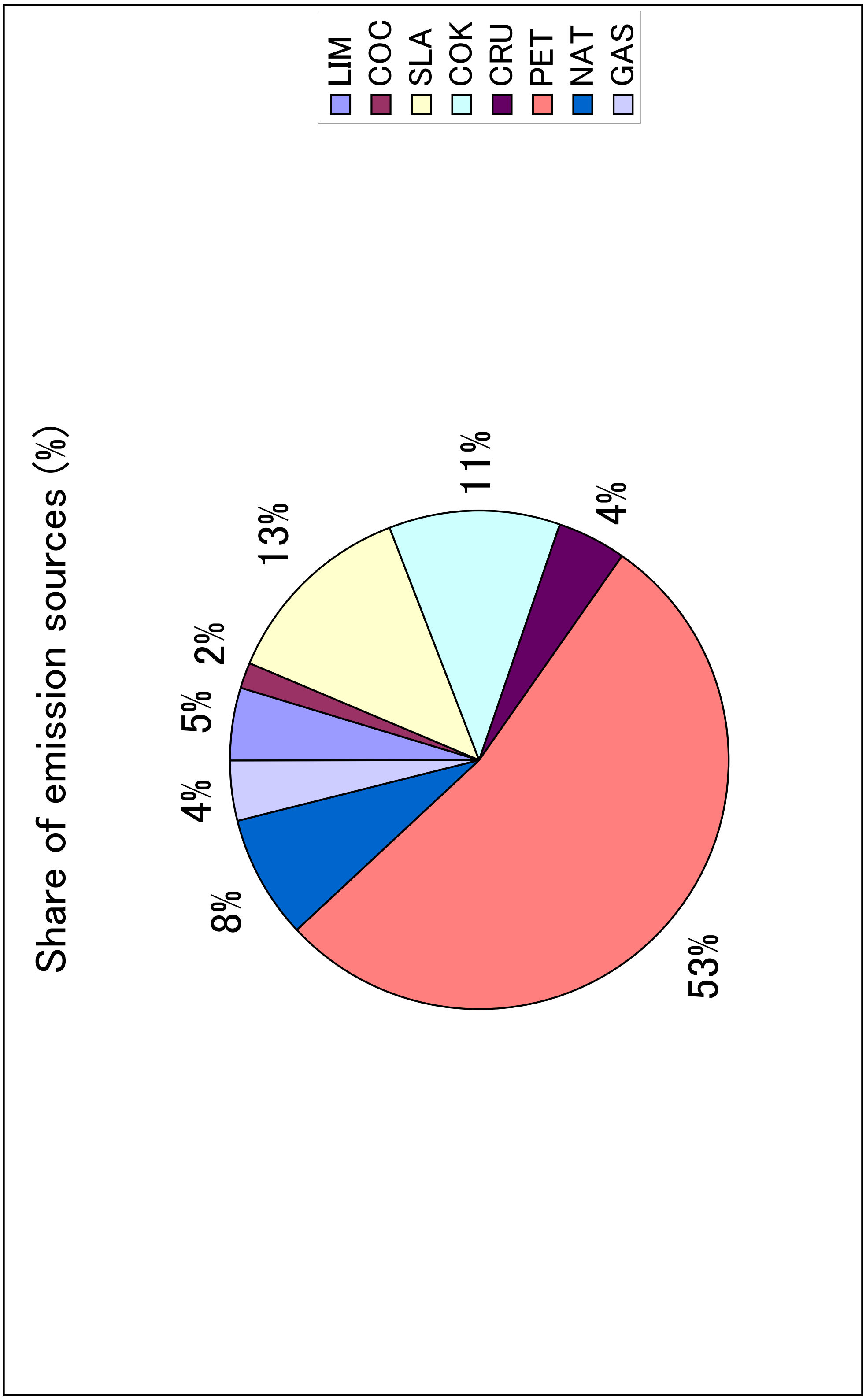

Figure 7: Emission shares by emission source (%)

Table 16: Emission share by emission source (%)

	INT	FCON	Total
LIM	5.4		4.7
COC	1.9		1.6
SLA	15.0		13.0
CRU	4.9		4.3
NAT	9.3		8.1
PET	48.2	87.2	53.4
COK	12.8		11.1
GAS	2.5	12.8	3.9
Total	100.0	100.0	100.0

INT is emissions from intermediate input and FCON is emissions from final consumption.

Table 17: Emission share and carbon intensity by sector

	Emission share (%)	Carbon intensity (tC/mil. yen)
AGR	1.7	0.35
FOO	1.3	0.10
TET	0.4	0.12
PPP	1.6	0.29
CHM	3.3	0.40
CSC	7.0	2.29
IAM	15.0	1.14
ELE	26.3	5.01
COM	1.2	0.04
TCB	18.2	0.89
SER	5.7	0.05
MIN	0.1	0.12
PAC	1.2	0.37
MAO	1.6	0.03
CRE	1.4	0.05
GSW	0.8	0.26
FCON	13.35	

FCON is final consumption.

4.11.3 Tax data

In this subsection, we review taxes in the benchmark dataset.[19] Table 18 presents taxes on production. As taxes on production, the model includes labor tax (LAB), capital tax (CAP) and indirect tax on production. Table 18 does not include indirect tax on production because their rate is constant in the simulation.

