A Supplement to "Economic Growth and Carbon Emissions with Endogenous Carbon Taxes"

Shiro Takeda*

Department of Economics, Kanto Gakuen University
200 Fujiakucho, Ota city, Gunma, 373–8515, Japan.

e-mail: <zbc08106@park.zero.ad.jp>

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Abstract

This paper is a supplement to the paper "Economic Growth and CO_2 Emissions with Endogenous Carbon Taxes". In this paper, we present the algebraic model description, parameterizations, and GAMS code for the simulation.

Contents

*corresponging author, email: <zbc08106@park.zero.ad.jp>.

Table 1: The list of regions in the model.

Region identifier	Region
USA	United States *
CAN	Canada *
WEU	Western Europe *
JPN	Japan *
AUS	Australia and New Zealand *
EFS	Eastern Europe and Former Soviet Union *
MEX	Mexico
CHN	China
IND	India
ASI	Other Asian countries
MIE	Middle East and Turkey
CSA	Central and Southern America
ROW	Rest of the world

Note: asterisk indicates Annex I regions.

1 Description of the Model

In this appendix, we present the complete model description.

1.1 Overview of the Model

The model is a multisector, multiregion recursive dynamic model. The periods range from 1997 (the benchmark year) to 2020. The structure of the model within a period is based on the GTAP–EG (Rutherford and Paltsev, 2000) and the GTAP standard model (Hertel, 1997).

The world is divided into 13 regions, and each economy has eight production sectors. The lists of regions and sectors of the model are provided in Table 1 and 2. Regions are chosen to be compatible with those employed in *International Energy Outlook 2001* (EIA, 2001, hereafter, EIA dataset). There are six Annex I regions and seven non-Annex I regions. The sectors in the model have been chosen to highlight the following two aspects: (i) the detailed description of energy sectors, and (ii) the difference in energy intensity among non-energy sectors. In the context of (i), the energy sectors are disaggregated into five different sectors: crude oil (CRU), coal (COL), gas (GAS), refined oil (OIL), and electricity (ELE). By distinguishing energy sectors in this way, we can observe the differences in carbon intensity of different energy sources. Moreover, to take account of (ii), non-energy sectors are divided into an energy-intensive sector (EIS), a non-energy-intensive sector (Y), and a savings goods sector (CGD).

We divide production sectors into two broad categories: fossil fuel production sectors (CRU, COL, and GAS) and non-fossil fuel production sectors (all other sectors). We assume that all production functions in all sectors have the nested CES form, but that the fossil fuel and non-fossil fuel sectors have different structures. There are three primary production factors, labor, capital, and fossil fuel resources. Fossil fuel resources are used only in the fossil fuel production sectors. All factors are assumed to be internationally immobile and fossil fuel resources are assumed to be specific to the fossil fuel sectors. All markets in the model are perfectly competitive and equilibrium prices are determined so as to clear all markets.

The demand side of each region is represented by the optimizing behavior of a representative agent. The agent's utility is derived from savings and final goods consumption (final demand),

Table 2: The list of sectors in the model.

Sector identifier	Sector
Y	Other manufactures and services
EIS	Energy-intensive sectors
COL	Coal
OIL	Petroleum and coal products (refined)
CRU	Crude oil
GAS	Natural gas
ELE	Electricity
CGD	Saving goods (investment goods)

which are chosen by the agent to maximize utility subject to the budget constraint. Agent income consists of factor income and tax revenue which is transfered from the government in a lump-sum fashion. Since we include government expenditure in the consumption of the representative agent, government does not appear in the model explicitly.

All regions in this model are connected to each other through international trade in goods. We assume that goods produced in different regions are regarded as being qualitatively distinct (Armington, 1969). It follows that trade in goods is characterized by trade flows between pairs of countries. Moreover, We assume that, to ship goods internationally, it is necessary to use transportation services. The policy instruments in this model appear in the form of taxes and subsidies including consumption taxes, intermediate input taxes, output taxes, export taxes, and import taxes.

Investment behavior is modeled under the rate-of-return assumption in the standard GTAP model (Hertel, 1997, p. 54). That is, regional investment is determined so that changes in the expected rates of return from capital stock are equalized across all regions. Investment in a period is transformed to capital stock in the next period.

In the following subsections, We present a more detailed description of individual components of the model (production structures, demand structure, and so on). For this, we define the following sets:

- $I \cdots$ A set of all goods and sectors.
- FE = COL, GAS, OIL. $\cdots$ Final energy.
- XE = COL, GAS, CRU $\cdots$ Fossil fuels.
- NXE $= I \setminus$ XE $\cdots$ Non-fossil fuels.
- ELE = ELE $\cdots$ Electricity.
- COL = COL. $\cdots$ Coal.
- LQD = OIL, GAS. $\cdots$ Liquidity energy.
- ENE = EC $\cup$ ELE $\cdots$ Energy goods.
- NENE = $I \setminus$ ENE $\cdots$ Non-energy goods.

In Figure **??**, the flows of goods, primary factors, taxes, and emissions are graphically depicted.

1.2 Production Side

Production structure and the specification of elasticity parameters are almost the same as Rutherford and Paltsev (2000). There are five energy production sectors: crude oil (CRU), coal (COL), gas (GAS), refined oil (OIL), and electricity (ELE). Crude oil is produced domestically or imported, and is used to produce refined oil, which is used as an input to production and final demand. Electricity is not

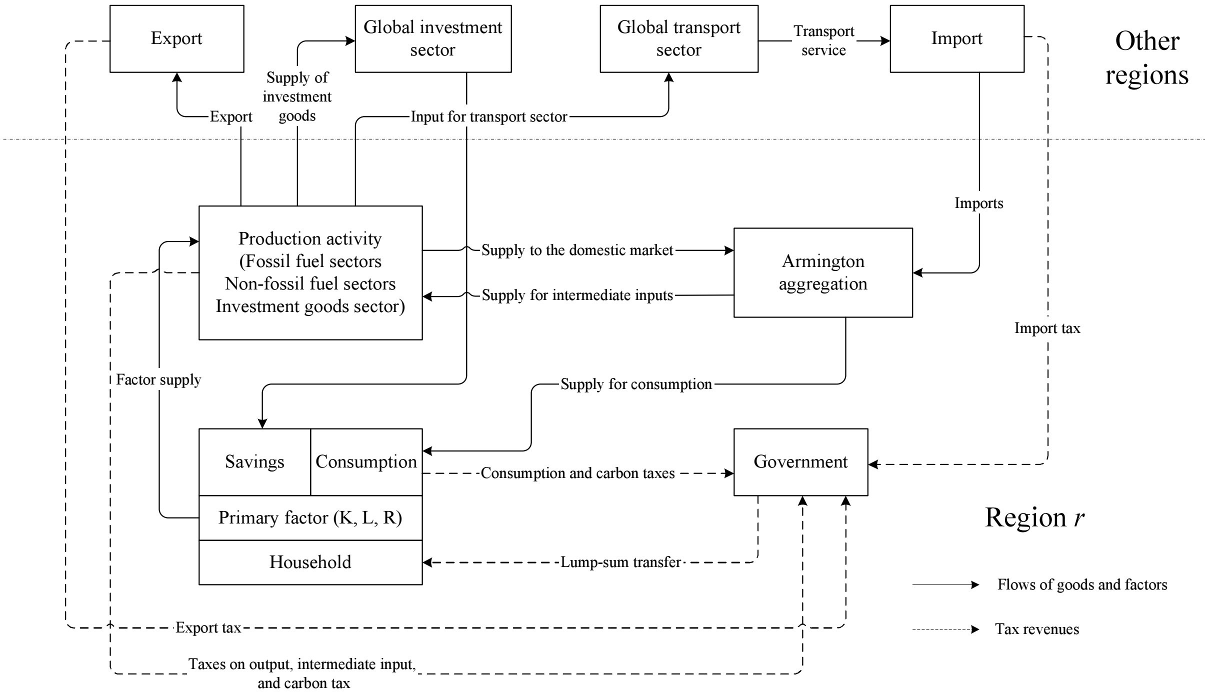

Figure 1: Flows of goods, primary factors, taxes, and emissions

traded and is produced using coal, oil, gas, and non-fossil intermediates. Final energy products (refined oil, gas, and coal) are supplied as inputs to both production and final demand.

All production sectors have a nested CES type structure, reflecting differences in substitutability between various inputs. We also assume that goods produced for the domestic market and goods produced for the export market are differentiated, and that there is a constant elasticity of transformation (CET) relationship between domestic and exported goods.

As already mentioned, the production sectors are divided into two types, fossil fuel and non-fossil fuel sectors, and we assume that these have different production structures. We explain each in turn.

1.2.1 Fossil Fuel Sectors

The fossil fuel production function is assumed to have nested CES structure represented by Figure 2. The input structure is explained as follows. First, in the second stage, labor and intermediate inputs are aggregated into a non-resource input composite through a Leontief function. Second, in the top stage, output of fossil fuel sector $i \in \text{XE}$ is a CES aggregation of non-resource input composite and natural resources with an elasticity of σ_{Rir}.

Let Y_{ir}^{XE} denote the output of fossil fuel sector $i \in \text{XE}$, and let R_{ir} and Q_{ir}^{IL} denote resource input and non-resource input composite respectively. Then the above relation is expressed as follows:

$$\begin{aligned} Y_i^{\text{XE}} &= f_i^{\text{XE}}(R_{ir}, Q_{ir}^{\text{IL}}) \\ &= \left[\alpha_{ir}^{\text{XER}} (R_{ir})^{\frac{\sigma_{Rir}-1}{\sigma_{Rir}}} + (1-\alpha_{ir}^{\text{XER}}) (Q_{ir}^{\text{IL}})^{\frac{\sigma_{Rir}-1}{\sigma_{Rir}}} \right]^{\frac{\sigma_{Rir}}{\sigma_{Rir}-1}} \end{aligned} \tag{1}$$

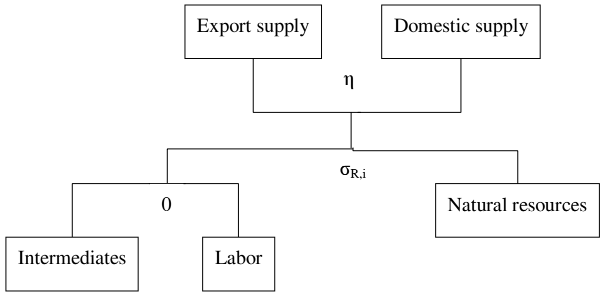

Figure 1: The nesting structure of fossil fuel sector.

Figure 2: Fossil fuel sector.

Since Q_{ir}^{IL} is a Leontief aggregation of labor and intermediate inputs, it is expressed as

$$Q_{ir}^{\mathrm{IL}} = f_{ir}^{\mathrm{IL}}(\{Q_{jir}^{I}\}, L_{ir}) = \min\left[\left\{\frac{Q_{jir}^{I}}{a_{jir}^{I}}\right\}, \frac{L_{ir}}{a_{ir}^{L}}\right]$$

where Q_{jir}^{I} is intermediate input of good j and L_{ir} is labor input.

When the production function is represented by multi-stage CES function, we can consider input choice in each stage separately. Based on this property of CES functions, we define price indices for inputs. First, consider Eq. (1). Producers who try to maximize their profits choose combination of inputs to minimize costs. From this cost minimizing behavior, we can define unit cost for production. Let p_{ir}^{R} denote the price of resource inputs and p_{ir}^{IL} denote the price index of non-resource input composite. Then, the unit cost is given by

$$\begin{aligned} c_{ir}^{Y} &\equiv \min\left[p_{ir}^{R} R + p_{ir}^{\mathrm{IL}} Q^{\mathrm{IL}} \mid f_{ir}^{\mathrm{XE}}(R, Q^{\mathrm{IL}}) = 1\right] \\ &= \left[(\alpha_{ir}^{R})^{\sigma_{Rir}}(p_{ir}^{R})^{1-\sigma_{Rir}} + (1-\alpha_{ir}^{R})^{\sigma_{Rir}}(p_{ir}^{\mathrm{IL}})^{1-\sigma_{Rir}}\right]^{\frac{1}{1-\sigma_{Rir}}} \end{aligned}$$

Similarly, the combination of intermediate inputs and labor is determined so as to minimize cost. Thus, we can express the price index of non-resource input composite as follows:

$$\begin{aligned} p_{ir}^{\mathrm{IL}} &\equiv \min\left[\sum_j \tilde{p}_{Ijir}^{A} Q_j^{I} + p_r^{L} L \mid f_{ir}^{\mathrm{IL}}(\{Q_j^{I}\}, L) = 1\right] \\ &= \sum_j a_{jir}^{I} \tilde{p}_{Ijr}^{A} + a_{ir}^{L} p_r^{L} \end{aligned}$$

where $\tilde{p}_{Ijir}^{A} = (1 + t_{ji}^{I}) p_{jr}^{A}$ is the producer price of intermediate input j and p_r^{L} is the price of labor.

