Memo by Shiro Takeda.

Date: 2012/12/06

This note shows the structure of GTAP model implemented in GAMS.

1 Note by S. Takeda

- See `README.txt` in the GAMS code archive for the details of simulations.
- To implemente a model in GAMS, it is necessary to represent the model in levels form. However, some of equations in the original GTAP model cannot be represented in levels form. To handle this, I slightly modify the model so that all equations are represented in levels form. For example, in equation PRICGDS, PRIMFACTPR, and PRIMFACTPRWLD in the original GTAP model, Divisia type price indices are used. Divisia type price indices cannot be represented in levels form. So, my model uses the Tornqvist index that is a discrete approximation to a continuous Divisia index. This means that my model is slightly different from the original GTAP model.
- **Most of the expository texts for variables, coefficients, and equations in this document are taken from `gtap.tab`.**
- Equations with (O) on the left end are original GTAP equations (normally in linearized form) and equations with (L) are levels form equations.
- Equations with prefix (A) indicate equations that appear only in the original GTAP code and do not appear in levels form. Similarly, equations with prefix (B) indicate equations that appear only in levels forms and do not appear in the original GTAP code.
- GTAP model is based on `gtap.tab` Version 6.2.
- Type of variables:
  - Not specified: linearized and percentage change variables.
  - L: levels variables.
  - C: linearized and change variables.
- References:
  - GTAP model: Hertel (1997), McDougall (2003).

Contents

2 Introduction

[Note]: Most of the expository texts for the model are taken from `gtap.tab` (ver. 6.2).

For documentation see:

1. Hertel, T.W. and M.E. Tsigas, "Structure of the Standard GTAP Model", Chapter 2 in T.W. Hertel (editor) "Global Trade Analysis: Modeling and Applications", Cambridge University Press, 1997.
2. R.A. McDougall, "A New Regional Household Demand System for GTAP", GTAP Technical Paper #20, Center for Global Trade Analysis, Purdue University

3. Hertel T.W., K. Itakura and R.A. McDougall, "GTAP.TAB: The Standard GTAP Model" (yet to be drafted), GTAP Technical Paper, Center for Global Trade Analysis, Purdue University, `http://www.agecon.purdue.edu/gtap/techpapr/`.
   This technical paper documents all of the changes since the GTAP book TAB file.

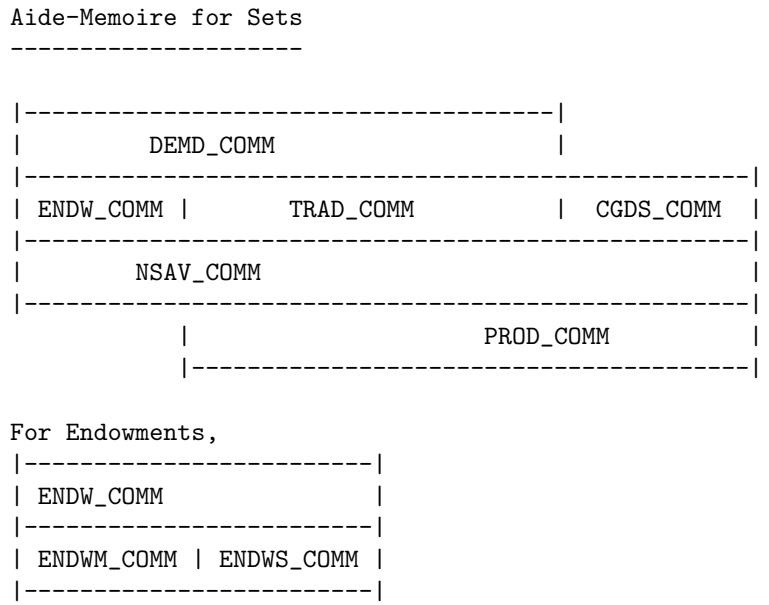
Version 6.0 replaces version 5.0 used in the August 2000 short course, but never formally released. The subsequent changes have been quite significant (primarily focused on final demand and welfare decomposition), so we decided to give it a new version number. This also prevents confusion with the soon to be released version 5.0 GTAP data set.

The differences with the previous publicly released version of GTAP.TAB, Version 4.1, dated November 1998, are numerous. One goal of this version is to reorganize the TAB file which had not be redesigned since its inception. In addition, there is a long list of substantive changes that have been made between Version 4.1 and 6.0. These include:

1. provision for multiple margins sectors (to accommodate post version 5 GTAP data)
2. introduction of the new theory of final demand following McDougall's GTAP Technical Paper #20
3. modification of the welfare decomposition following McDougall's Technical Paper #20
4. correction to the treatment of technical change, ao, in the presence of ESUBT > 0 (affects equations VADEMAND and INTDEMAND)
5. addition of import-augmenting "technical change" in the Armington nest: ams which can be used to lower the effective price of imported products
6. specification of Baldwin-type capital accmulation effects following Francois' Technical Paper #7
7. introduction of uniform consumption tax instrument, tp(r)