All tax rates except consumption tax are derived implicitly by dividing tax payments by the tax base. Let V_i^K denote payments to capital in sector i (net of tax), T_i^K denote capital tax payments in sector i, V_i^L denote payments to labor (net of tax), and let T_i^L denote labor tax payments. Then the capital tax rate (τ_i^K) and the labor tax rate (τ_i^L) in sector i are given by:

$$\tau_i^K = \frac{T_i^K}{V_i^K} \qquad \tau_i^L = \frac{T_i^L}{V_i^L}$$

This method of deriving tax rates yields tax rates that differ between sectors.

Table 19 presents taxes on households. In our model, we consider three taxes on households, that is, labor income tax (LIN), capital income tax (CIN), and consumption tax (CTX). In practice, income taxes depend on the household's income class and its type. However since we assume a representative household, we cannot consider differences in tax rates between households. To simplify the model, we assume that the income tax imposed corresponds to the average tax rate for a representative household. This average tax rate is derived implicitly by dividing tax payments by tax base.

In contrast to other taxes, payment of consumption tax is derived by multiplying tax base by tax rate. We assume that consumption tax rate is 3%. The value of consumption tax payment is about 8.3 trillion yen, which is slightly larger than actual consumption tax payment in 1995 (5.8 trillion yen). There are two reasons for this.

[1] The tax base of consumption tax (i.e. consumption expenditure) includes hypothetical expenditure such as imputed rent and is larger than actual tax base.

[2] Tax profits, which arise in reality, are not taken into account.

Table 20 presents values and shares for all taxes. CAP, and LAB are taxes on producers, and LIN, CIN, and CTX are taxes on households. In terms of the tax base, CAP and CIN are taxes on capital, and LAB and LIN are taxes on labor.

[19] For the details of Japanese tax system, see, for example, MOF (2002)

Table 18: Taxes on production

	Capital tax (CAP)		Labor tax (LAB)	
	Value (bil. yen)	Rate (%)	Value (bil. yen)	Rate (%)
AGR	21.7	0.34	73.0	3.81
OMI	40.3	15.37	18.6	8.46
LIM	5.3	15.37	5.0	10.11
SLA	2.0	15.37	3.2	10.54
NAT	5.3	15.37	1.1	10.49
FOO	627.9	17.61	368.5	7.61
TET	76.2	8.54	212.0	8.24
PPP	395.1	21.63	279.5	9.23
CHM	757.6	21.63	257.9	8.82
PET	84.0	21.63	15.0	8.07
OPP	10.6	21.63	1.9	8.07
COK	19.3	21.63	3.9	8.07
CSC	272.5	21.63	181.9	9.31
IAM	482.0	10.75	646.7	9.06
MAC	1930.7	16.36	1870.6	9.28
OIP	1001.4	32.94	617.2	8.49
CON	2189.3	40.84	2342.1	8.67
ELE	521.4	9.56	126.7	8.42
GAS	47.9	9.56	39.4	9.72
SWW	209.1	9.56	179.8	7.79
COM	3317.2	28.01	3618.3	7.62
RES	631.4	1.30	158.1	6.57
TCB	905.7	9.56	2043.0	10.42
PUB	0.0	0.00	2307.5	15.96
SER	3260.7	8.54	7363.4	8.50
Total	16814.6		22734.3	

Table 19: Taxes on households

	Rate (%)	Value (tril. yen)
CTX	3.00	8.30
LIN	9.21	23.40
CIN	3.99	6.38

CTX is consumption tax, LIN is labor income tax, and CIN is capital income tax.

Table 20: Total tax revenue

	Share (%)	Value (tril. yen)
CTX	7.4	8.3
CAP	14.9	16.8
LAB	20.2	22.7
LIN	20.8	23.4
CIN	5.7	6.4
Others	31.1	35.0
Total		112.6

5 Model for Computation

We have already explained the model structure in Section 1. However, in GAMS programs for the simulation, the model is described using calibrated share forms. Here, we present the model in accordance with GAMS programs.

[1] All function are written in calibrated share form (Rutherford, 1998).

[2] Reference prices are omitted for notational simplification.

[3] Slack variable associated to each equation is given in parentheses on the right end.

5.1 Notations

Activity levels

Notations	Description	Program
Q_{si}	Activity level for production	`q_q(s,i)`
A_{si}	Armington aggregation	`q_a(s,i)`
X_{si}	Export activity	`q_x(s,i)`
M_{si}	Import activity	`q_m(s,i)`
$\bar{C}_s$	Consumption aggregation activity	`q_cc(s)`
W_s	Period utility	`q_w(s)`
G_s	Government expenditure at each period	`q_g(s)`
J_s	Net investment	`q_j(s)`
L_s^S	Labor supply	`q_sl(s)`
U	Lifetime utility	`q_u`
K_s	Capital stock	`q_k(s)`
G^L	Government lifetime expenditure	`q_gle`
CNC_s	Carbon emissions ($i \in \mathrm{ES}$)	`q_cnc(s,e)`

Unit cost

Notations	Description	Program
c_s^{CC}	Unit cost of consumption aggregation	`c_cc(s)`
c_{si}^{A}	Unit cost of Armington aggregation	`c_a(s,i)`
c_{si}^{W}	Unit cost of period utility (expenditure function)	`c_w(s)`
c^{U}	Unit cost of lifetime utility (expenditure function)	`c_u`
c_{si}^{Q}	Unit cost of production	`c_q(s,i)`
c_{si}^{E}	Unit cost of emission sources for combustion purpose ($i \in \mathrm{ES}$)	`c_e(s,e)`