Next, let us consider the output side of the production. We assume that goods produced for domestic use and goods produced for export are differentiated, and that they are allocated through a CET (constant elasticity of transformation) function. Thus, domestic supply D_{ir}, export supply X_{ir}, and output Y_{ir}^{XE} have the following relation.

$$Y_{ir}^{\mathrm{XE}} = f_i^{O}(X_{ir}, D_{ir}) = \left[\alpha_i^{X}(X_{ir})^{\frac{1+\eta}{\eta}} + (1-\alpha_i^{X})(D_{ir})^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}$$

where η denotes constant elasticity of transformation. Given the price of domestic good p_{ir}^{D} and the price of export goods p_{ir}^{X}, profit-maximizing producers allocate output to domestic and export

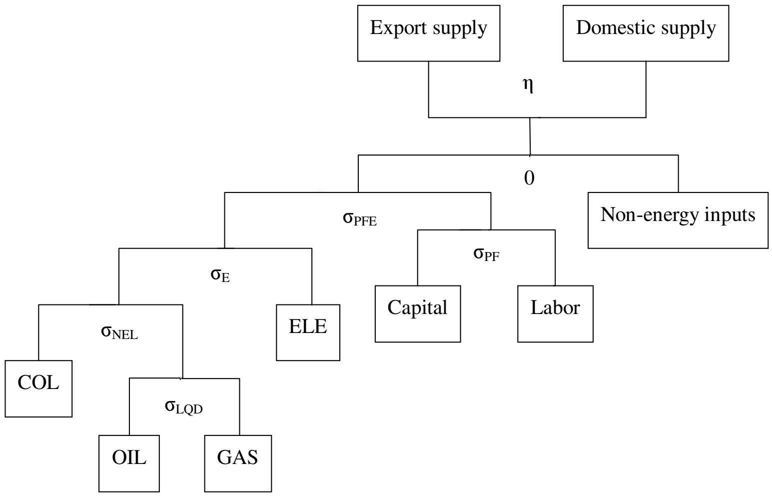

Figure 2: The nesting structure of non-fossil fuel sector.
Figure 3: Production function of non-fossil fuel sector.

markets so as to maximize their revenues. Thus, we can define the price index of output as follows.

$$
\begin{aligned}
p_{ir}^{Y} &\equiv \max_{X,D} \left[p_{ir}^{X} X + p_{ir}^{D} D \mid f_i^{O}(X, D) = 1 \right] \\
&= \left[(\alpha_i^X)^{-\eta} (p_{ir}^X)^{1+\eta} + (1-\alpha_i^X)^{-\eta} (p_{ir}^D)^{1+\eta} \right]^{\frac{1}{1+\eta}}
\end{aligned} \tag{2}
$$

This price index p_{ir}^Y indicates maximum revenues derived from one unit of output. Since we assume that the ad valorem tax t_i^Y is imposed on output, unit revenue for producers is given by $(1 - t_i^Y) p_{ir}^Y$.

From the above arguments, we can express zero profit condition for fossil fuel sector $i \in \text{XE}$ as follows.

$$
(1 - t_i^Y) p_{ir}^Y = c_{ir}^Y
$$

1.2.2 Non-fossil fuel sectors

Non-fossil fuel production (including electricity and refined oil) has a different structure from that of the fossil fuel sectors. Fig. 3 illustrates the nesting and elasticities of substitution employed in non-fossil fuel production sectors. Non-fossil fuel output is produced with fixed coefficient (Leontief) aggregation of non-energy intermediates and an energy-primary factor composite. The energy composite and primary factor composite are aggregated through a CES function with an elasticity of σ_{PFE}. The primary factor composite is a CES aggregation of labor and capital stock with an elasticity of σ_{PF}. The energy composite is a CES aggregation of electricity and a non-electric energy input composite with an elasticity of substitution of σ_E. The non-electric energy composite is a CES aggregation of coal and liquid energy composites. Finally, the liquid energy composite is a CES aggregation of gas and refined oil.

Let Y_{ir}^{NXE} denote output of non-fossil fuel sector $i \in \text{NXE}$ and let Q_{jir}^I and Q_{ir}^{PFE} denote non-energy intermediate inputs and energy-primary factor composite respectively. Then, Y_{ir}^{NXE} has a following relation:

$$
Y_{ir}^{\text{NXE}} = f_{ir}^{\text{NXE}}(\{Q_{jir}^I\}_{j \in \text{NENE}}, Q_{ir}^{\text{PFE}}) = \min \left[\left\{ \frac{Q_{jir}^I}{a_{jir}^I} \right\}_{j \in \text{NENE}}, \frac{Q_{ir}^{\text{PFE}}}{a_{ir}^{\text{PFE}}} \right] \tag{3}
$$

Since Q_{ir}^{PFE} is a CES aggregation of primary factor composite Q_{ir}^{PF} and energy composite Q_{ir}^{EA}, the following relation holds:

$$\begin{aligned} Q_{ir}^{\text{PFE}} &= f_{ir}^{\text{PFE}}(Q_{ir}^{\text{PF}}, Q_{ir}^{\text{EA}}) \\ &= \left[\alpha_{ir}^{\text{PF}}(Q_{ir}^{\text{PF}})^{\frac{\sigma_{\text{PFE}}-1}{\sigma_{\text{PFE}}}} + (1-\alpha_{ir}^{\text{PF}})(Q_{ir}^{\text{EA}})^{\frac{\sigma_{\text{PFE}}-1}{\sigma_{\text{PFE}}}}\right]^{\frac{\sigma_{\text{PFE}}}{\sigma_{\text{PFE}}-1}} \end{aligned} \tag{4}$$

Primary factor composite Q_{ir}^{PF} is a CES aggregation of labor L_{ir} and capital K_{ir}. Thus, we have

$$\begin{aligned} Q_{ir}^{\text{PF}} &= f_{ir}^{\text{PF}}(L_{ir}, K_{ir}) \\ &= \left[\alpha_{ir}^{L}(L_{ir})^{\frac{\sigma_{\text{PF}}-1}{\sigma_{\text{PF}}}} + (1-\alpha_{ir}^{L})(K_{ir})^{\frac{\sigma_{\text{PF}}-1}{\sigma_{\text{PF}}}}\right]^{\frac{\sigma_{\text{PF}}}{\sigma_{\text{PF}}-1}} \end{aligned} \tag{5}$$

On the other hand, energy composite is a CES aggregation of electricity $Q_{\text{ELE},ir}^{\text{EI}}$ and non-electricity energy composite Q_{ir}^{NEL}.

$$\begin{aligned} Q_{ir}^{E} &= f_{ir}^{E}(Q_{\text{ELE},ir}^{\text{EI}}, Q_{ir}^{\text{NEL}}) \\ &= \left[\alpha_{ir}^{\text{ELE}}(Q_{\text{ELE},i,r}^{\text{EI}})^{\frac{\sigma_E-1}{\sigma_E}} + (1-\alpha_{ir}^{\text{ELE}})(Q_{ir}^{\text{NEL}})^{\frac{\sigma_E-1}{\sigma_E}}\right]^{\frac{\sigma_E}{\sigma_E-1}} \end{aligned} \tag{6}$$

Non-electricity energy composite is a CES aggregation of coal $Q_{\text{COL},ir}^{\text{EI}}$ and liquidity energy composite Q_{ir}^{LQD}:

$$\begin{aligned} Q_{ir}^{\text{NEL}} &= f_{ir}^{\text{COL}}(Q_{\text{COL},ir}^{\text{EI}}, Q_{ir}^{\text{LQD}}) \\ &= \left[\alpha_{ir}^{\text{COL}}(Q_{\text{COL},i,r}^{\text{EI}})^{\frac{\sigma_{\text{COL}}-1}{\sigma_{\text{COL}}}} + (1-\alpha_{ir}^{\text{COL}})(Q_{ir}^{\text{LQD}})^{\frac{\sigma_{\text{COL}}-1}{\sigma_{\text{COL}}}}\right]^{\frac{\sigma_{\text{COL}}}{\sigma_{\text{COL}}-1}} \end{aligned} \tag{7}$$

Finally, liquidity energy composite is a CES aggregation of refined oil and gas:

$$Q_{ir}^{\text{LQD}} = f_{ir}^{\text{LQD}}(\{Q_{jir}^{\text{EI}}\}_{j\in\text{LQD}}) = \left[\sum_{j\in\text{LQD}} \alpha_{jir}^{\text{LQD}}(Q_{jir}^{\text{EI}})^{\frac{\sigma_{\text{LQD}}-1}{\sigma_{\text{LQD}}}}\right]^{\frac{\sigma_{\text{LQD}}}{\sigma_{\text{LQD}}-1}}$$

Next, as in fossil fuel sectors, we define unit cost and price indices for non-fossil fuel sectors. First, from (3), the unit cost function for non-fossil fuel sector $i \in \text{NXE}$ is given by

$$\begin{aligned} c_{ir}^{Y} &\equiv \min\left[\sum_{j\in\text{NENE}} \tilde{p}_{Ijir}^{A} Q_j^{I} + p_{ir}^{\text{PFE}} Q^{\text{PFE}} \mid f_{ir}^{\text{NXE}}(\{Q_j^{I}\}_{j\in\text{NENE}}, Q^{\text{PFE}}) = 1\right] \\ &= \sum_{j\in\text{NENE}} a_{jir}^{I} \tilde{p}_{Ijir}^{A} + a_{ir}^{\text{PFE}} p_{ir}^{\text{PFE}} \end{aligned}$$

where $\tilde{p}_{Ijir}^{A} = (1+t_{jir}^{I})p_{jr}^{A}$ is the producer price of intermediate input j and p_{ir}^{PFE} is the price index of energy-primary factor composite.

Next, from (4), the price index of energy-primary factor composite is given by

$$\begin{aligned} p_{ir}^{\text{PFE}} &\equiv \min\left[p_{ir}^{\text{PF}} Q^{\text{PF}} + p_{ir}^{\text{EA}} Q^{E} \mid f_{ir}^{\text{PFE}}(Q^{\text{PF}}, Q^{\text{EA}}) = 1\right] \\ &= \left[(\alpha_{ir}^{\text{PF}})^{\sigma_{\text{PFE}}}(p_{ir}^{\text{PF}})^{1-\sigma_{\text{PFE}}} + (1-\alpha_{ir}^{\text{PF}})^{\sigma_{\text{PFE}}}(p_{ir}^{\text{EA}})^{1-\sigma_{\text{PFE}}}\right]^{\frac{1}{1-\sigma_{\text{PFE}}}} \end{aligned}$$

where p_{ir}^{PF} is the price index of primary factor composite and p_{ir}^{EA} is the price index of energy composite.

From (5), the price index of primary factor composite is derived as follows:

$$\begin{aligned} p_{ir}^{\mathrm{PF}} &\equiv \min\left[p_r^L L + r_r^K K \mid f_{ir}^{\mathrm{PF}}(L,K)=1\right] \\ &= \left[(\alpha_{ir}^L)^{\sigma_{\mathrm{PF}}}(p_r^L)^{1-\sigma_{\mathrm{PF}}} + (1-\alpha_{ir}^L)^{\sigma_{\mathrm{PF}}}(r_r^K)^{1-\sigma_{\mathrm{PF}}}\right]^{\frac{1}{1-\sigma_{\mathrm{PF}}}} \end{aligned}$$

where p_r^L and r_r^K are wage rate and rental price respectively.

On the other hand, the price index of energy composite is derived from (6):

$$\begin{aligned} p_{ir}^{\mathrm{EA}} &\equiv \min\left[\tilde{p}_{I,\mathrm{ELE},ir}^A Q_{\mathrm{ELE}}^{\mathrm{EI}} + p_{ir}^{\mathrm{NEL}} Q^{\mathrm{NEL}} \mid f_{ir}^{\mathrm{EA}}(Q_{\mathrm{ELE}}^{\mathrm{EI}}, Q^{\mathrm{NEL}})=1\right] \\ &= \left[(\alpha_{ir}^{\mathrm{ELE}})^{\sigma_E}(\tilde{p}_{I,\mathrm{ELE},ir}^A)^{1-\sigma_E} + (1-\alpha_{ir}^{\mathrm{ELE}})^{\sigma_E}(p_{ir}^{\mathrm{NEL}})^{1-\sigma_E}\right]^{\frac{1}{1-\sigma_E}} \end{aligned}$$

where $\tilde{p}_{I,\mathrm{ELE},ir}^A = (1+t_{\mathrm{ELE},i,r}^I)p_{\mathrm{ELE},r}^A$ is the producer price of electricity and p_{ir}^{NEL} is the price index of non-electricity energy composite.

Similarly, from (7), p_{ir}^{NEL} is given by

$$\begin{aligned} p_{ir}^{\mathrm{NEL}} &\equiv \min\left[p_{\mathrm{COL},ir}^E Q_{\mathrm{COL}}^{\mathrm{EI}} + p_{ir}^{\mathrm{LQD}} Q^{\mathrm{LQD}} \mid f_{ir}^{\mathrm{NEL}}(Q_{\mathrm{COL}}^{\mathrm{EI}}, Q^{\mathrm{LQD}})=1\right] \\ &= \left[(\alpha_{ir}^{\mathrm{COL}})^{\sigma_{\mathrm{COL}}}(p_{\mathrm{COL},ir}^E)^{1-\sigma_{\mathrm{COL}}} + (1-\alpha_{ir}^{\mathrm{COL}})^{\sigma_{\mathrm{COL}}}(p_{ir}^{\mathrm{LQD}})^{1-\sigma_{\mathrm{COL}}}\right]^{\frac{1}{1-\sigma_{\mathrm{COL}}}} \end{aligned}$$

where $p_{\mathrm{COL},ir}^E$ is the producer price of coal and p_{ir}^{LQD} is the price index of liquidity energy composite.

Finally, from (8), p_{ir}^{LQD} is derived as follows:

$$\begin{aligned} p_{ir}^{\mathrm{LQD}} &\equiv \min\left[\sum_{j\in\mathrm{LQD}} p_{jir}^E Q_j^{\mathrm{EI}} \mid f_{ir}^{\mathrm{LQD}}(\{Q_j^{\mathrm{EI}}\}_{j\in\mathrm{LQD}})=1\right] \\ &= \left[\sum_{j\in\mathrm{LQD}} (\alpha_{jir}^{\mathrm{LQD}})^{\sigma_{\mathrm{LQD}}}(p_{jir}^E)^{1-\sigma_{\mathrm{LQD}}}\right]^{\frac{1}{1-\sigma_{\mathrm{LQD}}}} \end{aligned}$$

where $p_{jir}^E (j\in\mathrm{LQD})$ is the price of liquidity energy.

As to output side, the price index of output is defined in the same way as (2).