2.1 Overview of the GTAP.TAB Structure

- Preliminaries:
  1. FILES
  2. SETS
  3. READ statements of Base Data
  4. Common VARIABLES
  5. Common COEFFICIENTS
- Modules:
  1. Government Consumption
  2. Private Consumption
  3. Firms
  4. Investment, Global Bank, and Savings
  5. International Trade
  6. International Transport Services
  7. Regional Household
  8. Equilibrium Conditions
- Appendices:
  1. Summary Indices
  2. Equivalent Variation
  3. Welfare Decomposition
  4. Terms of Trade Decomposition



3 Preliminaries

3.1 File

Symbol	Description
GTAPSETS	file with set specification
GTAPDATA	file containing all base data
GTAPPARM	file containing behavioral parameters

3.2 SET definition

Symbol	Abbreviation	Description
VERNUM		Version of GTAP data.
REG	R	Regions in the model.
TRAD_COMM	T	Traded commodities.
MARG_COMM	M	Margin commodities .
NMARG_COMM	NM	Non-margin commodities = TRAD_COMM - MARG_COMM.
CGDS_COMM	C	Capital goods commodities = CGDS.
ENDW_COMM	E	Endowment commodities.
PROD_COMM	P	Produced commodities = TRAD_COMM $\cup$ CGDS_COMM.
DEM_COMM	D	Demanded commodities = ENDW_COMM $\cup$ TRAD_COMM.
NSAV_COMM	NS	Non-saving commodities = DEM_COMM $\cup$ CGDS_COMM.
ENDWS_COMM	ES	sluggish endowment commodities
ENDWC_COMM	EC	mobile endowment commodities
ENDWM_COMM	EM	capital endowment commodity

3.2.1 Coefficient:

Symbol	Description
SLUG(i)	Sluggish primary factor endowments.

3.3 READ Statements of Base Data

We read in here almost all the base data, and define variables and coefficients associated with them. A few data arrays used each in a single module are read in those modules: VKB, VTMFSD, and DPARSUM.

The READ statements are divided into six sections:

- Saving
- Government Consumption
- Private Consumption
- Firms
- Global Bank
- International Trade and Transport

Since these are invariant for each solution of the model, they are termed coefficients. Coefficients are assigned upper case to distinguish them from variables. (This is purely cosmetic, as GEMPACK is not case-sensitive.) Variables in GEMPACK are assigned lower case labels to denote the fact that they are percentage changes. In some cases, original levels values for selected variables are defined to permit the user to compare post-simulation levels values across several simulations.

The updating command indicates how the new level of the coefficient will be computed based on the previous solution of the linearized equations. Note that the notation used in the update commands is a shorthand for total differentials of these coefficient values. Thus, w * v indicates that we want to take the total differential of W * V, plug in the calculated values of w and v, and add this to the base level in order to obtain a revised value for this product.

3.4 Saving

3.4.1 Variables:

Symbol	Description
psave(r)	Price of savings in r.
qsave(r)	Regional demand for NET savings

3.4.2 Coefficient:

Symbol	Description	Update
SAVE(r)	Expenditure on NET savings in r valued at agent's prices.	psave(r) * qsave(r)

3.5 Government consumption

3.5.1 Variables:

Symbol	Description
pgd(i,s)$_{i\in T}$	Price of domestic i in government consumption in s.
qgd(i,s)$_{i\in T}$	Government hhld demand for domestic i in s.
pm(i,r)$_{i\in NS}$	Market price of commodity i in r.
pgm(i,s)$_{i\in T}$	Price of imports of i in government consumption in s.
qgm(i,s)$_{i\in T}$	Government hhld demand for imports of i in s.
pim(i,r)$_{i\in T}$	Market price of composite import i in r.

3.5.2 Coefficients:

Symbol	Description	Update
VDGA(i,r)$_{i\in T}$	Government consumption expenditure on domestic i in r (agent's price).	pgd(i,r) * qgd(i,r)
VDGM(i,r)$_{i\in T}$	Government consumption expenditure on domestic i in r (market price).	pm(i,r) * qgd(i,r)
VIGA(i,r)$_{i\in T}$	Government consumption expenditure on imported i in r (agent's price).	pgm(i,r) * qgm(i,r)
VIGM(i,r)$_{i\in T}$	Government consumption expenditure on imported i in r (market price).	pim(i,r) * qgm(i,r)

3.6 Private consumption

3.6.1 Variables:

Symbol	Description
ppd(i,s)$_{i\in T}$	Price of domestic i to private households in s.
qpd(i,s)$_{i\in T}$	Private hhld demand for domestic i in s.
ppm(i,s)$_{i\in T}$	Price of imports of i by private households in s.
qpm(i,s)$_{i\in T}$	Private hhld demand for imports of i in s.