Price (index)

Notations	Description	Program
p_{si}^{Q}	Price index (unit revenue) for output	`p_q(s,i)`
p_{si}^{F}	Price index of primary factor composite	`p_f(s,i)`
p_{si}^{EE}	Price index of energy composite in production	`p_ee(s,i)`
p_{si}^{FE}	Price index of primary factor-energy composite	`p_fe(s,i)`
p_{s}^{C}	Price index of non-energy composite in consumption	`p_c(s)`
p_{s}^{EC}	Price index of energy composite in consumption	`p_ec(s)`
p_{s}^{I}	Price index of investment good	`p_i(s)`
p_{si}^{E}	Price index of emission sources for combustion purpose ($i \in \mathrm{ES}$)	`p_e(s,e)`
p_{s}^{CC}	Price index of aggregate consumption	`p_cc(s)`
p_{si}^{D}	Price of domestic good	`p_d(s,i)`
p_{si}^{X}	Price of export good	`p_x(s,i)`
p_{si}^{M}	Price of import good	`p_m(s,i)`
p^{FX}	Exchange rate	`p_fxv`
p_{s}^{L}	Wage rate for producer	`p_l(s)`
p_{s}^{LE}	Wage rate for household	`p_le(s)`
p_{s}^{W}	Price of period utility	`p_w(s)`
p^{U}	Price of lifetime utility	`p_u`
p_{si}^{A}	Price of Armington good	`p_a(s,i)`
p_{s}^{G}	Price of government expenditure	`p_g(s)`
p_{s}^{K}	Shadow price of capital	`p_k(s)`
r_{s}^{K}	Rental price for industry	`p_rk(s)`
r_{s}^{KE}	Rental price for household	`p_rke(s)`
p^{GL}	Price index for government lifetime consumption	`p_gle`
t_{s}^{CA}	Carbon tax	`p_ca(s)`
p_{s}^{KA}	Adjustment premium	`p_ka(s)`

Unit demand and supply

Notations	Description	Program
a_{si}^{X}	Supply to export market	`a_x(s,i)`
a_{si}^{D}	Supply to domestic market	`a_d(s,i)`
a_{si}^{L}	Labor demand	`a_l(s,i)`
a_{si}^{K}	Capital demand	`a_k(s,i)`
a_{sji}^{E}	Demand for emission sources (exc. COK and LIM) in production ($j \in \mathrm{EC}$)	`a_e(s,ec,i)`
a_{si}^{EC}	Consumption demand for energy goods ($i \in \mathrm{ENE}$)	`a_ec(s,ene)`
a_{si}^{C}	Consumption demand for non-energy goods ($i \in \mathrm{NENE}$)	`a_c(s,nene)`
a_{s}^{CC}	Demand for aggregate consumption	`a_cc(s)`
a_{s}^{LE}	Demand for leisure `a_lei(s)`	
a_{s}^{U}	Demand for period utility	`a_u(s)`
a_{si}^{AD}	Demand for domestic good from Armington activity	`a_ad(s,i)`
a_{si}^{AM}	Demand for imported good from Armington activity	`a_am(s,i)`
$\bar{a}_{ij}^{\mathrm{NEN}}$	Input coefficient for non-energy intermediate goods (exc. COL and LIM)	
$\bar{a}_{ij}^{\mathrm{CL}}$	Input coefficient for COL and LIM for combustion purpose	
$\bar{a}_{ij}^{\mathrm{NC}}$	Input coefficient for emission sources for non-combustion purpose	
$\bar{a}_{i}^{I}$	Investment demand	
$\bar{a}_{i}^{G}$	Government demand	

Income

Notations	Description	Program
Y^H	Extended lifetime income of household	`v_inc_h`
Y^G	Government income (lifetime)	`v_inc_gl`
M_s^G	Government income (period)	`v_inc_g(s)`
K_{T+1}	Post-terminal capital stock	`q_tcap`
τ	Tax multiplier	`m_tau`

Other variables

Notations	Description	Program
R_s	Discount factor	`p_disc(s)`
r_s	Interest rate	`1/p_intr(s) - 1`

Share variables

Notations	Description	Program
θ_i^X	Share of export in output	`sh_x(i)`
θ_i^{FE}	Share of primary factor and energy in production cost	`sh_fe(i)`
θ_i^F	Share of primary factor in PF-E aggregation	`sh_f(i)`
θ_i^{FL}	Share of labor in primary factor	`sh_fl(i)`
θ_{ji}^E	Share of energy goods in production ($j \in \mathrm{ENE}$)	`sh_e(ene)`
$\theta_{ji}^{\mathrm{NEN}}$	Share of non-energy intermediate inputs ($j \in \mathrm{NEN}$)	`sh_ee(j,nen)`
$\theta_{ji}^{\mathrm{NC}}$	Share of energy for non-combustion purpose ($j \in \mathrm{ES}$)	`sh_nc(e,i)`
$\theta_{ji}^{\mathrm{CL}}$	Share of COK and LIM for combustion purpose ($j \in \mathrm{CL}$)	`sh_cl(cl,i)`
θ^{NENE}	Share of non-energy goods in consumption	`sh_nene`
θ_i^C	Share of each good in non-energy consumption ($i \in \mathrm{NENE}$)	`sh_c(nene)`
θ_i^{EC}	Share of each good in energy consumption. ($j \in \mathrm{ENE}$)	`sh_ec(ene)`
θ^{LE}	Share of leisure	`sh_lei`
θ_s^U	Share of period utility	`sh_u(s)`
θ_i^G	Share of each good in government expenditure	`sh_g(i)`
θ_i^{AD}	Share of domestic good in Armington aggregation	`sh_ad(i)`
θ_i^{INV}	Share of input in investment activity	`sh_i(i)`