1.2.3 Demand and supply

In this section, demands for production inputs are derived. Since we have already defined unit cost function and price indices, we can easily derive demands for inputs by using Shephard's lemma. First, let us consider fossil fuel sectors. Demand for resource inputs is derived by differentiating the cost function with the price of resource inputs:

$$R_{ir}^D = \frac{\partial c_{ir}^Y}{\partial p_{ir}^R} Y_{ir}^{\mathrm{XE}} = \left[\frac{\alpha_{ir}^R c_{ir}^Y}{p_{ir}^R}\right]^{\sigma_{Rir}} Y_{ir}^{\mathrm{XE}} \tag{8}$$

Similarly, demands for intermediate inputs and labor are given by

$$\begin{aligned} Q_{jir}^I &= \frac{\partial p_{ir}^{\mathrm{IL}}}{\partial p_{Ijir}^A}\frac{\partial c_{ir}^Y}{\partial p_{ir}^{\mathrm{IL}}} Y_{ir}^{\mathrm{XE}} = a_{jir}^I \left[\frac{\alpha_{ir}^R c_{ir}^Y}{p_{ir}^{\mathrm{IL}}}\right]^{\sigma_{Rir}} Y_{ir}^{\mathrm{XE}} \\ L_{ir}^D &= \frac{\partial p_{ir}^{\mathrm{IL}}}{\partial p_r^L}\frac{\partial c_{ir}^Y}{\partial p_{ir}^{\mathrm{IL}}} Y_{ir}^{\mathrm{XE}} = a_{ir}^L \left[\frac{\alpha_{ir}^R c_{ir}^Y}{p_{ir}^{\mathrm{IL}}}\right]^{\sigma_{Rir}} Y_{ir}^{\mathrm{XE}} \end{aligned}$$

Next, consider supply functions. As demand functions, we can derive supply functions with Shephard's lemma, that is, differentiating the price index of output (unit revenue function) with supply prices. For example, export supply is given by

$$X_{ir}^{S} = \frac{\partial p_{ir}^{Y}}{\partial p_{ir}^{X}} Y_{ir}^{\text{XE}} = \left[\frac{p_{ir}^{X}}{\alpha_{i}^{X} p_{ir}^{Y}} \right]^{\eta} Y_{ir}^{\text{XE}}$$

Similarly, domestic supply is

$$D_{ir}^{S} = \frac{\partial p_{ir}^{Y}}{\partial p_{ir}^{D}} Y_{ir}^{\text{XE}} = \left[\frac{p_{ir}^{D}}{(1-\alpha_{i}^{X}) p_{ir}^{Y}} \right]^{\eta} Y_{ir}^{\text{XE}}$$

Next, we consider demand and supply of non-fossil fuel sectors. Since the top level of the production function of non-fossil fuel sectors is Leontief technology, demand for non-energy intermediate good j is given by $a_{jir}^{I} Y_{ir}^{\text{NXE}}$. Demand for energy intermediate goods and primary factors are derived as follows.

By Shephard's lemma, demand for primary factors is

$$L_{ir}^{D} = \frac{\partial p_{ir}^{\text{PF}}}{\partial p_{r}^{L}} \frac{\partial p_{ir}^{\text{PFE}}}{\partial p_{ir}^{\text{PF}}} \frac{\partial c_{ir}^{Y}}{\partial p_{ir}^{\text{PFE}}} Y_{ir}^{\text{NXE}} = \left[\frac{\alpha_{ir}^{L} p_{ir}^{\text{PF}}}{p_{r}^{L}} \right]^{\sigma_{\text{PF}}} \left[\frac{\alpha_{ir}^{\text{PF}} p_{ir}^{\text{PFE}}}{p_{ir}^{\text{PF}}} \right]^{\sigma_{\text{PFE}}} a_{ir}^{\text{PFE}} Y_{ir}^{\text{NXE}}$$

$$K_{ir}^{D} = \frac{\partial p_{ir}^{\text{PF}}}{\partial r_{r}^{K}} \frac{\partial p_{ir}^{\text{PFE}}}{\partial p_{ir}^{\text{PF}}} \frac{\partial c_{ir}^{Y}}{\partial p_{ir}^{\text{PFE}}} Y_{ir}^{\text{NXE}} = \left[\frac{(1-\alpha_{ir}^{L}) p_{ir}^{\text{PF}}}{r_{r}^{K}} \right]^{\sigma_{\text{PF}}} \left[\frac{\alpha_{ir}^{\text{PF}} p_{ir}^{\text{PFE}}}{p_{ir}^{\text{PF}}} \right]^{\sigma_{\text{PFE}}} a_{ir}^{\text{PFE}} Y_{ir}^{\text{NXE}}$$

Following the same procedure, we can get demands for energy intermediate goods. First, demand for electricity is given by

$$\begin{aligned} Q_{\text{ELE},ir}^{\text{EI}} &= \frac{\partial p_{ir}^{\text{EA}}}{\partial \tilde{p}_{I,\text{ELE},ir}^{A}} \frac{\partial p_{ir}^{\text{PFE}}}{\partial p_{ir}^{\text{EA}}} \frac{\partial c_{ir}^{Y}}{\partial p_{ir}^{\text{PFE}}} Y_{ir}^{\text{NXE}} \\ &= \left[\frac{\alpha_{ir}^{\text{ELE}} p_{ir}^{\text{EA}}}{\tilde{p}_{I,\text{ELE},ir}^{A}} \right]^{\sigma_E} \left[\frac{(1-\alpha_{ir}^{\text{PF}}) p_{ir}^{\text{PFE}}}{p_{ir}^{\text{EA}}} \right]^{\sigma_{\text{PFE}}} a_{ir}^{\text{PFE}} Y_{ir}^{\text{NXE}} \end{aligned}$$

Demand for coal is given by

$$\begin{aligned} Q_{\text{COL},ir}^{\text{EI}} &= \frac{\partial p_{ir}^{\text{NEL}}}{\partial p_{\text{COL},ir}^{E}} \frac{\partial p_{ir}^{\text{EA}}}{\partial p_{ir}^{\text{NEL}}} \frac{\partial p_{ir}^{\text{PFE}}}{\partial p_{ir}^{\text{EA}}} \frac{\partial c_{ir}^{Y}}{\partial p_{ir}^{\text{PFE}}} Y_{ir}^{\text{NXE}} \\ &= \left[\frac{\alpha_{ir}^{\text{COL}} p_{ir}^{\text{NEL}}}{p_{\text{COL},ir}^{E}} \right]^{\sigma_{\text{COL}}} \left[\frac{\alpha_{ir}^{\text{NEL}} p_{ir}^{\text{EA}}}{p_{ir}^{\text{NEL}}} \right]^{\sigma_E} \left[\frac{(1-\alpha_{ir}^{\text{PF}}) p_{ir}^{\text{PFE}}}{p_{ir}^{\text{EA}}} \right]^{\sigma_{\text{PFE}}} a_{ir}^{\text{PFE}} Y_{ir}^{\text{NXE}} \end{aligned}$$

Finally, demands for liquidity energy intermediate $j \in \text{LQD}$ are

$$\begin{aligned} Q_{jir}^{\text{EI}} &= \frac{\partial p_{ir}^{\text{LQD}}}{\partial p_{jir}^{E}} \frac{\partial p_{ir}^{\text{NEL}}}{\partial p_{ir}^{\text{LQD}}} \frac{\partial p_{ir}^{\text{EA}}}{\partial p_{ir}^{\text{NEL}}} \frac{\partial p_{ir}^{\text{PFE}}}{\partial p_{ir}^{\text{EA}}} \frac{\partial c_{ir}^{Y}}{\partial p_{ir}^{\text{PFE}}} Y_{ir}^{\text{NXE}} \\ &= \left[\frac{\alpha_{jir}^{\text{LQD}} p_{ir}^{\text{LQD}}}{p_{jir}^{E}} \right]^{\sigma_{\text{LQD}}} \left[\frac{(1-\alpha_{ir}^{\text{COL}}) p_{ir}^{\text{NEL}}}{p_{ir}^{\text{LQD}}} \right]^{\sigma_{\text{COL}}} \\ &\qquad \left[\frac{\alpha_{ir}^{\text{NEL}} p_{ir}^{\text{EA}}}{p_{ir}^{\text{NEL}}} \right]^{\sigma_E} \left[\frac{(1-\alpha_{ir}^{\text{PF}}) p_{ir}^{\text{PFE}}}{p_{ir}^{\text{EA}}} \right]^{\sigma_{\text{PFE}}} a_{ir}^{\text{PFE}} Y_{ir}^{\text{NXE}} \end{aligned}$$

Supply functions of non-fossil fuel sectors are the same as those of fossil fuel sectors.

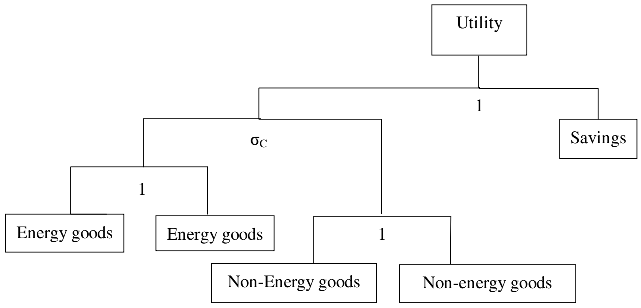

Figure 3: Utility function.

Figure 4: Utility function.

1.3 Demand side

In this section, we explain the demand side. To represent the demand side of each region, we assume a representative agent. The utility of a representative agent in a region depends on consumption and savings and has a structure in Fig 4. Utility is a nested Cobb–Douglas aggregation of savings and final goods consumption. The Cobb–Douglas specification means that the shares of savings and expenditure on goods in total expenditure are constant. Final consumption is a CES aggregation of a non-energy composite and an energy composite with an elasticity of substitution of σ_C. The non-energy composite is a Cobb–Douglas aggregate of non-energy goods, and the energy composite is a Cobb–Douglas aggregate of final energy (refined oil, gas, and coal) and electricity.

The structure of utility function is represented algebraically as follows. Let $\tilde{C}_r$ and S_r denote aggregate consumption and savings respectively. Then, utility of a representative agent in region r is

$$U_r = U(\tilde{C}_r, S_r) = (\tilde{C}_r)^{\alpha_r^C} (S_r)^{1-\alpha_r^C} \tag{9}$$

The aggregate consumption is a CES aggregation of energy consumption composite and non-energy consumption composite. Thus, we have

$$\tilde{C}_r = \tilde{C}_r(C_r^{\text{EC}}, C_r^{\text{CNE}}) = \left[\alpha_r^{\text{EC}} (C_r^{\text{EC}})^{\frac{\sigma_{\text{CC}}-1}{\sigma_{\text{CC}}}} + (1 - \alpha_r^{\text{EC}}) (C_r^{\text{CNE}})^{\frac{\sigma_{\text{CC}}-1}{\sigma_{\text{CC}}}} \right]^{\frac{\sigma_{\text{CC}}}{\sigma_{\text{CC}}-1}} \tag{10}$$

where C_r^{EC} is the energy consumption composite and C_r^{CNE} is the non-energy consumption composite.

Since C_r^{EC} is a Cobb–Douglas aggregation of energy consumption goods, it is represented as follows:

$$C_r^{\text{EC}} = C_r^{\text{EC}}(\{C_{ir}\}_{i \in \text{ENE}}) = \prod_{i \in \text{ENE}} (C_{ir})^{\alpha_{ir}^C} \tag{11}$$

Similarly, C_r^{CNE} is a Cobb–Douglas aggregation of non-energy consumption.

$$C_r^{\text{CNE}} = C_r^{\text{CNE}}(\{C_{ir}\}_{i \in \text{NENE}}) = \prod_{i \in \text{NENE}} (C_{ir})^{\alpha_{ir}^C} \tag{12}$$

1.3.1 Expenditure Minimizing Behavior

From the duality, utility maximizing behavior is captured in terms of expenditure minimizing behavior. Below, we see the agent's behavior in terms of expenditure minimizing behavior. As presented in the previous paragraphs, utility function is a multi-stage CES function. In this case, we can understand optimizing behaviors in each stage separately. That is, we can divide utility maximizing behavior into the following three choices: (1) choice between consumption and savings, (2) choice between energy consumption and non-energy consumption, (3) choice between consumption goods. Using this property, we consider the household behavior in each stage separately.

First, let us consider the top stage. The relation between aggregate consumption $\tilde{C}_r$ and savings S_r is given by (9). Under this relation, utility-maximizing household chooses the combination of aggregate consumption and savings that minimizes expenditure. From this expenditure minimization, we can define the price index for utility as follows:

$$p_r^U \equiv \min \left[p_r^C \tilde{C} + p^S S \mid U(\tilde{C}, S) = 1 \right] = \left[\frac{p_r^C}{\alpha_r^C} \right]^{\alpha_r^C} \left[\frac{p^S}{1 - \alpha_r^C} \right]^{1-\alpha_r^C}$$

where p_r^C is the price index of aggregate consumption and p^S is the price of savings which is uniform across all regions.

From (10), the price index of aggregate consumption is given by

$$\begin{aligned} p_r^C &\equiv \min \left[p_r^{\text{EC}} C^{\text{EC}} + p_r^{\text{CNE}} C^{\text{CNE}} \mid \tilde{C}_r(C^{\text{EC}}, C^{\text{CNE}}) = 1 \right] \\ &= \left[(\alpha_r^{\text{EC}})^{\sigma_{\text{CC}}} (p_r^{\text{EC}})^{1-\sigma_{\text{CC}}} + (1 - \alpha_r^{\text{EC}})^{\sigma_{\text{CC}}} (p_r^{\text{CNE}})^{1-\sigma_{\text{CC}}} \right]^{\frac{1}{1-\sigma_{\text{CC}}}} \end{aligned}$$

Since energy consumption composite is defined by (11), the price index for it is

$$\begin{aligned} p_r^{\text{EC}} &\equiv \min \left[\tilde{p}^A_{C,\text{ELE},r} C_{\text{ELE}} + \sum_{i \in \text{ENE}} p^E_{Cir} C_i \mid C_r^{\text{EC}}(\{C_i\}_{i \in \text{ENE}}) = 1 \right] \\ &= \left[\frac{\tilde{p}^A_{C,\text{ELE},r}}{\alpha^C_{\text{ELE},r}} \right]^{\alpha^C_{\text{ELE},r}} \prod_{i \in \text{FE}} \left[\frac{p_{ir}^{\text{ED}}}{\alpha_{ir}^C} \right]^{\alpha_{ir}^C} \end{aligned}$$

where $\tilde{p}^A_{Cir} = (1 + t^C_{ir}) p^A_{ir}$ is the consumer price of good i and $p_{ir}^{\text{ED}} (i \in \text{FE})$ is the consumer price of final energy.