3.6.2 Coefficients:

Symbol	Description	Update
VDPA(i,r)$_{i\in T}$	Private consumption expenditure on domestic i in r (agent's price).	ppd(i,r) * qpd(i,r)
VDPM(i,r)$_{i\in T}$	Private consumption expenditure on domestic i in r (market price).	pm(i,r) * qpd(i,r)
VIPA(i,r)$_{i\in T}$	Private consumption expenditure on imported i in r (agent's price).	ppm(i,r) * qpm(i,r)
VIPM(i,r)$_{i\in T}$	Private consumption expenditure on imported i in r (market price).	pim(i,r) * qpm(i,r)

3.7 Firms

3.7.1 Variables:

Symbol	Description
ps(i,r)$_{i\in NS}$	Supply price of commodity i in r.
qo(i,r)$_{i\in NS}$	Industry output of commodity i in r.
pfe(i,j,r)$_{i\in E,j\in P}$	Firms' price for endowment commodity i in ind. j, r.
qfe(i,j,r)$_{i\in E,j\in P}$	Demand for endowment i for use in ind. j in r.
pfd(i,j,s)$_{i\in T,j\in P}$	Price index for domestic purchases of i by j in s.
qfd(i,j,s)$_{i\in T,j\in P}$	Domestic good i demanded by industry j in s.
pfm(i,j,s)$_{i\in T,j\in P}$	Price index for imports of i by j in s.
qfm(i,j,s)$_{i\in T,j\in P}$	Demand for imported i by industry j in s.
pmes(i,j,s)$_{i\in E,j\in P}$	Market price of sluggish endowment i used by industry j in r.

3.7.2 Coefficients:

Symbol	Description	Update
EVOA(i,r)$_{i\in E}$	Value of commodity i output in r (agent's price).	ps(i,r) * qo(i,r)
EVFA(i,j,r)$_{i\in E,j\in P}$	Producer expenditure on i by j in r (agent's price)	pfe(i,j,r) * qfe(i,j,r)
VDFA(i,j,r)$_{i\in T,j\in P}$	Purchases of domestic i for use by j in r (agent's price).	pfd(i,j,r) * qfd(i,j,r)
VIFA(i,j,r)$_{i\in T,j\in P}$	Purchases of imported i for use by j in r (agent's price).	pfm(i,j,r) * qfm(i,j,r)
VFM(i,j,r)$_{i\in E,j\in P}$	Producer expenditure on i by j (market price).	pmes(i,j,r) * qfe(i,j,r)
VIFM(i,j,r)$_{i\in T,j\in P}$	Purchase of imports i for use by j in r (market price).	pim(i,r) * qfm(i,j,r)
VDFM(i,j,r)$_{i\in T,j\in P}$	Purchase of domestic i for use by industry j in r (market price).	pm(i,r) * qfd(i,j,r)

3.8 Global Bank

3.8.1 Variables:

Symbol	Description
kb(r)	Beginning-of-period capital stock in r.
pcgds(r)	Price of investment goods = ps("cgds",r).

3.8.2 Coefficients:

Symbol	Description	Update
VDEP(r)	Value of capital depreciation in r (exogenous).	kb(r) * pcgds(r)

3.9 International Trade and Transport

3.9.1 Variables:

Symbol	Description
pms(i,r,s)$_{i\in T}$	Domestic price for good i supplied from r to s.
qxs(i,r,s)$_{i\in T}$	Export sales of commodity i from r to s.
pcif(i,r,s)$_{i\in T}$	CIF world price of commodity i supplied from r to s.
pfob(i,r,s)$_{i\in T}$	FOB world price of commodity i supplied from r to s.
qst(m,r)$_{m\in M}$	Sales of m from r to international transport

3.9.2 Coefficients:

Symbol	Description	Update
VIMS(i,r,s)$_{i\in T}$	Imports of i from r to s valued at domestic mkt prices	pms(i,r,s) * qxs(i,r,s)
VIWS(i,r,s)$_{i\in T}$	Imports of i from r to s valued at CIF prices (tradeables only)	pcif(i,r,s) * qxs(i,r,s)
VXWD(i,r,s)$_{i\in T}$	Exports of i from r to s valued FOB (tradeables only)	pfob(i,r,s) * qxs(i,r,s)
VXMD(i,r,s)$_{i\in T}$	Exports of i from r to s valued at mkt prices (tradeables only)	pm(i,r) * qxs(i,r,s)
VST(m,s)$_{m\in M}$	Exports of m from r for international transport valued at mkt prices (tradeables only)	pm(m,r) * qst(m,r)

3.10 Common Variables

Common variables are defined as variables which are used in more than one module. For example, the variable y(r) is used in the Goverment Consumption, Private Consumption, Firms, Regional Household and Investment, Global Bank and Savings modules. Appendices, e.g., Summary Indices, are not included in this definition.