Policy variables

Notations	Description	Program
t_i^L	Labor tax rate in production	`tl(i)`
t_i^K	Capital tax rate in production	`tk(i)`
t^C	Consumption tax rate	`tc`
t^I	Labor income tax rate	`tli`
t^A	Capital income tax rate	`tci`
t_i^Q	Indirect tax rate on production	`tq(i)`
t_i^M	Import tariff	`tm(i)`
s_i^C	Subsidy rate on consumption	`-ts(i)`

Elasticity parameters

Notations	Description	Program
η	Elasticity of transformation between domestic supply and export supply	`eta`
$\sigma_{F,i}$	Elasticity of substitution between labor and capital	`sigpf(i)`
σ_{FE}	Elasticity of substitution between energy and primary factor	`sigpfe`
σ_{EE}	Elasticity of substitution between energy goods	`sigee`
$\sigma_{A,i}$	Armington elasticity	`siga(i)`
σ_{LC}	Elasticity of substitution between consumption and leisure	`sigcl`
σ_{CC}	Elasticity of substitution between energy and non-energy goods	`sigcc`
σ_{EC}	Elasticity of substitution between energy goods	`sigec`
σ_C	Elasticity of substitution between non-energy goods	`sigc`
σ_U	Intertemporal elasticity of substitution	`sigu`

Indices for sectors and goods

Notations	Description	Production
I	= A set of all goods and sectors	`i,ii`
ES	$= \{\mathrm{COC}, \mathrm{SLA}, \mathrm{COK}, \mathrm{CRU}, \mathrm{PET}, \mathrm{NAT}, \mathrm{GAS}, \mathrm{LIM}\}$ $\cdots$ Emission sources	`e`
EC	$= \{\mathrm{COC}, \mathrm{SLA}, \mathrm{CRU}, \mathrm{PET}, \mathrm{NAT}, \mathrm{GAS}\}$	`ec`
CL	$= \{\mathrm{COK}, \mathrm{LIM}\}$.	`cl`
ELE	$= \{\mathrm{ELE}\}$ $\cdots$ Electricity	`ele`
ENE	$= \mathrm{EC} \cup \mathrm{ELE}$ $\cdots$ Energy goods	`ene`
NENE	$= I \setminus \mathrm{ENE}$ $\cdots$ non-energy goods	`nene`
EN	$= \mathrm{ES} \cup \mathrm{ELE}$ $\cdots$ Energy goods + COL + LIM	`en`
NEN	$= I \setminus \mathrm{EN}$ $\cdots$ Non-Energy goods – COL – LIM	`nen`

5.2 Model

5.2.1 Unit cost

Unit cost of consumption aggregation:

$$c_s^{\mathrm{CC}} = \left[\theta^{\mathrm{NENE}} \left(p_s^C\right)^{1-\sigma_{\mathrm{CC}}} + (1-\theta^{\mathrm{NENE}}) \left(p_s^{\mathrm{EC}}\right)^{1-\sigma_{\mathrm{CC}}}\right]^{\frac{1}{1-\sigma_{\mathrm{CC}}}} \qquad \{c_s^{\mathrm{CC}}\}$$

Unit cost of Armington aggregation:

$$c_{si}^A = \left[\theta_i^{\mathrm{AD}} \left(p_{si}^D\right)^{1-\sigma_A} + (1-\theta_i^{\mathrm{AD}}) \left(p_{si}^M\right)^{1-\sigma_A}\right]^{\frac{1}{1-\sigma_A}} \qquad \{c_{si}^A\}$$

Unit cost of period utility (expenditure function):

$$c_s^W = \left[\theta^{\mathrm{LE}} \left[p_s^{\mathrm{LE}}\right]^{1-\sigma_{\mathrm{LC}}} + (1-\theta^{\mathrm{LE}}) \left[(1+t_s^C) p_s^{\mathrm{CC}}\right]^{1-\sigma_{\mathrm{LC}}}\right]^{\frac{1}{1-\sigma_{\mathrm{LC}}}} \qquad \{c_s^W\}$$

Unit cost of lifetime utility (expenditure function):

$$c^U = \left[\sum_{s=t}^{T} \theta_s^U \left(p_s^W\right)^{1-\sigma_U}\right]^{\frac{1}{1-\sigma_U}} \qquad \{c^U\}$$

Unit cost of production:

$$c_{si}^{Q} = \left[\sum_{j \in \mathrm{NEN}} \theta_{ji}^{\mathrm{NEN}} p_{sj}^{A} + \sum_{j \in \mathrm{ES}} \theta_{ji}^{\mathrm{NC}} p_{sj}^{A} + \sum_{j \in \mathrm{CL}} \theta_{ji}^{\mathrm{CL}} p_{sj}^{\mathrm{EE}} + \theta_{i}^{\mathrm{FE}} p_{si}^{\mathrm{FE}} \right] \qquad \{c_{si}^{Q}\}$$