Similarly, from (12), the price index of non-energy consumption composite is

$$\begin{aligned} p_r^{\text{CNE}} &\equiv \min \left[\sum_{i \in \text{NENE}} \tilde{p}^A_{Cir} C_i \mid C_r^{\text{CNE}}(\{C_i\}_{i \in \text{NENE}}) = 1 \right] \\ &= \prod_{i \in \text{NENE}} \left[\frac{\tilde{p}^A_{Cir}}{\alpha^C_{ir}} \right]^{\alpha^C_{ir}} \end{aligned}$$

Using the price index of utility defined above, utility that the agent achieves is represented by the following relation.

$$U_r = H_r / p_r^U$$

where H_r is the representative agent's income that will be defined later.

1.3.2 Compensated Demand

In this section, demands for consumption and leisure are derived. Since we have so far defined price indices (expenditure functions), we consider compensated demand functions (Hicksian demand functions). Compensated demand functions can be derived by using Shephard's lemma. The approach here is the same as the one in deriving demands for production inputs.

Demand for savings is

$$S_r = \frac{\partial p_r^U}{\partial p_r^S} U_r = \frac{(1 - \alpha_r^C) p_r^U}{p_r^S} U_r$$

Demand for non-energy consumption goods is

$$C_{ir}^{D} = \frac{\partial p_r^{\text{CNE}}}{\partial \tilde{p}_{Cir}^{A}} \frac{\partial p_r^{C}}{\partial p_r^{\text{CNE}}} \frac{\partial p_r^{U}}{\partial p_r^{C}} U_r = \left[\frac{\alpha_{ir}^{C} p_{ir}^{\text{CNE}}}{\tilde{p}_{Cir}^{A}} \right] \left[\frac{(1-\alpha_{ir}^{\text{EC}}) p_{ir}^{C}}{p_{ir}^{\text{CNE}}} \right]^{\sigma_{\text{CC}}} \frac{\alpha_r^{C} p_r^{U}}{p_r^{C}} U_r$$

Consumption demand for electricity is

$$C_{ir}^{D} = \frac{\partial p_r^{\text{EC}}}{\partial \tilde{p}_{\text{C,ELE},r}^{A}} \frac{\partial p_r^{C}}{\partial p_r^{\text{EC}}} \frac{\partial p_r^{U}}{\partial p_r^{C}} U_r = \left[\frac{\alpha_{\text{ELE},r}^{C} p_{ir}^{\text{EC}}}{\tilde{p}_{\text{C,ELE},r}^{A}} \right] \left[\frac{\alpha_{ir}^{\text{EC}} p_{ir}^{C}}{p_{ir}^{\text{EC}}} \right]^{\sigma_{\text{CC}}} \frac{\alpha_r^{C} p_r^{U}}{p_r^{C}} U_r$$

Consumption demand for final energy ($i \in \text{FE}$) is

$$C_{ir}^{D} = \frac{\partial p_r^{\text{EC}}}{\partial p_{Cir}^{E}} \frac{\partial p_r^{C}}{\partial p_r^{\text{EC}}} \frac{\partial p_r^{U}}{\partial p_r^{C}} U_r = \left[\frac{\alpha_{ir}^{C} p_{ir}^{\text{EC}}}{p_{Cir}^{E}} \right] \left[\frac{\alpha_{ir}^{\text{EC}} p_{ir}^{C}}{p_{ir}^{\text{EC}}} \right]^{\sigma_{\text{CC}}} \frac{\alpha_r^{C} p_r^{U}}{p_r^{C}} U_r$$

1.4 International trade

Like other CGE analyses, we use the Armington assumption (Armington, 1969). The Armington assumption implies that goods produced in different regions are imperfect substitutes. We assume that goods produced in different regions are aggregated through a CES function. This aggregation is conducted in two stages. First, imports from different regions are aggregated into an import composite. Second, an import composite and domestic goods are aggregated into a composite (an Armington composite).

Let M_{ir} denote the import composite of region r and let MM_{isr} denote the import from region s to region r. Then, M_{ir} is represented as follows:

$$M_{ir} = M_{ir}(\{\text{MM}_{isr}\}) = \left[\sum_{s} \alpha_{isr}^{\text{MM}} (\text{MM}_{isr})^{\frac{\sigma_M - 1}{\sigma_M}} \right]^{\frac{\sigma_M}{\sigma_M - 1}}$$

Similarly, the Armington composite A_{ir} is expressed as

$$A_{ir} = A_{ir}(D_{ir}, M_{ir}) = \left[\alpha_{ir}^{\text{AD}} (D_{ir})^{\frac{\sigma_A - 1}{\sigma_A}} + (1-\alpha_{ir}^{\text{AD}})(M_{ir})^{\frac{\sigma_A - 1}{\sigma_A}} \right]^{\frac{\sigma_A}{\sigma_A - 1}}$$

where D_{ir} is the amount of domestic goods. Armington goods are used both for intermediate inputs and final consumption.

It is assumed that the combination of imports from different regions are chosen so as to minimize cost. Thus, we can define the price index of import composite as follows:

$$\begin{aligned} p_{ir}^{M} &\equiv \min \left[\sum_{s} p_{isr}^{\text{MM}} \text{MM}_{is} \mid M_{ir}(\{\text{MM}_{is}\}) = 1 \right] \\ &= \left[\sum_{s} (\alpha_{isr}^{\text{MM}})^{\sigma_M} (p_{isr}^{\text{MM}})^{1-\sigma_M} \right]^{\frac{1}{1-\sigma_M}} \end{aligned}$$

where p_{isr}^{MM} is the price of import from region s to region r. This import price includes export tax, import tax and transport cost. Let t_{isr}^{X} and t_{isr}^{M} denote export tax and import tax imposed on good i from region s to region r. Moreover, let τ_{isr} denote the amount of transport services required to ship one unit of good i from region s to region r and let p^T denote the price of transport services. Then, p_{isr}^{MM} is written as follows:

$$p_{isr}^{\text{MM}} = (1 + t_{isr}^{M})[(1 + t_{isr}^{X}) p_{is}^{X} + p^T \tau_{isr}]$$

Similarly, we assume that domestic goods and import composite are chosen to minimize cost. Thus, the price index of Armington goods is defined as follows:

$$\begin{aligned} p_{ir}^{A} &\equiv \min \left[p_{ir}^{D} D + p_{ir}^{M} M | A_{ir}(D, M) = 1 \right] \\ &= \left[\alpha_{ir}^{\text{AD}} (p_{ir}^{D})^{1-\sigma_A} + (1-\alpha_{ir}^{\text{AD}})(p_{ir}^{M})^{1-\sigma_A} \right]^{\frac{1}{1-\sigma_A}} \end{aligned}$$

From the price indices defined above, we can derive demand functions for domestic goods and imports. First, region r's demand for import of good i from region s is

$$\text{MM}_{isr}^{D} = \frac{\partial p_{ir}^{M}}{\partial p_{isr}^{\text{MM}}} M_{ir} = \left[\frac{\alpha_{isr}^{\text{MM}} p_{ir}^{M}}{p_{isr}^{\text{MM}}} \right]^{\sigma_M} M_{ir}$$

Demands for domestic goods and import composite are given by

$$\begin{aligned} D_{ir}^{\text{AD}} &= \frac{\partial p_{ir}^{A}}{\partial p_{ir}^{D}} A_{ir} = \left[\frac{\alpha_{ir}^{\text{AD}} p_{ir}^{A}}{p_{ir}^{D}} \right]^{\sigma_A} A_{ir} \\ M_{ir}^{D} &= \frac{\partial p_{ir}^{A}}{\partial p_{ir}^{M}} A_{ir} = \left[\frac{(1-\alpha_{ir}^{\text{AD}}) p_{ir}^{A}}{p_{ir}^{M}} \right]^{\sigma_A} A_{ir} \end{aligned}$$

Finally, demand for transport services is given by

$$T_{isr}^{D} = \tau_{isr} \text{MM}_{isr}$$

1.5 International transport sector

International transport sector supplies transport services. Transport services are produced from export goods supplied from different regions under a Leontief technology. Let Y^T is the amount of transport services and X_{ir}^{T} is the export good i from region r used to produce transport services. Then, we have

$$Y^T = \min \left[\left\{ \frac{X_{ir}^{T}}{a_{ir}^{T}} \right\} \right]$$

From this, the price of transport services is expressed as follows:

$$p^T = \sum_{i,r} a_{ir}^{T} p_{ir}^{X}$$

1.6 Carbon emissions and carbon taxes

We assume that carbon emissions are generated only from final energies (that is, OIL, COL, and GAS) and that they are generated only from final consumption and intermediate inputs in non-fossil fuel sectors. This means that carbon emissions are not generated from the consumption of CRU and from intermediate inputs in fossil fuel sectors.

Let δ_{jir}^{I} denote the amount of carbon emissions in sector $i \in$ NXE per unit of intermediate input $j \in$ FE and let t_r^{CA} denote carbon tax. Then, the price of final energy j that sector i faces is given by

$$p_{jir}^{E} = (1 + t_{jir}^{I}) p_{jr}^{A} + \delta_{jir}^{I} t_r^{\text{CA}}$$

The price of final energy is the sum of the price of Armington goods and carbon tax.

Similarly, let δ^{ED}_{ir} denote the amount of carbon emissions per unit of final consumption $i \in \text{FE}$. Then, the consumer price of final energy i is given by

$$p^{\text{ED}}_{ir} = (1 + t^{\text{C}}_{ir})p^{A}_{ir} + \delta^{\text{C}}_{ir} t^{\text{CA}}_{r}$$

Total carbon emissions in region r are given by

$$\text{CA}^{T}_{r} = \sum_{j \in \text{FE}} \left[\sum_{i \in \text{NXE}} \delta^{I}_{jir} Q^{\text{EI}}_{jir} + \delta^{\text{ED}}_{jr} C^{D}_{jr} \right]$$

1.7 Investment

Since our model is an open model and we assume that money can move freely across regions, investment and savings in a region need not be equalized. As already described, savings are determined through utility maximization of a representative agent. On the other hand, we assume that regional investment (investment in a region) is determined in the same way as the standard GTAP model (Hertel, 1997, p. 54). That is, regional investment is determined so that the rate of changes in the expected rate of return from capital stock are equalized across regions.[1] In the rate-of-return model of GTAP, regional investment and the world investment are determined as follows.

First, we define the end-of-period capital stock of region r.

$$K^{E}_{r} = (1 - \phi_r)K_r + I_r = K_r + I^{N}_{r}$$

where ϕ_r is the depreciation rate, I_r and $I^{N}_{r} = I_r - \phi_r K_r$ are the gross and net investment of region r respectively. Note that I_r is equal to $Y_{\text{CGD},r}$.

Next, we define the current net rate of return on fixed capital in region r:

$$r^{C}_{r} = \frac{r^{K}_{r}}{p^{I}_{r}} - \phi_r$$

where p^{I}_{r} is the price of investment goods in region r and is equal to $p^{D}_{\text{CGD},r}$.

We assume that investors are cautious in assessing the effects of net investment in a region. They behave as if they expect that region's rate of return in the next period (r^{E}_{r}) to decline with positive additions to the capital stock. The rate at which this decline is expected is a function of the flexibility parameter $\beta_r > 0$. The expected net rate of return on capital stock in region r:

$$r^{E}_{r} = r^{C}_{r} \left[\frac{K^{E}_{r}}{K_r} \right]^{-\beta_r}$$

Then, we assume that regional investment is determined so that the rates of change in the expected rates of return on capital stock are equalized in all regions:

$$r^{G} = \xi_r r^{E}_{r}$$

where r^{G} is the global rate of return. r^{G} is determined so that the value of global saving is equal to the value of global investment:

$$p^{S} I^{G} = \sum_{r} p^{I}_{r} I^{N}_{r}$$

where global net investment is the sum of regional net investment.

$$I^{G} = \sum_{r} I^{N}_{r}$$

[1] In the standard GTAP model, there is another type of investment decision. That is, regional investment is determined so that share of each region's capital stock is fixed. In this chapter, we do not employ this approach.

1.8 Market clearing conditions

In this section, equilibrium conditions for goods and factor markets are presented.