3.10.1 Variables:

Symbol	Type	Description
y(r)		Regional household income in r
pop(r)		Regional population
qoes(i,j,r)$_{i\in E,j\in P}$		Supply of sluggish endowment i used by j in r.
endwslack(i,r)$_{i\in E}$		Slack variable in endowment market clearing condition.
pgov(r)		Price index for government hhld expenditure in r
yg(r)		Regional (total) government consumption expenditure in r
ug(r)		Per capita utility from government expenditure in r
ppriv(r)		Price index for private consumption expenditure in r
uepriv(r)		Elasticity of cost wrt utility from private consumption in r
yp(r)		Regional private consumption expenditure in r
up(r)		Per capita utility from private expenditure in r
to(i,r)$_{i\in NS}$		Output (or income) tax in r
qim(i,s)$_{i\in T}$		Aggregate imports of i in s, market price weights.
globalcgds		Global supply of capital goods for NET investment.
pcgdswld		World average price of capital goods (net investment weights).
del_taxrgc(r)	C	Change in ratio of government consumption tax to INCOME
del_taxrpc(r)	C	Change in ratio of private consumption tax to INCOME
del_taxriu(r)	C	Change in ratio of tax on intermediate usage to INCOME
del_taxrfu(r)	C	Change in ratio of tax on primary factor usage to INCOME
del_taxrout(r)	C	Change in ratio of output tax to INCOME
del_taxrexp(r)	C	Change in ratio of export tax to INCOME
del_taxrimp(r)	C	Change in ratio of import tax to INCOME
del_taxrinc(r)	C	Change in ratio of income tax to INCOME

Note on endwslack(i,r) This is exogenous, unless the user wishes to employ a partial equilibrium closure in which the price of one or more of the primary factors is fixed.

Note on to(i,r) Note: It is important that the user NOT shock the tax on capital goods output, as this will cause an inconsistency in the update relationship for VOM(cgds).

Note on del_* The following variables could be dropped when converting to levels equation for income. They are only needed for the linearized equation. The idea here is to look at the ratio of taxes to income in order to preserve homogeneity in prices. (We could also look at changes in tax revenue, but then a uniform price increase would change this variable.) Obviously a simple percentage change variable doesn't work, since many taxes are initially zero. The basic logic of this approach is as follows:

Let R be the ratio of taxes to income: R = T/Y, then: dR = d(T/Y) = R(t - y)/100 multiply through by Y to get: YdR = dT - Ty/100 This ratio change is computed for each tax type and for total taxes. Then the change in tax revenue itself may be computed as: dT = YdR + Ty/100 in order to determine regional income.

3.11 Common coefficients

Key Derivatives of the Base Data, Regional Expenditure and Income, Indirect Tax Receipts, Miscellaneous Coefficients.

Common Coefficients are defined as coefficients which are used in more than one module. For example, ESUBD(i) is used in the Goverment Household, Private Household, and Firms modules.

3.11.1 Coefficients:

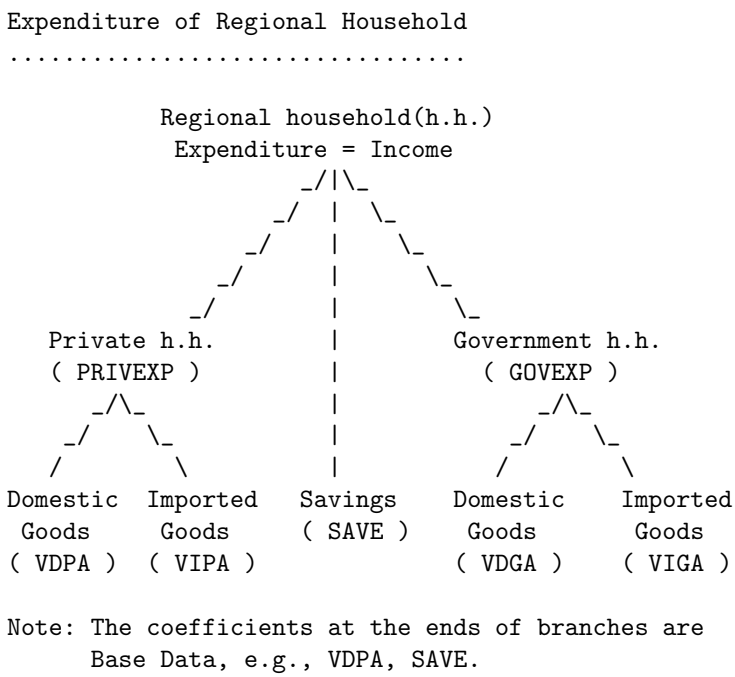
Symbol	Description
VFA(i,j,r)$_{i\in D,j\in P}$	Procuder expenditure on i by j in r valued at agent's prices
VOA(i,r)$_{i\in NS}$	Value of commodity i output in region r at agent's prices
VDM(i,r)$_{i\in T}$	Domestic sales of i in r at mkt prices (tradeables only).
VOM(i,r)$_{i\in NS}$	Value of commodity i output in region r at mrk prices.