Unit cost of emission sources for combustion purpose:

$$c_{si}^{E} = p_{si}^{A} + \gamma_i p_{s}^{\mathrm{CA}} \qquad \{c_{si}^{E}\}$$

5.2.2 Price (index)

Price index (unit revenue) for output:

$$p_{si}^{Q} = \left[\theta_{i}^{X} \left(p_{si}^{X} \right)^{1+\eta} + (1 - \theta_{i}^{X}) \left(p_{si}^{D} \right)^{1+\eta} \right]^{\frac{1}{1+\eta}} \qquad \{p_{si}^{Q}\}$$

Price index of primary factor composite:

$$p_{si}^{F} = \left[\theta_{i}^{\mathrm{FL}} \left[(1 + t_{i}^{L}) p_{s}^{L} \right]^{1-\sigma_{F,i}} + (1 - \theta_{i}^{\mathrm{FL}}) \left[(1 + t_{i}^{K}) r_{s}^{K} \right]^{1-\sigma_{F,i}} \right]^{\frac{1}{1-\sigma_{F,i}}} \qquad \{p_{si}^{F}\}$$

Price index of energy composite in production:

$$p_{si}^{\mathrm{EE}} = \left[\sum_{j \in \mathrm{EC}} \theta_{ji}^{\mathrm{EE}} \left(p_{sj}^{E} \right)^{1-\sigma_{\mathrm{EE}}} + \theta_{\mathrm{ELE},i}^{\mathrm{EE}} \left(p_{s,\mathrm{ELE}}^{A} \right)^{1-\sigma_{\mathrm{EE}}} \right]^{\frac{1}{1-\sigma_{\mathrm{EE}}}} \qquad \{p_{si}^{\mathrm{EE}}\}$$

Price index of primary factor-energy composite:

$$p_{si}^{\mathrm{FE}} = \left[\theta_{i}^{F} \left(p_{si}^{F} \right)^{1-\sigma_{\mathrm{FE}}} + (1 - \theta_{i}^{F}) \left(p_{si}^{\mathrm{EE}} \right)^{1-\sigma_{\mathrm{FE}}} \right]^{\frac{1}{1-\sigma_{\mathrm{FE}}}} \qquad \{p_{si}^{\mathrm{FE}}\}$$

Price index of energy composite in consumption:

$$p_{s}^{C} = \left[\sum_{i \in \mathrm{NENE}} \theta_{i}^{C} \left[(1 - s_{i}^{C}) p_{si}^{A} \right]^{1-\sigma_{C}} \right]^{\frac{1}{1-\sigma_{C}}} \qquad \{p_{s}^{C}\}$$

Price index of energy composite in consumption:

$$p_{s}^{\mathrm{EC}} = \left[\sum_{i \in \mathrm{EC}} \theta_{i}^{\mathrm{EC}} \left[(1 - s_{i}^{C}) p_{si}^{E} \right]^{1-\sigma_{\mathrm{EC}}} + \theta_{\mathrm{ELE}}^{\mathrm{EC}} \left[(1 - s_{\mathrm{ELE}}^{C}) p_{s,\mathrm{ELE}}^{A} \right]^{1-\sigma_{\mathrm{EC}}} \right]^{\frac{1}{1-\sigma_{\mathrm{EC}}}} \qquad \{p_{s}^{\mathrm{EC}}\}$$

Price index of investment good:

$$p_{s}^{I} = \bar{p}_{t}^{I} \left[\sum_{i} \theta_{i}^{\mathrm{INV}} p_{si}^{A} \right] \qquad \{p_{s}^{I}\}$$

Adjustment premium:

$$p_{s}^{\mathrm{KA}} = p_{s}^{I} \frac{\phi}{2} \left[\frac{J_s}{K_s} \right]^{2} \qquad \{p_{s}^{\mathrm{KA}}\}$$

Wage rate for household:

$$p_{s}^{\mathrm{LE}} = (1 - t^{L}) p_{s}^{L} \qquad \{p_{s}^{\mathrm{LE}}\}$$

Rental price for household:

$$r_{s}^{\mathrm{KE}} = (1 - t^{A}) r_{s}^{K} \qquad \{r_{s}^{\mathrm{KE}}\}$$

Discount factor:

$$R_s = \left\{ \begin{array}{cc} 1 & s = t \\ \prod_{l=t}^{s} (1 + r_s)^{-1} & s > t \end{array} \right. \qquad \{R_s\}$$

5.2.3 Zero profit conditions

Zero profit for production activity:

$$c_{si}^{Q} \geq (1 - t_i^{Q}) p_{si}^{Q} \qquad \{Q_{si}\}$$

Zero profit for Armington aggregation activity:

$$c_{si}^{A} \geq p_{si}^{A} \qquad \{A_{si}\}$$

Zero profit for export activity:

$$p_{si}^{X} \geq p^{\mathrm{FX}} \qquad \{X_{si}\}$$

Zero profit for import activity:

$$(1 + t_i^{M}) p^{\mathrm{FX}} \geq p_{si}^{M} \qquad \{M_{si}\}$$

Zero profit for consumption aggregation:

$$c_s^{\mathrm{CC}} \geq p_s^{\mathrm{CC}} \qquad \{\bar{C}_s\}$$

Zero profit for period utility:

$$R_s c_s^{W} \geq p_s^{W} \qquad \{W_s\}$$

Zero profit for lifetime utility:

$$c^{U} \geq p^{U} \qquad \{U\}$$

Zero profit for government expenditure:

$$\bar{p}_t^{G} \left[\sum_i \theta_i^{G} p_{si}^{A} \right] \geq p_s^{G} \qquad \{G_s\}$$