1.8.1 Markets for domestic goods

Domestic goods are supplied by production sectors and demanded by Armington aggregation activity.

$$D_{ir} = D_{ir}^{\mathrm{AD}}$$

1.8.2 Markets for export goods

Export goods are supplied by production sectors and demanded by other regions and global transport sector.

$$X_{ir} = \sum_{s} \mathrm{MM}_{irs}^{D} + a_{ir}^{T} T$$

1.8.3 Markets for import composite

Market for import composite:

$$M_{ir} = M_{ir}^{D}$$

1.8.4 Markets for Armington goods

Market for non-energy goods ($i \in \mathrm{NENE}$):

$$A_{ir} = \sum_{j \in \mathrm{XE}} Q_{ijr}^{I} + \sum_{j \in \mathrm{NXE}} a_{ijr}^{I} Y_{jr} + C_{ir}^{D} \qquad i \in \mathrm{NENE}$$

Market for energy goods ($i \in \mathrm{ENE}$):

$$A_{ir} = \sum_{j \in \mathrm{XE}} Q_{ijr}^{I} + \sum_{j \in \mathrm{NXE}} Q_{ijr}^{\mathrm{EI}} + C_{ir}^{D} \qquad i \in \mathrm{NENE}$$

Market for transport services:

$$Y^{T} = \sum_{i,r,s} T_{irs}^{D}$$

Market for labor. Labor is supplied by the representative agent and demanded by production sectors:

$$L_{r} = \sum_{i} L_{ir}^{D}$$

Market for renting capital. Capital is supplied by the representative agent and demanded by production sectors:

$$K_{r} = \sum_{i \in \mathrm{NXE}} K_{ir}^{D}$$

Market for natural resources. Note that natural resources are used only in fossil fuel sectors and they are sector-specific. Thus, their prices come to differ across sectors:

$$R_{ir} = R_{ir}^{D}$$

Market for savings. Savings goods are supplied by global investment sector and demanded by each region:

$$I^G = \sum_r S_r$$

1.9 Income

Income of a representative agent is the sum of factor income and tax revenue minus depreciation.

$$\begin{aligned} H_r = & \, p_r^L L_r + r_r^K K_r + \sum_{i \in \mathrm{XE}} p_{ir}^R R_{ir} \\ & + \sum_{i \in \mathrm{XE}} \sum_j t_{jir}^I p_{jr}^A Q_{jir}^I + \sum_{i \in \mathrm{NXE}} \sum_{j \in \mathrm{NENE}} t_{jir}^I p_{jr}^A Q_{jir}^I \\ & + \sum_{i,s} t_{irs}^X p_{ir}^X \mathrm{MM}_{irs}^D + \sum_{i,s} t_{isr}^M [(1 + t_{isr}^X) p_{ir}^X + p^T \tau_{isr}] \mathrm{MM}_{isr}^D \\ & + \sum_{i \in \mathrm{XE}} t_{ir}^Y p_{ir}^Y Y_{ir}^{\mathrm{XE}} + \sum_{i \in \mathrm{NXE}} t_{ir}^Y p_{ir}^Y Y_{ir}^{\mathrm{NXE}} + \sum_i t_{ir}^C p_{ir}^A C_{ir}^D + t_r^{\mathrm{CA}} \mathrm{CA}_r^T \\ & - p_r^I \phi_r K_r \end{aligned}$$

1.10 Recursive structure of the model

Our model is a recursive one and solved successively for each period. In each period, the model is solved according to the following procedure:

[1] Given the beginning-of-period capital stock K_r, the model for that period is solved.

[2] Then, we can calculate the end-of-period capital stock K_r^E as follows:

$$K_r^E = (1 - \phi_r) K_r + I_r$$

[3] Set K_r^E to the beginning-of-period capital stock in the next period and solve the model for the next period.

Following the same procedure, equilibria in subsequent periods are solved successively over time span of our model (1997 to 2020).

2 Determination of $\hat{a}$ and $\hat{b}$

In Section **??**, we have explained the way for calculating $\hat{a}$ and $\hat{b}$. Here, we explain it in more details.

We assume that the following relation holds between the level of carbon tax and per capita income:

$$\begin{aligned} t_r^{\mathrm{CA}} &= a + b y_r \\ y_r &= Y_r / (p_r^U n_r) \end{aligned}$$

where n_r is the population and y_r is real per capita income.

In the simulation, a and b are estimated by OLS using the values of t_r^{CA} and y_r realized in 2010 and then the following equation is incorporated into the model.

$$t_r^{\mathrm{CA}} = \hat{a} + \hat{b} Y_r / (p_r^U n_r) + \hat{e}_r$$

This relation determines carbon tax endogenously according to per capita income level.

Table 3: Population (mil.)

	1997	2000	2005	2010	2015	2020
USA	268.4	275.6	288.0	300.2	312.6	325.2
CAN	30.3	31.1	32.6	33.9	35.3	36.6
WEU	386.4	388.2	389.7	389.1	387.3	384.7
JPN	126.0	126.7	127.5	127.3	126.1	126.7
AUS	26.1	26.9	28.1	29.3	30.4	31.5
EFS	413.0	412.8	413.1	414.5	415.3	414.2
MEX	94.3	98.9	106.1	112.9	119.2	125.0
CHN	1,244.2	1,277.6	1,326.4	1,372.9	1,417.7	1,454.5
IND	966.2	1,013.7	1,087.5	1,152.2	1,211.7	1,272.2
ASI	910.7	965.5	1,049.7	1,131.7	1,212.2	1,288.1
MIE	229.4	244.5	268.4	295.4	322.9	349.9
CSA	397.5	416.3	447.4	477.9	507.6	535.7
ROW	731.4	784.4	875.9	973.3	1,077.8	1,187.4

Source: EIA (2001).

3 Parameters

In this section, we explain parameters used in the model. Parameters here mean variables which are not determined endogenously in the model. Most of the parameters are given exogenously, but some of them are calibrated. Parameters presented here are:

[1] Population.

[2] Labor growth rate.

[3] Technology growth rate.

[4] Elasticity parameters.

[5] Other parameters (depreciation rate etc.).

3.1 Population

In the simulation, we consider per capita income in each region. To derive per capita income, it is necessary to project population of each region. For this, we use the population projection in EIA dataset. Projected population is displayed in Table 3[2].

3.2 Labor growth

Labor endowment of each region grows at the exogenously given growth rate. We assume that the growth rate of labor endowment is equal to the population growth rate. Table 4 displays the annual growth rates of population and labor.

3.3 Technology growth

We assume that technology improvement is modeled as labor and capital augmented technology change and that this technology improvement occurs only in non-fossil fuel sectors. Moreover, we assume that technology improvement is uniform across all non-fossil fuel sectors in a region and

[2]The spreadsheet of the dataset in EIA (2001) is available at EIA (Energy Information Agency) website. `http://www.eia.doe.gov/`.

Table 4: Annual growth rate of population (labor) (%)

	1997–99	2000–04	2005–09	2010–14	2015–19
USA	0.89	0.88	0.83	0.81	0.80
CAN	0.97	0.89	0.83	0.80	0.75
WEU	0.16	0.08	−0.03	−0.09	−0.14
JPN	0.18	0.12	−0.02	−0.20	0.10
AUS	0.95	0.89	0.82	0.78	0.73
EFS	−0.02	0.01	0.07	0.04	−0.05
MEX	1.60	1.43	1.24	1.09	0.95
CHN	0.89	0.75	0.69	0.64	0.51
IND	1.61	1.42	1.16	1.01	0.98
ASI	1.97	1.69	1.52	1.38	1.22
MIE	2.14	1.89	1.93	1.80	1.62
CSA	1.55	1.45	1.33	1.21	1.08
ROW	2.36	2.23	2.13	2.06	1.96

Source: EIA (2001).

Table 5: Growth rate of technology parameter ζ_r (%)

	%
USA	1.3
CAN	0.9
WEU	1.4
JPN	0.4
AUS	0.8
EFS	3.0
MEX	1.9
CHN	3.3
IND	1.9
ASI	1.2
MIE	1.8
CSA	1.5
ROW	1.2

that the rate of technology improvement is constant over time span of the model. In the production function below, technology improvement is represented by the increase in ζ_r.

$$Q_{ir}^{\text{PF}} = \left[\alpha_{ir}^{L} (\zeta_r L_{ir})^{\frac{\sigma_{\text{PF}}-1}{\sigma_{\text{PF}}}} + (1-\alpha_{ir}^{L})(\zeta_r K_{ir})^{\frac{\sigma_{\text{PF}}-1}{\sigma_{\text{PF}}}} \right]^{\frac{\sigma_{\text{PF}}}{\sigma_{\text{PF}}-1}}$$

The rate of growth in ζ_r is determined so that the GDP growth rates derived from the model become close to the GDP growth rates projected in EIA dataset (EIA, 2001). The rate of technology growth is given in Table 5.

3.4 Elasticity of substitution

Table 6 presents values of elasticity of substitution. Most of them are taken from Rutherford and Paltsev (2000). As to σ_{NEL} (elasticity of substitution between coal and liquidity energy) and σ_{LQD} (elasticity of substitution between oil and gas), we use slightly higher values than the original values. They are adjusted so that emission shares of final energy derived from our model become close to projected shares in EIA dataset.

Table 6: Value of elasticity parameters*

Notation	Description	Value
η	EOT between domestic supply and export supply	4
σ_U	EOS between consumption and savings	1
σ_C	EOS between energy and non-energy consumption goods	0.5
σ_{CNE}	EOS between non-energy consumption goods	1
σ_{CE}	EOS between energy consumption goods	1
σ_{PFE}	EOS between primary factor-energy composite	0.5
σ_{PF}	EOS between primary factors	1
σ_E	EOS between electricity and non-electricity	0.1
σ_{NEL}	EOS between coal and liquidity energy	1.5
σ_{LQD}	EOS between oil and gas	4
σ_{Rir}	EOS between fossil fuel resources and other inputs in fossil fuel sectors†	
σ_A	EOS between domestic and import goods	4
σ_M	EOS between imports from different regions	8
σ_T	EOS between inputs in international transport sector	0

∗ EOS is elasticity of substitution and EOT is elasticity of transformation.
† σ_{Rir} is calibrated from fossil fuel supply elasticity.

3.5 Calibration of elasticity of substitution between resource input and non-resource inputs

Elasticity of substitution between fossil fuel resources and non-resource inputs in fossil fuel sectors ($\sigma_{R,i}$, i = CRU, COL, GAS) is calibrated from a benchmark value of the fossil fuel supply elasticity (ε_i^S). Calibration is conducted in the following way.

First, the supply elasticity of fossil fuel is defined by

$$\varepsilon_{ir}^S \equiv \frac{\partial Y_{ir}^{\text{XE}}}{\partial p_{ir}^Y} \frac{p_{ir}^Y}{Y_{ir}^{\text{XE}}} = \frac{\partial Y_{ir}^{\text{XE}}}{\partial \tilde{p}_{ir}^Y} \frac{\tilde{p}_{ir}^Y}{Y_{ir}^{\text{XE}}} \qquad i \in \text{XE} \tag{13}$$

where $\tilde{p}^Y \equiv (1 - t^Y) p^Y$.

From the chain rule, we have

$$\frac{\partial Y_{ir}^{\text{XE}}}{\partial \tilde{p}_{ir}^Y} = \frac{\partial Y_{ir}^{\text{XE}}}{\partial Q_{ir}^{\text{IL}}} \frac{\partial Q_{ir}^{\text{IL}}}{\partial \tilde{p}_{ir}^Y} \tag{14}$$

To obtain the value of the RHS, we use the condition of profit maximization. The production function of fossil fuel is defined as follows (subscript i and r are omitted for notational simplification).

$$Y^{\text{XE}} = \left[\alpha^{\text{XER}} (R)^{\frac{\sigma_R - 1}{\sigma_R}} + (1 - \alpha^{\text{XER}}) (Q^{\text{IL}})^{\frac{\sigma_R - 1}{\sigma_R}} \right]^{\frac{\sigma_R}{\sigma_R - 1}} \tag{15}$$

From this, we can derive the condition of profit maximization for fossil fuel sectors.

$$\tilde{p}^Y \frac{\partial Y^{\text{XE}}}{\partial Q^{\text{IL}}} = p^{\text{IL}} \tag{16}$$

Given the price of fossil fuel $\tilde{p}^Y$, the price index of non-resource input p^{IL}, and resource input R, this condition determines the optimal input of non-fossil fuel Q^{IL}. Totally differentiating (16), we have

$$\frac{\partial Y^{\text{XE}}}{\partial Q^{\text{IL}}} d\tilde{p}^Y + \frac{\partial^2 Y^{\text{XE}}}{\partial (Q^{\text{IL}})^2} dQ^{\text{IL}} = 0$$

From this, $\partial Q^{\text{IL}}/\partial p^Y$ is given by

$$\frac{\partial Q^{\text{IL}}}{\partial \tilde{p}^Y} = -\frac{\partial Y^{\text{XE}}/\partial Q^{\text{IL}}}{\tilde{p}^Y \partial^2 Y^{\text{XE}}/\partial (Q^{\text{IL}})^2} \tag{17}$$

(17) includes the first and second derivatives of Y^{XE}. Two derivatives can be derived from (15), but we use the calibrated share form of the production function instead of the original production function. The calibrated share form associated to (15) is given by

$$Y^{\text{XE}} = \bar{Y}^{\text{XE}} \left[\theta^{\text{XER}} \left(\frac{R}{\bar{R}} \right)^{\frac{\sigma_R - 1}{\sigma_R}} + (1 - \theta^{\text{XER}}) \left(\frac{Q^{\text{IL}}}{\bar{Q}^{\text{IL}}} \right)^{\frac{\sigma_R - 1}{\sigma_R}} \right]^{\frac{\sigma_R}{\sigma_R - 1}} \tag{18}$$

where a bar over a variable denotes the benchmark value of that variable and θ^{XER} is the benchmark share of resource inputs in total cost:

$$\theta^{\text{XER}} \equiv \frac{\bar{p}^R \bar{R}}{\bar{p}^R \bar{R} + \bar{p}^{\text{IL}} \bar{Q}^{\text{IL}}}$$

From (18), the first and second derivatives are derived as follows.

$$\begin{aligned}
\frac{\partial Y^{\text{XE}}}{\partial Q^{\text{IL}}} &= (\bar{Y}^{\text{XE}})^{\frac{\sigma_R - 1}{\sigma_R}} (Y^{\text{XE}})^{\frac{1}{\sigma_R}} (1 - \theta^{\text{XER}}) \left[\frac{Q^{\text{IL}}}{\bar{Q}^{\text{IL}}} \right]^{-\frac{1}{\sigma_R}} \frac{1}{\bar{Q}^{\text{IL}}} \\
\frac{\partial^2 Y^{\text{XE}}}{\partial (Q^{\text{IL}})^2} &= (\bar{Y}^{\text{XE}})^{\frac{2(\sigma_R - 1)}{\sigma_R}} (Y^{\text{XE}})^{\frac{2 - \sigma_R}{\sigma_R}} \frac{(1 - \theta^{\text{XER}})^2}{\sigma_R} \left[\frac{Q^{\text{IL}}}{\bar{Q}^{\text{IL}}} \right]^{-\frac{2}{\sigma_R}} \left[\frac{1}{\bar{Q}^{\text{IL}}} \right]^2 \\
&\quad - (\bar{Y}^{\text{XE}})^{\frac{\sigma_R - 1}{\sigma_R}} (Y^{\text{XE}})^{\frac{1}{\sigma_R}} \frac{1 - \theta^{\text{XER}}}{\sigma_R} \left[\frac{Q^{\text{IL}}}{\bar{Q}^{\text{IL}}} \right]^{-\frac{1 + \sigma_R}{\sigma_R}} \left[\frac{1}{\bar{Q}^{\text{IL}}} \right]^2
\end{aligned}$$