3.12 Regional expenditure and income

3.12.1 Coefficients:

Symbol	Description
VGA(i,r)$_{i\in T}$	Government consumption expenditure on i in r at agent's prices
GOVEXP(r)	Government expenditure in region r
VPA(i,r)$_{i\in T}$	Private hhld expenditure on i in r valued at agent's prices.
PRIVEXP(r)	Private consumption expenditure in region r.
INCOME(r)	Level of expenditure, which equals NET income in region r.
DGTAX(i,r)$_{i\in T}$	Tax on government consumption of domestic good i in region r.
IGTAX(i,r)$_{i\in T}$	Tax on government consumption of imported good i in region r
TGC(r)	Government consumption tax payments in r
DPTAX(i,r)$_{i\in T}$	Tax on private consumption of domestic good i in r
IPTAX(i,r)$_{i\in T}$	Tax on private consumption of imported good i in r
TPC(r)	Private consumption tax payments in r
DFTAX(i,j,r)$_{i\in T,j\in P}$	Tax on use of domestic intermediate good i by j in r
IFTAX(i,j,r)$_{i\in T,j\in P}$	Tax on use of imported good i by j in r
TIU(r)	Firms' tax payments on intermediate good usage in r
ETAX(i,j,r)$_{i\in E,j\in P}$	Tax on use of endowment good i by industry j in r
TFU(r)	Firms' tax payments on primary factor usage in r.
PTAX(i,r)$_{i\in NS}$	Output tax on good i in region r
TOUT(r)	Production tax payments in r.
XTAXD(i,r,s)$_{i\in T}$	Tax on exports of good i from source r to destination s
TEX(r)	Export tax payments in r
MTAX(i,r,s)$_{i\in T}$	Tax on imports of good i from source r in destination s
TIM(r)	Import tax payments in r

Note on regional income Regional income is allocated between private consumption expenditure, government consumption expenditure, and saving.



Note: The coefficients at the ends of branches are Base Data, e.g., VDPA, SAVE.

3.13 Miscellaneous coefficients

3.13.1 Coefficients:

Symbol	Description
ESUBD(i)$_{i\in T}$	Region-generic eos domestic/imported for all agents
REGINV(r)	Regional GROSS investment in r (value of ”cgds” output)
NETINV(r)	Regional NET investment in r
GLOBINV	Global expenditure on net investment
CONSHR(i,r)$_{i\in T}$	Share of private hhld consumption devoted to good i in r
INCPAR(i,r)$_{i\in T}$	Expansion parameter in the CDE minimum expenditure function
UELASPRIV(r)	Elasticity of cost wrt utility from private consumption

4 Modules

1. Government Consumption
2. Private Consumption
3. Firms
4. Investment, Global Bank, and Savings
5. International Trade
6. International Transport Services
7. Regional Household
8. Equilibrium Conditions

4.1 Government consumption

4.1.1 Variables:

Symbol	Description
pg(i,r)$_{i\in T}$	Government consumption price for commodity i in region r
qg(i,r)$_{i\in T}$	Total government hhld demand for commodity i in region r
tgd(i,r)$_{i\in T}$	Tax on domestic i purchased by government hhld in r
tgm(i,r)$_{i\in T}$	Tax on imported i purchased by government hhld in r

4.1.2 Coefficients:

Symbol	Description
GMSHR(i,s)$_{i\in T}$	Share of imports for government hhld at agent's prices

4.2 Private consumption modules

4.2.1 Variables:

Symbol	Description
pp(i,r)$_{i\in T}$	Private consumption price for commodity i in r
qp(i,r)$_{i\in T}$	Private hhld demand for commodity i in region r
tp(r)	Comm.-, source-gen. shift in tax on private cons.
tpd(i,r)$_{i\in T}$	Comm.-, source-spec. shift in tax on private cons of dom.
atpd(i,r)$_{i\in T}$	Power of tax on domestic i purchased by private hhld in r.

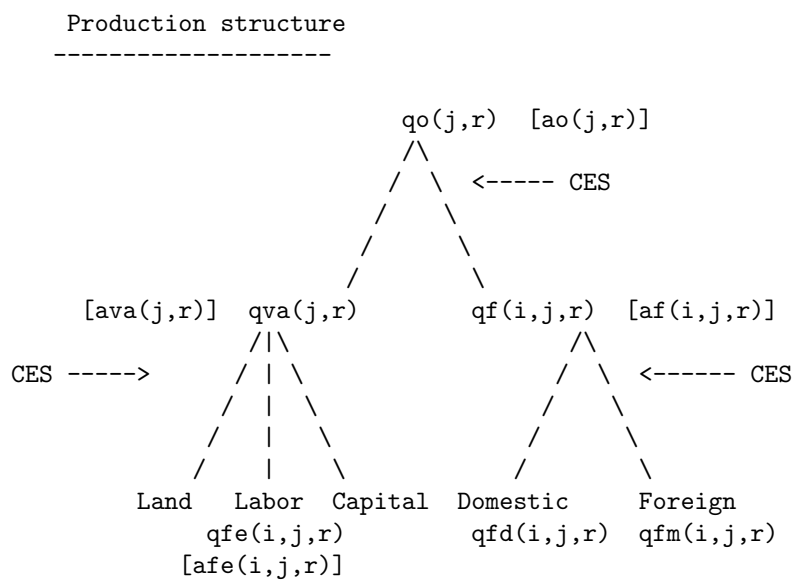
Note on tp(r) The variable tp(r) can be swapped with del_ttaxr(r) in order to generate a tax replacement scenario, whereby taxes remain a constant share of national income.