Zero profit for investment activity:

$$p_s^{I} \left[1 + \phi \left(\frac{J_s}{K_s} \right) \right] \geq p_s^{K} \qquad \{J_s\}$$

Zero profit for capital accumulation:

$$(1 + r_s) p_{s-1}^{K} \geq r_s^{\mathrm{KE}} + (1 - \delta) p_s^{K} + p_s^{\mathrm{KA}} \qquad \{K_s\}$$

Zero profit for government lifetime expenditure:

$$\bar{p}^{\mathrm{GL}} \left[\sum_{s=t}^{T} \theta_s^{G} p_s^{G} \right] \geq p^{\mathrm{GL}} \qquad \{G^{L}\}$$

Carbon emissions:

$$c_{si}^{E} \geq p_{si}^{E} \qquad \{\mathrm{CNC}_i\}_{i \in \mathrm{ES}}$$

5.2.4 Unit demand and supply

Supply to export and domestic markets:

$$a_{si}^{X} = \bar{a}_{ti}^{X} \left[\frac{p_{si}^{X}}{p_{si}^{Q}}\right]^{\eta} \qquad \{a_{si}^{X}\}$$

$$a_{si}^{D} = \bar{a}_{ti}^{D} \left[\frac{p_{si}^{D}}{p_{si}^{Q}}\right]^{\eta} \qquad \{a_{si}^{D}\}$$

Labor demand:

$$a_{si}^{L} = \bar{a}_{ti}^{L} \left[\frac{p_{si}^{F}}{(1+t_i^{L})p_s^{L}}\right]^{\sigma_{\text{FL}}} \left[\frac{p_{si}^{\text{FE}}}{p_{si}^{F}}\right]^{\sigma_{\text{FE}}} \qquad \{a_{si}^{L}\}$$

Capital demand:

$$a_{si}^{K} = \bar{a}_{ti}^{K} \left[\frac{p_{si}^{F}}{(1+t_i^{K})r_s^{K}}\right]^{\sigma_{\text{FL}}} \left[\frac{p_{si}^{\text{FE}}}{p_{si}^{F}}\right]^{\sigma_{\text{FE}}} \qquad \{a_{si}^{K}\}$$

Demand for emission sources (exc. COK and LIM) in production:

$$a_{sij}^{E} = \bar{a}_{tij}^{E} \left[\frac{p_{sj}^{\text{EE}}}{p_{si}^{E}}\right]^{\sigma_{\text{EE}}} \left[\frac{p_{sj}^{\text{FE}}}{p_{sj}^{\text{EE}}}\right]^{\sigma_{\text{FE}}} \qquad \{a_{sij}^{E}\}_{i\in\text{EC}}$$

Demand for electricity in production:

$$a_{sij}^{E} = \bar{a}_{tij}^{E} \left[\frac{p_{sj}^{\text{EE}}}{p_{si}^{A}}\right]^{\sigma_{\text{EE}}} \left[\frac{p_{sj}^{\text{FE}}}{p_{sj}^{\text{EE}}}\right]^{\sigma_{\text{FE}}} \qquad \{a_{si}^{E}\}_{i\in\text{ELE}}$$

Consumption demand for emission sources (exc. LIM and COK):

$$a_{si}^{\text{EC}} = \bar{a}_{ti}^{\text{EC}} \left[\frac{p_{s}^{\text{EC}}}{(1-s_i^{C})p_{si}^{E}}\right]^{\sigma_{\text{EC}}} \left[\frac{p_{s}^{\text{CC}}}{p_{s}^{\text{EC}}}\right]^{\sigma_{\text{CC}}} \qquad \{a_{si}^{\text{EC}}\}_{i\in\text{EC}}$$

Consumption demand for electricity:

$$a_{si}^{\text{EC}} = \bar{a}_{ti}^{\text{EC}} \left[\frac{p_{s}^{\text{EC}}}{(1-s_i^{C})p_{si}^{A}}\right]^{\sigma_{\text{EC}}} \left[\frac{p_{s}^{\text{CC}}}{p_{s}^{\text{EC}}}\right]^{\sigma_{\text{CC}}} \qquad \{a_{si}^{\text{EC}}\}_{i\in\text{ELE}}$$

Consumption demand for non-emission sources (exc. electricity):

$$a_{si}^{C} = \bar{a}_{ti}^{C} \left[\frac{p_{s}^{C}}{(1-s_i^{C})p_{si}^{A}}\right]^{\sigma_{C}} \left[\frac{p_{s}^{\text{CC}}}{p_{s}^{C}}\right]^{\sigma_{\text{CC}}} \qquad \{a_{si}^{C}\}_{i\in\text{NEN}}$$

Consumption demand for COK and LIM:

$$a_{si}^{C} = \bar{a}_{ti}^{C} \left[\frac{p_{s}^{C}}{(1-s_i^{C})p_{si}^{E}}\right]^{\sigma_{C}} \left[\frac{p_{s}^{\text{CC}}}{p_{s}^{C}}\right]^{\sigma_{\text{CC}}} \qquad \{a_{si}^{C}\}_{i\in\text{CL}}$$

Demand for aggregate consumption:

$$a_{s}^{\text{CC}} = \bar{a}_{t}^{\text{CC}} \left[\frac{p_{s}^{W}}{(1+t^{C})p_{s}^{\text{CC}}}\right]^{\sigma_{\text{LC}}} \qquad \{a_{s}^{\text{CC}}\}$$