Since we want to derive the value of supply elasticity at the benchmark, we evaluate the above equations with the benchmark values. So, we set $Y^{\text{XE}} = \bar{Y}^{\text{XE}}$, $R = \bar{R}$, and $Q^{\text{IL}} = \bar{Q}^{\text{IL}}$:

$$\frac{\partial Y^{\text{XE}}}{\partial Q^{\text{IL}}} = \bar{Y}^{\text{XE}} (1 - \theta^{\text{XER}}) \frac{1}{\bar{Q}^{\text{IL}}} \tag{19}$$

$$\frac{\partial Y^{\text{XE}}}{\partial Q^{\text{IL}}} = -\bar{Y}^{\text{XE}} \frac{(1 - \theta^{\text{XER}})}{\sigma_R} \left[\frac{1}{\bar{Q}^{\text{IL}}} \right]^2 \theta^{\text{XER}} \tag{20}$$

From (13), (14), and (17), we have

$$\varepsilon^S = \frac{\tilde{p}^Y}{Y^{\text{XE}}} \frac{\partial Y^{\text{XE}}}{\partial Q^{\text{IL}}} \frac{\partial Q^{\text{IL}}}{\partial \tilde{p}^Y} = -\frac{1}{Y^{\text{XE}}} \frac{\left[\frac{\partial Y^{\text{XE}}}{\partial Q^{\text{IL}}} \right]^2}{\frac{\partial^2 Y^{\text{XE}}}{\partial (Q^{\text{IL}})^2}}$$

Inserting (19) and (20) into the above equations, the benchmark value of supply elasticity is derived.

$$\varepsilon^S = \frac{(1 - \theta^{\text{XER}}) \sigma_R}{\theta^{\text{XER}}}$$

Solving this equation with respect to σ_R, we have

$$\sigma_{Rir} = \frac{\theta_{ir}^{\text{XER}}}{1 - \theta_{ir}^{\text{XER}}} \varepsilon_i^S$$

Given θ_{ir}^{XER} and ε_i^S, we can calibrate the value of σ_{Rir} from this relation.

The benchmark shares of resource inputs θ_{ir}^{XER} are derived from the benchmark dataset. As the benchmark supply elasticity, we assume the values in Table 7. The benchmark values of the fossil fuel supply elasticity ε_i^S are determined so that carbon emissions of each regions become close to projected emissions in EIA dataset.

Table 7: Benchmark price elasticity of fossil fuel supply

Fossil fuel	Elasticity
CRU	0.6
COL	2.2
GAS	0.1

Table 8: Other parameters

Notation	Description	Value
ϕ_r	Depreciation rate	0.04
β_r	Flexibility parameter	10

3.6 Other parameters

There are two more parameters we need to determine. That is, the depreciation rate ϕ_r and flexibility parameter β_i. For these parameters, we assume values in Table 8. These values are taken from the standard GTAP model.

4 Overview of the Benchmark Dataset

For the benchmark dataset, we use a global economic-energy dataset, GTAP–EG provided by Rutherford and Paltsev (2001). We aggregate the GTAP–EG dataset to eight sectors and 13 regions, using the aggregation routine program provided by GTAP–EG. The benchmark year for the data is 1997. In this section, we overview this benchmark dataset.

4.1 GDP

Table 9 reports value and share of GDP in the benchmark year.

4.2 Emission data in the benchmark year

Table 10 reports carbon emissions at the benchmark year by region and by sector. It shows that carbon emissions from Annex I regions and Non-Annex I regions in 1997 are $3,681$ and $2,318$ MtC respectively and the world total emissions are $5,999$ MtC. It also shows that the most carbon generating sector is electricity (ELE) and that emissions from Annex I regions account for 60% in total. Table 9 also includes carbon intensity of each region (that is, the amount of carbon emissions (kg) required to generate one US$ dollar of GDP). From the table, we can see that Annex I regions tend to have the lower carbon intensity than Non-Annex I regions. This is partly because Annex I regions have more efficient production technology that non-Annex I regions.

Figure 5 and Table 11 report the volume and share of carbon emissions by source. They show that non-Annex I regions tend to use more carbon intensive energy (that is, coal) than Annex I regions. This also explains why Annex I regions tend to have the lower carbon intensity than Non-Annex I regions.

Table 9: GDP at the benchmark year

	Value (bil. $)	Share (%)
USA	7,916.2	29.9
CAN	545.4	2.1
WEU	7,436.0	28.0
JPN	3,639.9	13.7
AUS	410.0	1.5
EFS	792.5	3.0
MEX	343.8	1.3
CHN	843.4	3.2
IND	351.3	1.3
ASI	1,388.7	5.2
MIE	627.9	2.4
CSA	1,468.7	5.5
ROW	749.8	2.8
Annex I	20,740.0	78.2
Non-Annex I	5,773.7	21.8
World	26,513.7	0.0

Table 10: Carbon emissions (MtC) and carbon intensity (kg/$)

	Y	EIS	ELE	FD*	total	Share (%)	kg/$
USA	582.3	196.7	605.2	98.2	1,482.4	24.7	0.19
CAN	68.0	25.6	29.5	12.6	135.7	2.3	0.25
WEU	375.0	158.7	257.0	109.1	899.8	15.0	0.12
JPN	123.9	77.0	109.6	15.2	325.7	5.4	0.09
AUS	32.4	13.4	41.8	0.3	87.9	1.5	0.21
EFS	193.6	91.1	377.6	87.2	749.5	12.5	0.95
MEX	37.4	19.1	24.9	6.1	87.5	1.5	0.25
CHN	174.7	239.1	327.2	77.2	818.0	13.6	0.97
IND	59.7	45.8	114.7	13.3	233.6	3.9	0.66
ASI	168.7	75.7	121.8	23.2	389.4	6.5	0.28
MIE	133.7	37.0	85.3	28.2	284.1	4.7	0.45
CSA	121.0	48.3	35.6	15.9	220.7	3.7	0.15
ROW	129.9	34.1	109.7	11.7	285.4	4.8	0.38
Annex I	1,375.2	562.5	1,420.7	322.6	3,681.0	61.4	
Non-Annex I	825.1	499.0	819.2	175.5	2,318.8	38.6	
World	2,200.3	1,061.5	2,239.9	498.1	5,999.8		

∗ FD indicates emissions from final demand.

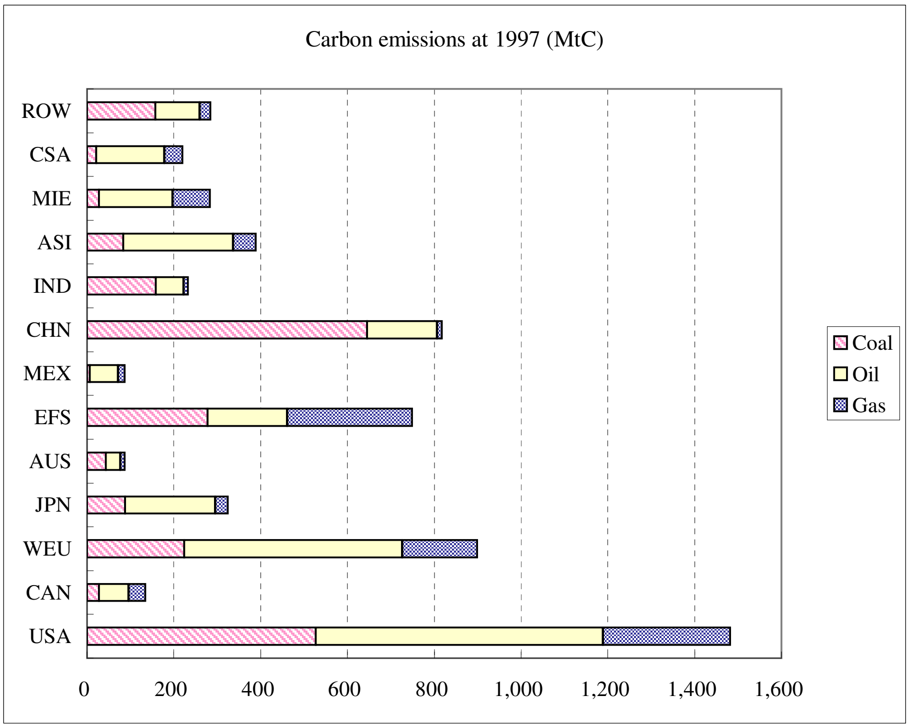

Figure 5: Carbon emissions at the benchmark year (MtC)

Table 11: Emission share by source (%)

	COL	OIL	GAS
USA	35.6	44.7	19.8
CAN	20.3	50.4	29.3
WEU	24.9	55.9	19.2
JPN	26.8	63.9	9.3
AUS	49.5	38.7	11.8
EFS	37.0	24.6	38.4
MEX	7.4	74.7	17.9
CHN	78.9	19.7	1.4
IND	68.0	27.5	4.5
ASI	21.5	65.0	13.5
MIE	9.7	60.0	30.4
CSA	9.5	71.6	18.9
ROW	55.0	36.0	9.1
Annex I	32.2	45.1	22.7
Non-Annex I	47.4	42.0	10.5
World	38.1	43.9	18.0

5 Model for the simulation

We have already explained the model structure. However, in GAMS programs for the simulation, the model is described using calibrated share forms. Here, we present the model in accordance with GAMS programs.

[1] All function are written in calibrated share form (Rutherford, 1998).

[2] Reference prices are omitted for notational simplification.

[3] Slack variable associated to each equation is given in parentheses on the right end.

All GAMS programs for the simulation is available from the author upon request.

5.1 Notations

Third columns in tables below display notations used in GAMS programs.

Set definitions

Notations	Description	In program
I	A set of all goods and sectors	`i,j`
FE	= {COL, GAS, OIL} $\cdots$ Final energy.	`fe`
XE	= {COL, GAS, CRU} $\cdots$ Fossil fuels.	`xe`
NXE	= $I \setminus$ XE $\cdots$ Non-fossil fuels.	`nxe`
ELE	= ELE $\cdots$ Electricity.	`ele`
COL	= COL. $\cdots$ Coal.	`col`
LQD	= OIL, GAS. $\cdots$ liquidity energy.	`lqd`
ENE	= EC $\cup$ ELE $\cdots$ Energy goods.	`eg`
NENE	= $I \setminus$ ENE $\cdots$ Non-energy goods.	`neg`

Activity variables

Notations	Description	In program
Y_{ir}	Level of production.	`y(i,r)`
E^A_{ir}	Energy aggregation for non-fossil fuel sectors ($i \in$ NXE).	`ea(i,r)`
A_{ir}	Armington aggregation.	`a(i,r)`
M_{ir}	Import aggregation.	`m(i,r)`
C_{ir}	Consumption aggregation.	`c(r)`
Y^T	Global transport sector.	`yt`
U_r	Utility.	`u(r)`

Unit cost functions

Notations	Description	In program
c^Y_{ir}	Unit cost of production.	`c_y(i,r)`
c^{EA}_{ir}	Unit cost of energy aggregation ($i\imath$NXE).	`c_ea(i,r)`
c^A_{ir}	Unit cost of Armington aggregation.	`c_a(i,r)`
c^M_{ir}	Unit cost of import aggregation.	`c_m(i,r)`
c^C_r	Unit cost of consumption aggregation.	`c_c(r)`
c^T	Unit cost of transport sector.	`c_t`
c^U_r	Unit cost of utility.	`c_u(r)`
c^G_r	Unit cost of utility.	`c_u(r)`

Price variables

Notations	Description	In program
p^Y_{ir}	Price index of output.	`p_y(i,r)`
p^X_{ir}	Price of export goods.	`p_x(i,r)`
p^D_{ir}	Price of domestic goods.	`p_d(i,r)`
p^{IL}_{ir}	Price index of non-resource inputs ($i \in$ XE).	`p_il(i,r)`
p^{PFE}_{ir}	Price index of energy-primary factor composite ($i \in$ NXE).	`p_pfe(i,r)`
p^{PF}_{ir}	Price index of primary factor composite ($i \in$ NXE).	`p_pf(i,r)`
p^{NEL}_{ir}	Price index of non-electricity energy composite ($i \in$ NXE).	`p_nel(i,r)`
p^{LQD}_{ir}	Price index of liquidity energy composite ($i \in$ NXE).	`p_lqd(i,r)`
p^{MM}_{irs}	Price of import including import tax.	`p_m(i,r,s)`
p^M_{ir}	Price of import composite.	`p_m(i,r)`
p^T	Price of transport services.	`p_t`
p^{EA}_{ir}	Price index of energy composite ($i \in$ NXE).	`p_ea(i,r)`
p^L_r	Wage rate.	`p_l(r)`
r^K_r	Rental price.	`r_k(r)`
p^A_{ir}	Price index of Armington goods.	`p_a(i,r)`
p^C_r	Price index of aggregate consumption.	`p_c(r)`
p^I_r	Price of investment goods.	`p_inv(r)`
p^U_r	Price of utility.	`p_u(r)`
p^S_r	Price of savings goods.	`p_s`
p^R_{ir}	Price of resource inputs ($i \in$ XE).	`p_r(i,r)`
p^{CNE}_r	Price index of non-energy consumption.	`p_cne(r)`
p^{CE}_r	Price index of energy consumption.	`p_ce(r)`
p^E_{ijr}	Price of final energy for intermediate inputs ($i \in$ FE, $j \in$ NXE).	`p_e(i,j,r)`
p^{ED}_{ir}	Price of final energy for final consumption ($i \in$ FE).	`p_efd(i,r)`

Unit demand and supply functions

Notations	Description	In program
a^{XER}_{ir}	Demand for resource inputs ($i \in$ XE).	`a_xer(i,r)`
a^{XEL}_{ir}	Demand for labor ($i \in$ XE).	`a_xel(i,r)`
a^{XEI}_{jir}	Demand for intermediate goods ($i \in$ XE).	`a_xei(j,i,r)`
a^F_{Kir}	Demand for capital ($i \in$ NXE).	`a_fk(i,r)`
a^F_{Lir}	Demand for labor ($i \in$ NXE).	`a_fl(i,r)`
a^{EA}_{Lir}	Demand for energy composite ($i \in$ NXE).	`a_ea(i,r)`
a^E_{jir}	Demand for energy goods ($j \in$ ENE, $i \in$ NXE).	`a_e(j,i,r)`
a^{AD}_{ir}	Demand for domestic goods.	`a_ad(i,r)`
a^{AM}_{ir}	Demand for aggregate import.	`a_am(i,r)`
a^{MM}_{irs}	Demand for import goods.	`a_mm(i,r,s)`
a^{CNE}_{ir}	Demand for non-energy consumption goods ($i \in$ NENE).	`a_cne(i,r)`
a^{CE}_{ir}	Demand for energy consumption goods ($i \in$ ENE).	`a_ce(i,r)`
a^X_{ir}	Supply of export goods.	`a_x(i,r)`
a^D_{ir}	Supply of domestic goods.	`a_d(i,r)`
a^T_{ir}	Demand from transport sector.	`a_t(i,r)`
a^C_r	Demand for aggregate consumption.	`a_cc(r)`
a^S_r	Demand for savings goods.	`a_s(r)`
$\bar{a}^I_{jir}$	Input coefficients for non-energy intermediate goods ($j \in$ NENE, $i \in$ NXE).	