4.2.2 Coefficients:

Symbol	Description
XWCONSHR(i,r)$_{i\in T}$	Expansion-parameter-weighted consumption share
SUBPAR(i,r)$_{i\in T}$	Substitution parameter in CDE minimum expenditure function
ALPHA(i,r)$_{i\in T}$	1 - sub. parameter in the CDE minumum expenditure function
APE(i,k,r)$_{i\in T}$	Allen partial elst. of sub. between composite i and k in r
EY(i,r)$_{i\in T}$	Income elasticity of private hhld demand for i in r (HT F4)
EP(i,k,r)$_{i\in T,k\in T}$	Uncomp. elas. of private hhld demand for i wrt price of k in r (HT F5)
PMSHR(i,s)$_{i\in T}$	Share of imports for priv hhld at agent's prices

4.3 Firms

We now turn to the behavioral equations for firms. The following picture describes factor demands. The first set of equations describe demands for primary factors. (See table 4 of Hertel and Tsigas.)



4.3.1 Variables:

Symbol	Description
pva(j,r)$_{j\in P}$	Firms' price of value added in industry j of r
qva(j,r)$_{j\in P}$	Valued added in industry j of region r
pf(i,j,r)$_{i\in T,j\in P}$	Firms' price for commodity i for use by j in r
qf(i,j,r)$_{i\in T,j\in P}$	Demand for commodity i for use by j in r
ao(j,r)$_{j\in P}$	Output augmenting technical chagne in sector j of r
ava(i,r)$_{i\in P}$	Value added augmenting tech change in sector i of r
af(i,j,r)$_{i\in T,j\in P}$	Composite intermed. input i augmenting tech change by j of r
afe(i,j,r)$_{i\in E,j\in P}$	Primary factor i augmenting tech change by j of r.
ams(i,r,s)$_{i\in T}$	Import i from region r augmenting tech change in region s
aosec(j)$_{j\in P}$	Output tech change of sector j, worldwide
aoreg(r)	Output tech change in region r
aoall(j,r)$_{j\in P}$	Output augmenting technical chagne in sector j of r
avasec(j)$_{j\in P}$	Value added tech change of sector j, worldwide j.
avareg(r)	Value added tech change in region r.
avaall(j,r)$_{j\in P}$	Value added augmenting technical change in sector j of r.
afcom(i)$_{i\in T}$	Intermeidate tech change of input i, worldwide.
afsec(j)$_{j\in P}$	Intermeidate tech change of sector j, worldwide.
afreg(r)	Intermeidate tech change in region r.
afall(i,j,r)$_{i\in T,j\in P}$	Intermeidate input i augmenting tech change by j in r.
tfd(i,j,r)$_{i\in T,j\in P}$	Tax on domestic i purchased by j in r
tfm(i,j,r)$_{i\in T,j\in P}$	Tax on imported i purchased by j in r
afecom(i)$_{i\in E}$	Factor input tech change of input i, worldwide
afesec(j)$_{j\in P}$	Factor input tech change of sector j, worldwide
afereg(r)	Factor input tech change in region r
afeall(i,j,r)$_{i\in E,j\in P}$	Primary factor i augmenting tech change sector j in r
profitslack(j,r)$_{j\in P}$	Slack variable in the zero profit equation

Note on profitslack(j,r) This is exogenous, unless the user wishes to specify output in a given region exogenously.

4.3.2 Coefficients:

Symbol	Description
ESUBT(j)$_{j\in P}$	Elst. of sub. among composite intermediate inputs in production.
FMSHR(i,j,s)$_{i\in T,j\in P}$	Share of firms' imports in dom. composite, agent's prices
tf(i,j,r)$_{i\in E,j\in P}$	Tax on primary factor i used by j in region r
VVA(j,r)$_{j\in P}$	Value added in activity j in region r
SVADEFAULT(i)$_{i\in E}$	Zerodiv default for SVA
SVA(i,j,r)$_{i\in E,j\in P}$	Share of i in total value added in j in r
ESUBVA(j)$_{j\in P}$	Elst. of sub. capital/labor/land, in production of value added in j
STC(i,j,r)$_{i\in D,j\in P}$	Share of i in total costs of j in r

4.4 Investment, Global Bank, Savings

Capital stock and rate of return equations follow. They correspond to the Investment Equations of Table 8 in Hertel and Tsigas.