Demand for leisure:

$$a_{s}^{\text{LE}} = \bar{a}_{t}^{\text{LE}} \left[\frac{p_{s}^{W}}{p_{s}^{\text{LE}}}\right]^{\sigma_{W}} \qquad \{a_{s}^{\text{LE}}\}$$

Demand for period utility:

$$a_s^U = \bar{a}_s^U \left[\frac{c^U}{p_s^W} \right]^{\sigma_U} \qquad \{a_s^U\}$$

Demand for domestic and import goods from Armington activity:

$$a_{si}^{\mathrm{AD}} = \bar{a}_{ti}^{\mathrm{AD}} \left[\frac{c_{si}^A}{p_{si}^D} \right]^{\sigma_A} \qquad \{a_{si}^{\mathrm{AD}}\}$$

$$a_{si}^{\mathrm{AM}} = \bar{a}_{ti}^{\mathrm{AM}} \left[\frac{c_{si}^A}{p_{si}^M} \right]^{\sigma_A} \qquad \{a_{si}^{\mathrm{AM}}\}$$

Labor supply:

$$L_s^S = \bar{L}_s - a_s^{\mathrm{LE}} W_s \qquad \{L_s^S\}$$

5.2.5 Market clearing

Market for aggregate consumption:

$$\bar{C}_s \geq a_s^{\mathrm{CC}} W_s \qquad \{p_s^{\mathrm{CC}}\}$$

Market for domestic goods:

$$a_{si}^D Q_{si} \geq a_{si}^{\mathrm{AD}} A_{si} \qquad \{p_{si}^D\}$$

Market for export goods:

$$a_{si}^X Q_{si} \geq X_{si} \qquad \{p_{si}^X\}$$

Market for import goods:

$$M_{si} \geq a_{si}^{\mathrm{AM}} A_{si} \qquad \{p_{si}^M\}$$

Current account (market for foreign exchange):

$$\sum_{s=t}^{T} \sum_i p_s^{\mathrm{RF}} X_{si} \geq \sum_{s=t}^{T} \sum_i p_s^{\mathrm{RF}} M_{si} \qquad \{p^{\mathrm{FX}}\}$$

Labor market:

$$L_s^S \geq \sum_i a_{si}^L Q_{si} \qquad \{p_s^L\}$$

Period utility:

$$W_s \geq a_s^U U \qquad \{p_s^W\}$$

Lifetime utility:

$$M^H \geq p^U U \qquad \{p^U\}$$

Armington goods:
Market for non-energy goods (exc. COK and LIM).

$$A_{si} \geq a_{si}^C \bar{C}_s + \sum_j \bar{a}_{ij}^Q Q_{sj} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad \{p_{si}^A\}_{i \in \mathrm{NENE}}$$

Market for LIM and COK.

$$A_{si} \geq a_{si}^{C}\bar{C}_s + \sum_j \bar{a}_{ij}^{\mathrm{CL}} Q_{sj} + \sum_j \bar{a}_{ij}^{\mathrm{NC}} Q_{sj} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad \{p_{si}^A\}_{i \in \mathrm{CL}}$$

Market for electricity.

$$A_{si} \geq a_{si}^{\mathrm{EC}}\bar{C}_s + \sum_j a_{sij}^{E} Q_{sj} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad \{p_{si}^A\}_{i \in \mathrm{ELE}}$$

Market for emission sources (exc. LIM and COK).

$$A_{si} \geq a_{si}^{\mathrm{EC}}\bar{C}_s + \sum_j a_{sij}^{E} Q_{sj} + \sum_j \bar{a}_{ij}^{\mathrm{NC}} Q_{sj} + \bar{a}_i^I I_s + \bar{a}_i^G G_s \qquad \{p_{si}^A\}_{i \in \mathrm{EC}}$$

Investment goods:

$$J_{s-1} + (1-\delta)K_{s-1} \geq K_s \qquad \{p_s^K\}_{s>t}$$

Market for renting capital:

$$K_s^S \geq \sum_i a_{ti}^K Q_{si} \qquad \{r_s^K\}$$

Government period expenditure:

$$G_s \geq \bar{a}_s^{\mathrm{GL}} G^L \qquad \{p^{\mathrm{GL}}\}$$

Government lifetime expenditure:

$$Y^G \geq p^{\mathrm{GL}} G^L \qquad \{p^{\mathrm{GL}}\}$$

Market for carbon emissions:

$$\mathrm{CA}_s \geq \sum_{i \in E} \gamma_i \mathrm{CNC}_{si} \qquad \{t_s^{\mathrm{CA}}\}$$

Price index of emission sources for combustion purpose:

$$\mathrm{CNC}_{si} \geq a_{si}^{C}\bar{C}_s + \sum_j \bar{a}_{ij}^{\mathrm{CL}} Q_{sj} \qquad \{p_{si}^E\}_{i \in \mathrm{CL}}$$

$$\mathrm{CNC}_{si} \geq a_{si}^{\mathrm{EC}}\bar{C}_s + \sum_j a_{sij}^{E} Q_{sj} \qquad \{p_{si}^E\}_{i \in \mathrm{EC}}$$

5.2.6 Income

Household lifetime income:

$$\begin{aligned} Y^H = & \left[(1-t^A)r_t^K + (1-\delta)p_t^K + p_t^{\mathrm{KA}}\right] K_t - R_T p_T^K K_{T+1} \\ & + \sum_{s=t}^{T} R_s \left[(1-t^I)p_s^L \bar{L}_s + p_s^G T_s\right] \qquad \{Y^H\} \end{aligned}$$