Other variables and parameters

Notations	Description	In program
H_r	Income of the representative agent.	`inc_ra(r)`
CA^E_{jir}	Emissions from intermediate inputs. ($j \in \text{FE}, i \in \text{NXE}$).	`d_c_e(j,i,r)`
$\text{CA}^{\text{ED}}_{ir}$	Emissions from final consumption ($i \in \text{FE}$).	`d_c_fd(i,r)`
CA^T_r	Total emissions of region r.	`d_ca(r)`
r^C_r	Current rate of return from capital stock.	`r_c(r)`
r^E_r	Expected rate of return from capital stock.	`r_e(r)`
r^G	Global rate of return from capital stock.	`r_g`
K^E_r	End-of-period capital stock.	`k_e(r)`
K_r	Beginning-of-period capital stock.	`k_b(r)`
I^G	Global net investment.	`ginv`
I_r	Regional gross investment.	`grossinv(r)`
I^N_r	Regional net investment.	`netinv(r)`
S_r	Regional savings.	`rsave(r)`
ϕ_r	Depreciation rate.	`depr(r)`
β_r	Flexibility parameter.	`rorflex(r)`

Tax variables

Notations	Description	In program
t^Y_{ir}	Production tax rates.	`t_y(i,r)`
t^I_{ijr}	Intermediate tax rates.	`t_i(i,j,r)`
t^C_{ir}	Consumption tax rates.	`t_c(i,r)`
t^M_{irs}	Import tax rates.	`t_m(i,r,s)`
t^X_{irs}	Export tax rates.	`t_x(i,r,s)`
t^{CA}_r	Carbon tax.	`c_tax(r)`
t^{CA}_r / p^U_r	Real carbon tax.	`rc_tax(r)`

Share parameters

Notations	Description	In program
θ_{ir}^{X}	Share of export in output	`sh_x(i,r)`
θ_{ir}^{XER}	Share of resource inputs ($i \in$ XE)	`sh_xer(i,r)`
θ_{jir}^{XE}	Share of labor and intermediate inputs ($i \in$ XE)	`sh_xe(*,i,r)`
$\theta_{jir}^{\text{NENE}}$	Share of non-energy intermediate inputs ($j \in$ NENE, $i \in$ NXE)	`sh_neg(j,i,r)`
θ_{ir}^{PFE}	Share of primary factor-energy composite ($i \in$ NXE)	`sh_pfe(i,r)`
θ_{ir}^{PF}	Share of primary factor ($i \in$ NXE)	`sh_pf(i,r)`
θ_{fir}^{F}	Share of labor and capital ($i \in$ NXE)	`sh_lab(i,r)`
θ_{ir}^{ELE}	Share of electricity ($i \in$ NXE)	`sh_ele(i,r)`
θ_{ir}^{COL}	Share of coal ($i \in$ NXE)	`sh_col(i,r)`
θ_{ir}^{LQD}	Share of liquidity energy ($i \in$ NXE)	`sh_lqd(lqd,i,r)`
θ_{ir}^{M}	Share of aggregate import	`sh_m(i,r)`
θ_{irs}^{MM}	Share of import goods from a region.	`sh_mm(i,r,s)`
γ_{irs}		`gamma_(i,s,r)`
θ_{r}^{CE}	Share of energy composite in consumption.	`sh_ce(r)`
$\theta_{ir}^{\text{CNEG}}$	Share of non-energy consumption goods.	`sh_cneg(i,r)`
θ_{ir}^{CEG}	Share of energy consumption goods.	`sh_ceg(i,r)`
θ_{r}^{CC}	Share of consumption composite.	`sh_cc(r)`
θ_{ir}^{T}	Share of each input in transport sector.	`sh_t(i,r)`

5.2 Model

5.2.1 Unit cost and price index

Price index of output ($i \in I$):

$$p_{ir}^{Y} = \left[\theta_{ir}^{X}(p_{ir}^{X})^{1+\eta} + (1-\theta_{ir}^{X})(p_{ir}^{D})^{1+\eta}\right]^{\frac{1}{1+\eta}} \qquad \{p_{ir}^{Y}\}$$

Unit cost of fossil fuel production ($i \in$ XE):

$$c_{ir}^{Y} = \left[\theta_{ir}^{\text{XER}}(p_{ir}^{R})^{1-\sigma_{Ri}} + (1-\theta_{ir}^{\text{XER}})(p_{ir}^{\text{IL}})^{1-\sigma_{Ri}}\right]^{\frac{1}{1-\sigma_{Ri}}} \qquad \{c_{ir}^{Y}\}$$

Price index of non-resource input composite in fossil fuel production ($i \in$ XE):

$$p_{ir}^{\text{IL}} = \theta_{Lir}^{\text{XE}} p_{r}^{L} + \sum_{j} \theta_{jir}^{\text{XE}} \tilde{p}_{Ijir}^{A} \qquad \{p_{ir}^{\text{IL}}\}$$

where $\tilde{p}_{Ijir}^{A} = (1+t_{jir}^{I})p_{jr}^{A}$.

Unit cost of non-fossil fuel production ($i \in$ NXE):

$$c_{ir}^{Y} = \sum_{j \in \text{NENE}} \theta_{jir}^{\text{NENE}} \tilde{p}_{Ijir}^{A} + \theta_{ir}^{\text{PFE}} p_{ir}^{\text{PFE}} \qquad \{c_{ir}^{Y}\}$$

where $\tilde{p}_{Ijir}^{A} = (1+t_{jir}^{I})p_{jr}^{A}$.

Price index of primary factor-energy composite ($i \in$ NXE):

$$p_{ir}^{\text{PFE}} = \left[\theta_{ir}^{\text{PF}}(p_{ir}^{\text{PF}})^{1-\sigma_{\text{PFE}}} + (1-\theta_{ir}^{\text{PF}})(p_{ir}^{\text{EA}})^{1-\sigma_{\text{PFE}}}\right]^{\frac{1}{1-\sigma_{\text{PFE}}}} \qquad \{p_{ir}^{\text{PFE}}\}$$

Price index of primary factor ($i \in$ NXE):

$$p_{ir}^{\mathrm{PF}} = \left[\theta_{Lir}^{F}(p_r^L)^{1-\sigma_{\mathrm{PF}}} + \theta_{Kir}^{F}(r_r^K)^{1-\sigma_{\mathrm{PF}}}\right]^{\frac{1}{1-\sigma_{\mathrm{PF}}}} \qquad \{p_{ir}^{\mathrm{PF}}\}$$

Unit cost of energy aggregation activity ($i \in$ NXE):

$$c_{ir}^{\mathrm{EA}} = \left[\theta_{ir}^{\mathrm{ELE}}(\tilde{p}_{I,\mathrm{ELE},ir}^{A})^{1-\sigma_E} + (1-\theta_{ir}^{\mathrm{ELE}})(p_{ir}^{\mathrm{NEL}})^{1-\sigma_E}\right]^{\frac{1}{1-\sigma_E}} \qquad \{c_{ir}^{\mathrm{EA}}\}$$

Price index of non-electricity energy ($i \in$ NXE):

$$p_{ir}^{\mathrm{NEL}} = \left[\theta_{ir}^{\mathrm{COL}}(p_{\mathrm{COL},ir}^{E})^{1-\sigma_{\mathrm{NEL}}} + (1-\theta_{ir}^{\mathrm{COL}})(p_{ir}^{\mathrm{LQD}})^{1-\sigma_{\mathrm{NEL}}}\right]^{\frac{1}{1-\sigma_{\mathrm{NEL}}}} \qquad \{p_{ir}^{\mathrm{NEL}}\}$$

Price index of liquidity energy ($i \in$ NXE):

$$p_{ir}^{\mathrm{LQD}} = \left[\sum_{j\in \mathrm{LQD}} \theta_{jir}^{\mathrm{LQD}}(p_{jir}^{E})^{1-\sigma_{\mathrm{LQD}}}\right]^{\frac{1}{1-\sigma_{\mathrm{LQD}}}} \qquad \{p_{ir}^{\mathrm{LQD}}\}$$

Unit cost of Armington aggregation:

$$c_{ir}^{A} = \left[\theta_{ir}^{M}(p_{ir}^{M})^{1-\sigma_A} + (1-\theta_{ir}^{M})(p_{ir}^{D})^{1-\sigma_A}\right]^{\frac{1}{1-\sigma_A}} \qquad \{c_{ir}^{A}\}$$

Unit cost of import aggregation:

$$c_{ir}^{M} = \left[\sum_{s} \theta_{isr}^{\mathrm{MM}}(p_{isr}^{\mathrm{MM}})^{1-\sigma_M}\right]^{\frac{1}{1-\sigma_M}} \qquad \{c_{ir}^{M}\}$$

Price index of imports from region s to region r:

$$p_{irs}^{\mathrm{MM}} = \gamma_{irs}(1+t_{irs}^{M})(1+t_{irs}^{X})p_{ir}^{X} + (1-\gamma_{irs})(1+t_{irs}^{M})p^{T}\tau_{irs} \qquad \{p_{irs}^{\mathrm{MM}}\}$$

Unit cost of consumption:

$$c_r^{C} = \left[\theta_r^{\mathrm{CE}}(p_r^{\mathrm{CE}})^{1-\sigma_C} + (1-\theta_r^{\mathrm{CE}})(p_r^{\mathrm{CNE}})^{1-\sigma_C}\right]^{\frac{1}{1-\sigma_C}} \qquad \{c_r^{C}\}$$

Price index of non-energy consumption composite:

$$p_r^{\mathrm{CNE}} = \left[\sum_{i\in \mathrm{NENE}} \theta_{ir}^{\mathrm{CNEG}}(\tilde{p}_{Cir}^{A})^{1-\sigma_{\mathrm{CNE}}}\right]^{\frac{1}{1-\sigma_{\mathrm{CNE}}}} \qquad \{p_r^{\mathrm{CNE}}\}$$

where $\tilde{p}_{Cir}^{A} = (1+t_{ir}^{C})p_{ir}^{A}$.

Price index of energy consumption composite.

$$p_r^{\mathrm{CE}} = \left[\theta_{\mathrm{ELE},r}^{\mathrm{CEG}}(\tilde{p}_{C,\mathrm{ELE},r}^{A})^{1-\sigma_{\mathrm{CE}}} + \sum_{i\in \mathrm{FE}} \theta_{ir}^{\mathrm{CEG}}(p_{ir}^{\mathrm{ED}})^{1-\sigma_{\mathrm{CE}}}\right]^{\frac{1}{1-\sigma_{\mathrm{CE}}}} \qquad \{p_r^{\mathrm{CE}}\}$$

Unit cost of utility.

$$c_r^{U} = \left[\theta_r^{\mathrm{CC}}(p_r^{C})^{1-\sigma_U} + (1-\theta_r^{\mathrm{CC}})(p_r^{S})^{1-\sigma_U}\right]^{\frac{1}{1-\sigma_U}} \qquad \{c_r^{U}\}$$

Unit cost of transport services.

$$c^T = \left[\sum_{i,r} \theta_{ir}^T (p_{ir}^X)^{1-\sigma_T} \right]^{\frac{1}{1-\sigma_T}} \qquad \{c^T\}$$

Price index of final energy for intermediate inputs ($i \in \text{FE}$, $j \in \text{NXE}$):

$$p_{ijr}^E = (1 + t_{ijr}^I) p_{ir}^A + \delta_{ijr}^I t_r^{\text{CA}} \qquad \{p_{ijr}^E\}$$

Price index of final energy for final consumption ($i \in \text{FE}$):

$$p_{ir}^{\text{ED}} = (1 + t_{ir}^C) p_{ir}^A + \delta_{ir}^C t_r^{\text{CA}} \qquad \{p_{ir}^{\text{ED}}\}$$

Price of investment goods:

$$p_r^I = p_{\text{CGD},r}^D \qquad \{p_r^I\}$$

5.2.2 Zero profit conditions

Production activity ($i \in I$):

$$c_{ir}^Y \geq (1 - t_{ir}^Y) p_{ir}^Y \qquad \{Y_{ir}\}$$

Energy aggregation activity ($i \notin \text{eg}$).

$$c_{ir}^{\text{EA}} \geq p_{ir}^{\text{EA}} \qquad \{E_{ir}^A\}$$

Armington aggregation activity:

$$c_{ir}^A \geq p_{ir}^A \qquad \{A_{ir}\}$$

Import aggregation activity:

$$c_{ir}^M \geq p_{ir}^M \qquad \{M_{ir}\}$$

Consumption aggregation activity:

$$c_r^C \geq p_r^C \qquad \{C_r\}$$

Utility production:

$$c_r^U \geq p_r^U \qquad \{U_r\}$$

Transport activity:

$$c^T \geq p^T \qquad \{Y^T\}$$

5.2.3 Demand and supply

Supply of export goods:

$$a_{ir}^X = \bar{a}_{ir}^X \left[\frac{p_{ir}^X}{p_{ir}^Y} \right]^\eta \qquad \{a_{ir}^X\}$$