4.4.1 Variables:

Symbol	Description
rental(r)	Rental rate on capital = ps("capital",r)
ke(r)	End-of-period capital stock
rore(r)	Expected net rate of return on capital stock in r
rorc(r)	Current net rate of return on capital stock in r
qcgds(r)	Output of capital goods sector = qo("cgds",r)
ksvces(r)	Capital services = qo("capital",r)
EXPAND(i,r)	Change in investment levels relative to endowment stock
rorg	Global net rate of return on capital stock
cgdslack(r)	Slack variable for qcgds(r)
psaveslack(r)	Slack variable for the savings price equation

Note on cgdslack(r) This is exogenous, unless the user wishes to exogenously fix the level of gross investment in a region, in which case it can be swapped with cgdslack.

Note on psaveslack(r) This is exogenous under the normal closure. However, in order to replicate simulations from the GTAP book, where there was a single savings price, psaveslack may be swapped with psave in order to fix the latter. In this case, pcgdswld must be the numeraire so that each region faces the same price of savings.

4.4.2 Coefficients:

Symbol	Description
VKB(r)	Value of beginning-of-period capital stock in region r
INVKERATIO(r)	Ratio of gross investment to end-of-period capital stock in r
GRNETRATIO(r)	Ratio of GROSS/NET rates of return on capital in r
RORFLEX(r)	Flexibility of expected net ROR on capital stock in r wrt investment
RORDELTA	Binary coefficient to switch mechanism of allocating investment funds

Note on VKB(r) VKB(r) is used only in this sub-module, so its data are read here. The associated variables, pcgds(r) and kb(r), are used across modules, so they have been defined as common variables.

Note on RORFLEX(r) RORFLEX is the flexibility of the expected net rate of return on the capital stock, in region r, with respect to investment. If a region's capital stock increases by 1%, then it is expected that the net rate of return on capital will decline by RORFLEX %. A larger absolute value for this parameter will reduce the tendency of international investment flows to respond to changes in expected rates of return.

Note on RORDELTA RORDELTA is a binary coefficient which determines the mechanism of allocating investment funds across regions. When RORDELTA = 1, investment funds are allocated across regions to equate the change in the expected rates of return (i.e., rore(r)). When RORDELTA = 0, investment funds are allocated across regions to maintain the existing composition of capital stocks.

4.5 International Trade

4.5.1 Variables:

Symbol	Description
tx(i,r)$_{i\in T}$	Dest.-gen. change in subsidy on exports of i from r
txs(i,r,s)$_{i\in T}$	Dest.-spec. change in subsidy on exports of i from r to s
tm(i,s)$_{i\in T}$	Source-gen. change in tax on imports of i into s
tms(i,r,s)$_{i\in T}$	Source-spec. change in tax on imports of i from r into s
pr(i,r)$_{i\in T}$	Ratio of domestic to imported prices in r

Note on txs(i,r,s) The variable txs captures changes in the power of bilateral export taxes. However, the presence of a destination-generic export subsidy shift (tx) also permits the user to swap a single export tax shock with another target variable. It is most naturally swapped with the variable qo to insulate domestic producers from the world market.

Note on tms(i,r,s) The variable tms captures changes in the power of bilateral import taxes. However, the presence of a source-generic import tariff shift (tm) also permits the user to swap a single import tariff shock with another target variable. In particular, to insulate domestic producers from import price changes, it may be swapped with the relative price variable pr – see below.

4.5.2 Coefficients:

Symbol	Description
MSHRS(i,r,s)$_{i\in T}$	Share of imports from r in import bill of s at mkt prices
ESUBM(i)$_{i\in T}$	Region-generic el. of sub. among imports of i in Armington structure

4.6 International transport services

4.6.1 Variables:

Symbol	Description
qtmfsd(m,i,r,s)$_{m\in M,i\in T}$	International usage margin m on i from r to s
atmfsd(m,i,r,s)$_{m\in M,i\in T}$	Tech change in m's shipping of i from region r to s
atm(m)$_{m\in M}$	Tech change in mode m, worldwide
atf(i)$_{i\in T}$	Tech change shipping of i, worldwide
ats(r)	Tech change shipping from region r
atd(s)	Tech change shipping to s
atall(m,i,r,s)$_{m\in M,i\in T}$	Tech change in m's shipping of i from region r to s
ptrans(i,r,s)$_{i\in T}$	Cost index for international transport of i from r to s
qtm(m)$_{m\in M}$	Global margin usage
pt(m)$_{m\in M}$	Price of composite margins services, type

Note on qtmfsd(m,i,r,s) International margin usage, by Margin, Freight, Source, and Destination, i.e., the percent change in usage of m in transport of i from r to s.

Note on atmfsd(m,i,r,s) Technical progress in shipping by Margin, Freight, Source, and Destination. This is endogenous and driven by the following mode-, product-, source-, and destination-specific determinants.