Government period income:

$$\begin{aligned} M_s^G = & \sum_i \left[t_i^L p_s^L a_{si}^L + t_i^K r_s^K a_{si}^K\right] Q_{si} + \sum_i t_i^Q p_{si}^Q Q_{si} \\ & + t^C p_s^{\mathrm{CC}} a_s^{\mathrm{CC}} W_s + t^I p_s^L L_s^S + t^A r_s^K K_s \\ & + \sum_i t_i^M p^{\mathrm{FX}} p_s^{\mathrm{RF}} M_{si} - \sum_i s_i^C p_{si}^A a_{si}^{\mathrm{CC}} \mathrm{CC}_s - p_s^G T_s \qquad \{M_s^G\} \end{aligned}$$

Government lifetime income:

$$Y^G = \sum_{s=t}^{T} R_s Y_s^G \qquad \{Y_s^G\}$$

Terminal conditions on investment:

$$\frac{W_T}{W_{T-1}} = \frac{J_T}{J_{T-1}} \qquad \{K_{T+1}\}$$

References

AIM Project Team (2002) "AIM/Trend Model". National Institute for Environmental Studies, (available at: `http://www-iam.nies.go.jp/aim/index.htm`).

Armington, Paul S. (1969) "A Theory of Demand for Products Distinguished by Place of Production". IMF Staff Papers 16.

Ballard, Charles L. (1990) "Marginal Welfare Cost Calculations: Differential Analysis vs. Balanced–Budget Analysis", *Journal of Public Economics*, Vol. 41, No. 2, pp. 263–267.

Bessho, Shunichiro, Nobuo Akai, and Masayoshi Hayashi (2003) "The Marginal Cost of Public Funds", *Nihon Keizai Kenkyu*, Vol. 47, pp. 1–19. (In Japanese).

Böhringer, Christoph, Andreas Pahlke, and Thomas F. Rutherford (1997) "Environmental Tax Reforms and the Prospect for a Double Dividend", January. Mimeo, (available at: `http://debrue.colorado.edu/`).

Inagaki, Mitsutaka ed. (2002) *Japanese Tax System*: Zaikei Shohosha. (*Zusetsu Nihon no Zaisei, Heisei 14 nendo ban, in Japanese*).

Kuroda, Masahiro, Kazunari Shimpo, Koji Nomura, and Nobuyuki Kobayashi (1997) *KEO database*: Keio University, Institute for Economic and Industry Studies (Keio Economic Observatory). Keio Economic Observatory Monograph Series, In Japanese.

Lau, Morten I., Andreas Pahlke, and Thomas F. Rutherford (2002) "Approximating Infinite-Horizon Models in a Complementarity Format: A Primer in Dynamic General Equilibrium Analysis", *Journal of Economic Dynamics and Control*, Vol. 26, pp. 577–609.

MCAG (1999) *1995 Input-Output Tables*: Management and Coordination Agency Government of Japan. (in Japanese).

MOF ed. (1996) *Japanese Tax System*: Zaikei Shohosha. (*Zusetsu Nihon no Zaisei, Heisei 11 nendo ban, in Japanese*).

MOF (1997) *Fiscal and Financial Statistical Monthly (540)*: MOF (Ministry of Finance). (*Zaisei Kinyu Tokei Geppo, in Japanese*).

——— (2002) *An Outline of Japanese Taxes 2001 · 2002*: Tax Bureau, Ministry of Finance, Japan.

Nansai, Keisuke, Yuichi Moriguchi, and Susumu Tohno (2002) *Embodied Energy and Emission Intensity Data for Japan Using Input-Output Table (3EID) – Inventory Data for LCA –*: Center for Global Environmental Research, National Institute for Environmental Studies, Japan.

NIPSSR (2002) "Projected Population of Japan (January 2002)". National Institute of Population and Security Research, (in Japanse).

NTA (1997) *National Tax Agency Annual Statistics Report: 1995*: National Tax Agency. (*Kokuzeicho Tokei Nenpo Sho: Heisei 7 Nendo Ban, in Japanese*).

Rutherford, Thomas F. (1998) “CES Preferences and Technology: A Practical Introduction”. in “Economic Equilibrium Modeling with GAMS: An Introduction to GAMS/MCP and GAMS/MPSGE (GAMS/MPSGE Solver Manual)”, pp. 89-115,
(available at: `http://www.gams.com/docs/solver/mpsge.pdf`).

Rutherford, Thomas F. and Miles K. Light (2002) “A General Equilibrium Model for Tax Policy Analysis in Colombia: The MEGATAX Model”. ARCHIVOS DE ECONOMÍA, Documento 188.

Rutherford, Thomas F., Miles K. Light, and Gustavo Hernández (2002) “A Dynamic General Equilibrium Model for Tax Policy Analysis in Colombia”. ARCHIVOS DE ECONOMÍA, Documento 189.

Shoven, John B. and John Whalley (1992) *Applying General Equilibrium*, New York: Cambridge University Press.

Yashiro, Naohiro, Takashi Oshio, Masako Ii, Mantaro Matsuya, Yasuhiro Terasaki, Yuuichi Yamagishi, Masayuki Miyamoto, and Yosiaki Igarashi (1997) “Economic Analysis of Aging”, *Keizai Bunseki*, Vol. 151, September. ESRI, (in Japanese).