Supply of domestic goods:

$$a_{ir}^D = \bar{a}_{ir}^D \left[\frac{p_{ir}^D}{p_{ir}^Y} \right]^\eta \qquad \{a_{ir}^D\}$$

Demand for natural resources of fossil fuel sectors ($i \in \text{XE}$):

$$a_{ir}^{\text{XER}} = \bar{a}_{ir}^{\text{XER}} \left[\frac{c_{ir}^{Y}}{p_{ir}^{R}} \right]^{\sigma_{R,i}} \qquad \{a_{ir}^{\text{XER}}\}$$

Demand for labor of fossil fuel sectors ($i \in \text{XE}$):

$$a_{ir}^{\text{XEL}} = \bar{a}_{ir}^{\text{XEL}} \left[\frac{c_{ir}^{Y}}{p_{ir}^{\text{IL}}} \right]^{\sigma_{R,i}} \qquad \{a_{ir}^{\text{XEL}}\}$$

Demand for intermediate inputs of fossil fuel sectors ($i \in \text{XE}$):

$$a_{jir}^{\text{XEI}} = \bar{a}_{jir}^{\text{XEI}} \left[\frac{c_{ir}^{Y}}{p_{ir}^{\text{IL}}} \right]^{\sigma_{R,i}} \qquad \{a_{jir}^{\text{XEI}}\}$$

Demand for primary factor of non-fossil fuel sectors ($i \in \text{NXE}$):

$$a_{Lir}^{F} = \bar{a}_{Lir}^{F} \left[\frac{p_{ir}^{\text{PF}}}{p_{r}^{L}} \right]^{\sigma_{\text{PF}}} \left[\frac{p_{ir}^{\text{PFE}}}{p_{ir}^{\text{PF}}} \right]^{\sigma_{\text{PFE}}} \qquad \{a_{Lir}^{F}\}$$

$$a_{Kir}^{F} = \bar{a}_{Kir}^{F} \left[\frac{p_{ir}^{\text{PF}}}{r_{r}^{K}} \right]^{\sigma_{\text{PF}}} \left[\frac{p_{ir}^{\text{PFE}}}{p_{ir}^{\text{PF}}} \right]^{\sigma_{\text{PFE}}} \qquad \{a_{Kir}^{F}\}$$

Demand for energy composite of non-fossil fuel sectors ($i \in \text{NXE}$):

$$a_{ir}^{\text{EA}} = \bar{a}_{ir}^{\text{EA}} \left[\frac{p_{ir}^{\text{PFE}}}{p_{ir}^{\text{EA}}} \right]^{\sigma_{\text{PFE}}} \qquad \{a_{ir}^{\text{EA}}\}$$

Demand for electricity of non-fossil fuel sectors ($j = \text{ELE}, i \in \text{NXE}$):

$$a_{jir}^{E} = \bar{a}_{jir}^{E} \left[\frac{c_{ir}^{\text{EA}}}{\tilde{p}_{Ijir}^{A}} \right]^{\sigma_{E}} \qquad \{a_{jir}^{E}\}$$

Demand for coal of non-fossil fuel sectors ($j = \text{COL}, i \in \text{NXE}$):

$$a_{jir}^{E} = \bar{a}_{jir}^{E} \left[\frac{p_{ir}^{\text{NEL}}}{p_{jir}^{E}} \right]^{\sigma_{\text{NEL}}} \left[\frac{c_{ir}^{\text{EA}}}{p_{ir}^{\text{NEL}}} \right]^{\sigma_{E}} \qquad \{a_{jir}^{E}\}$$

Demand for liquidity energy of non-fossil fuel sectors ($j \in \text{LQD}, i \in \text{NXE}$):

$$a_{jir}^{E} = \bar{a}_{jir}^{E} \left[\frac{p_{ir}^{\text{LQD}}}{p_{jir}^{E}} \right]^{\sigma_{\text{LQD}}} \left[\frac{p_{ir}^{\text{NEL}}}{p_{ir}^{\text{LQD}}} \right]^{\sigma_{\text{NEL}}} \left[\frac{c_{ir}^{\text{EA}}}{p_{ir}^{\text{NEL}}} \right]^{\sigma_{E}} \qquad \{a_{jir}^{E}\}$$

Demand for domestic goods of Armington aggregation:

$$a_{ir}^{\text{AD}} = \bar{a}_{ir}^{\text{AD}} \left[\frac{c_{ir}^{A}}{p_{ir}^{D}} \right]^{\sigma_{A}} \qquad \{a_{ir}^{\text{AD}}\}$$

Demand for import goods of Armington aggregation:

$$a_{ir}^{\text{AM}} = \bar{a}_{ir}^{\text{AM}} \left[\frac{c_{ir}^{A}}{p_{ir}^{M}} \right]^{\sigma_{A}} \qquad \{a_{ir}^{\text{AM}}\}$$

Demand for import i from region r to region s:

$$a_{isr}^{\text{MM}} = \bar{a}_{isr}^{\text{MM}} \left[\frac{p_{ir}^{M}}{p_{isr}^{\text{MM}}} \right]^{\sigma_M} \qquad \{a_{isr}^{\text{MM}}\}$$

Consumption demand for non-energy goods ($i \notin$ eg):

$$a_{ir}^{\text{CNE}} = \bar{a}_{ir}^{\text{CNE}} \left[\frac{p_r^{\text{CNE}}}{\tilde{p}_{Cir}^{A}} \right]^{\sigma_{\text{CE}}} \left[\frac{p_r^{C}}{p_r^{\text{CNE}}} \right]^{\sigma_C} \qquad \{a_{ir}^{\text{CNE}}\}$$

where $\tilde{p}_{Cir}^{A} = (1 + t_{ir}^{C}) p_{ir}^{A}$.

Consumption demand for energy goods ($i \in$ eg):

$$a_{ir}^{\text{CE}} = \bar{a}_{ir}^{\text{CE}} \left[\frac{p_r^{\text{CE}}}{p_{ir}^{\text{ED}}} \right]^{\sigma_{\text{CE}}} \left[\frac{c_r^{C}}{p_r^{\text{CE}}} \right]^{\sigma_C} \qquad \{a_{ir}^{\text{CE}}\}_{i \notin \text{ELE}}$$

$$a_{ir}^{\text{CE}} = \bar{a}_{ir}^{\text{CE}} \left[\frac{p_r^{\text{CE}}}{\tilde{p}_{Cir}^{A}} \right]^{\sigma_{\text{CE}}} \left[\frac{c_r^{C}}{p_r^{\text{CE}}} \right]^{\sigma_C} \qquad \{a_{ir}^{\text{CE}}\}_{i \in \text{ELE}}$$

Demand for aggregate consumption:

$$a_r^{C} = \bar{a}_r^{C} \left[\frac{p_r^{U}}{p_r^{C}} \right]^{\sigma_U} \qquad \{a_r^{C}\}$$

Demand for savings:

$$a_r^{S} = \bar{a}_r^{S} \left[\frac{p_r^{U}}{p^{S}} \right]^{\sigma_U} \qquad \{a_r^{S}\}$$

Demand for intermediate inputs of international transport sector:

$$a_{ir}^{T} = \bar{a}_{ir}^{T} \left[\frac{p^{T}}{p_{ir}^{X}} \right]^{\sigma_T} \qquad \{a_{ir}^{T}\}$$

Emissions from intermediate inputs ($j \in$ FE, $i \in$ NXE):

$$\text{CA}_{jir}^{E} = \delta_{jir}^{I} a_{jir}^{E} E_{ir}^{A} \qquad \{\text{CA}_{jir}^{E}\}$$

Emissions from final consumption ($i \in$ FE):

$$\text{CA}_{ir}^{\text{ED}} = \delta_{ir}^{\text{ED}} a_{ir}^{\text{CE}} C_r \qquad \{\text{CA}_{ir}^{\text{ED}}\}$$

Total emissions:

$$\text{CA}_r^{T} = \sum_{i \in \text{FE}} \left[\sum_{j \in \text{NXE}} \text{CA}_{ijr}^{E} + \text{CA}_{ir}^{\text{ED}} \right] \qquad \{\text{CA}_r^{T}\}$$

5.2.4 Market clearing conditions

Market for export goods:

$$a_{ir}^{X} Y_{ir} \geq \sum_{s} a_{irs}^{\text{MM}} M_{is} + a_{ir}^{T} Y^{T} \qquad \{p_{ir}^{X}\}$$

Market for domestic goods ($i \neq$ CGD):

$$a_{ir}^{D} Y_{ir} \geq a_{ir}^{\text{AD}} A_{ir} \qquad \{p_{ir}^{D}\}$$

Market for investment goods ($i =$ CGD):

$$a_{ir}^{D} Y_{ir} \geq I_r^N + \phi_r K_r \qquad \{p_{ir}^{D}\}$$

Market for aggregate imports:

$$M_{ir} \geq A_{ir}^{\text{AM}} A_{ir} \qquad \{p_{ir}^{M}\}$$

Market for transport service:

$$Y^T \geq \sum_{i,s,r} \tau_{isr} a_{isr}^{\text{MM}} M_{ir} \qquad \{p^T\}$$

Market for aggregate energy ($i \in$ NXE):

$$E_{ir}^{A} \geq a_{ir}^{\text{EA}} Y_{ir} \qquad \{p_{ir}^{\text{EA}}\}$$

Market for labor:

$$L_r \geq \sum_{i \in \text{XE}} a_{ir}^{\text{XEL}} Y_{ir} + \sum_{i \in \text{NXE}} a_{Lir}^{F} Y_{ir} \qquad \{p_r^L\}$$

Market for capital stock:

$$K_r \geq \sum_{i \in \text{NXE}} a_{Kir}^{F} Y_{ir} \qquad \{r_r^K\}$$

Market for sector specific natural resources ($i \in$ XE):

$$R_{ir} \geq a_{ir}^{\text{XER}} Y_{ir} \qquad \{p_{ir}^{R}\}$$

Market for Armington goods ($i \in$ NENE):

$$A_{ir} \geq \sum_{j \in \text{XE}} a_{ijr}^{\text{XEI}} Y_{jr} + \sum_{j \in \text{NXE}} \bar{a}_{ijr}^{I} Y_{jr} + a_{ir}^{\text{CNE}} C_r \qquad \{p_{ir}^{A}\}$$

Market for Armington goods ($i \in$ ENE):

$$A_{ir} \geq \sum_{j \in \text{XE}} a_{ijr}^{\text{XEI}} Y_{jr} + \sum_{j \in \text{NXE}} a_{ijr}^{E} E_j^A + a_{ir}^{\text{CE}} C_r \qquad \{p_{ir}^{A}\}$$

Market for aggregate consumption:

$$C_r \geq a_r^C U_r \qquad \{p_r^C\}$$

Market for utility:

$$H_r \geq p_r^U U_r \qquad \{p_r^U\}$$

5.2.5 Income

Income of the representative agent:

$$
\begin{aligned}
H_r = p^L_r L_r + r^K_r K_r + \sum_{i \in \text{XE}} p^R_{ir} R_{ir} \\
+ \sum_{i} t^Y_{ir} p^Y_{ir} Y_{ir} + \sum_{i \in \text{XE}} \sum_{j} t^I_{jir} p^A_{jr} a^{\text{XEI}}_{jir} Y_{ir} \\
+ \sum_{i \in \text{NXE}} \sum_{j \in \text{ENE}} t^I_{jir} p^A_{jr} a^E_{jir} Y_{ir} + \sum_{i \in \text{NXE}} \sum_{j \in \text{NENE}} t^I_{jir} p^A_{jr} a^I_{jir} Y_{ir} \\
+ \sum_{i,s} t^X_{irs} p^X_{ir} a^{\text{MM}}_{irs} M_{is} + \sum_{i,s} t^M_{isr} [(1 + t^X_{isr}) p^X_{is} + p^T \tau_{isr}] a^{\text{MM}}_{isr} M_{ir} \\
+ \sum_{i \in \text{NENE}} t^C_{ir} p^A_{ir} a^{\text{CNE}}_{ir} C_r + \sum_{i \in \text{ENE}} t^C_{ir} p^A_{ir} a^{\text{CE}}_{ir} C_r \\
- p^G_r G_r + t^{\text{CA}}_r \text{CA}^T_r \qquad \{H_r\}
\end{aligned}
$$

5.2.6 Investment and savings

Regional savings:

$$S_r = a^S_r U_r \qquad \{S_r\}$$

Global savings:

$$I^G = \sum_r S_r \qquad \{p^S\}$$

Global (net) investment:

$$I^G = \sum_r I^N_r \qquad \{I^G\}$$

Regional gross investment:

$$I_r = I^N_r + \phi_r K_r \qquad \{I_r\}$$

The current rate of return from capital stock:

$$r^C_r = \frac{r^K_r}{p^I_r} - \phi_r$$

The expected rate of return from capital stock:

$$r^E_r = r^C_r \left[\frac{K^E_r}{K_r} \right]^{-\beta_r}$$

The global rate of return:

$$\sum_r p^I_r I^N_r \geq p^S I^G \qquad \{r^G\}$$

The end-of-period capital stock:

$$K^E_r = K_r + I^N_r \qquad \{K^E_r\}$$

Regional net investment:

$$r^G = \xi_r r^E_r \qquad \{I^N_r\}$$

5.2.7 Carbon tax

Carbon tax:

$$\tilde{t}_r^{\mathrm{CA}} = \hat{a} + \hat{b} H_r / (p_r^U n_r) + \hat{e}_r \qquad \{t_r^{\mathrm{CA}}\}$$

Real carbon tax:

$$\tilde{t}_r^{\mathrm{CA}} = t_r^{\mathrm{CA}} / p_r^U \qquad \{\tilde{t}_r^{\mathrm{CA}}\}$$

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