Note on ptrans(i,r,s) average cost index for margin services used in getting i from r to s

Note on pt(m) price index for commodity m in margin services usage

4.6.2 Coefficients:

Symbol	Description
MSHRS(i,r,s)$_{i\in T}$	Share of imports from r in import bill of s at mkt prices
VTMFSD(m,i,r,s)$_{m\in M,i\in T}$	Int'l margin usage, by margin, freight, source, and destination
VTFSD(i,r,s)$_{i\in T}$	Aggregate value of svces in the shipment of i from r to s
VTMUSE(m)$_{m\in M}$	International margin services usage, by type
VTMPROV(m)$_{m\in M}$	International margin services provision
VTRPROV(r)	International margin supply, by region
VT	International margin supply
VTMUSESHR(m,i,r,s)$_{m\in M,i\in T}$	Share of i,r,s usage in global demand for m
VTSUPPSHR(m,r)$_{m\in M}$	Share of region r in global supply of margin m
VTUSE	International margin services usage
VTFSD_MSH(m,i,r,s)$_{m\in M,i\in T}$	Share of margin m in cost of getting i from r to s
VIWSCOST(i,r,s)$_{i\in T}$	Value of imports calculated as total cost of imports
FOBSHR(i,r,s)$_{i\in T}$	Fob share in VIW
TRNSHR(i,r,s)$_{i\in T}$	Transport share in VIW

4.7 Regional household

4.7.1 Variables:

Symbol	Type	Description
uelas(r)		Elasticity of cost of utility wrt utility
dppriv(r)		Private consumption distribution parameter
dpgov(r)		Government consumption distribution parameter
dpsave(r)		Saving distribution parameter
fincome(r)		Factor income at market prices net of depreciation
del_indtaxr(r)	C	Change in ratio of indirect taxes to INCOME in r
del_ttaxr(r)	C	Change in ratio of taxes to INCOME in r
incomeslack(r)		Slack variable in the expression for regional income
dpav(r)		Average distribution parameter shift, for EV calc.
p(r)		Price index for disposition of income by regional household
au(r)		Input-neutral shift in utility function
dpsum(r)		Sum of the distribution parameters
u(r)		Per capita utility from aggregate hhld expend. in region r

Note on incomeslack(r) This is exogenous, unless the user wishes to fix regional income.

4.7.2 Coefficients:

Symbol	Description
XSHRPRIV(r)	Private expenditure share in regional income
XSHRGOV(r)	Government expenditure share in regional income
XSHRSAVE(r)	Saving share in regional income
TINC(r)	Income tax payments in r
REVSHR(i,j,r)$_{i\in E,j\in P}$	
ETRAE(i)$_{i\in E}$	elst. of transformation for sluggish primary factor endowments (< 0)
FY(r)	Primary factor income in r net of depreciation
INDTAX(r)	Indirect tax receipts in r
DPARSUM(r)	Sum of distribution parameters
UTILELAS(r)	Elasticity of cost of utility wrt utility
DPARPRIV(r)	Private consumption distribution parameter
DPARGOV(r)	Government consumption distribution parameter
DPARSAVE(r)	Saving distribution parameter
UTILPRIV(r)	Utility from private consumption
UTILGOV(r)	Utility from government consumption
UTILSAVE(r)	Utility from saving

4.8 Market clearing conditions

4.8.1 Variables:

Symbol	Description
qds(i,r)$_{i\in T}$	Domestic sales of commodity i in r
tradslack(i,r)$_{i\in T}$	Slack variable in tradeables market clearing condition
walras_sup	Supply in omitted market–global supply of cgds composite
walras_dem	Demand in the omitted market–global demand for savings
walraslack	Slack variable in the omitted market

Note on tradslack(i,r) This is exogenous unless the user wishes to specify the price of tradeables exogenously, in which case the analysis becomes partial equilibrium and walraslack must be exogenized.

Note on walraslack This is endogenous under normal, GE closure. If the GE links are broken, then this must be swapped with the numeraire, thereby forcing global savings to explicitly equal global investment.

4.8.2 Coefficients:

Symbol	Description
SHRDFM(i,j,r)$_{i\in T,j\in P}$	Share of dom. prod. i used by sector j in r at mkt prices
SHRDPM(i,r)$_{i\in T}$	Share of domestic prod. of i used by private hhlds in r
SHRDGM(i,r)$_{i\in T}$	Share of imports of i used by gov't hhlds in r
SHRDM(i,r)$_{i\in T}$	Share of domestic sales of i in r
SHRST(m,r)$_{m\in M}$	Share of sales of m to global transport services in r
SHRXMD(i,r,s)$_{i\in T}$	Share of export sales of i to s in r
VIM(i,r)$_{i\in T}$	Value of imports of commodity i in r at domestic market prices
SHRIFM(i,j,r)$_{i\in T,j\in P}$	Share of import i used by sector j in r
SHRIPM(i,r)$_{i\in T}$	Share of import i used by private hhlds in r
SHRIGM(i,r)$_{i\in T}$	The share of import i used by gov't hhlds in r
SHREM(i,j,r)$_{i\in E,j\in P}$	Share of mobile endowment i used by sector j at mkt prices

5 Equation declaration

5.1 Government consumption

Symbol	Description
GPRICEINDEX	Definition of price index for aggregate gov't purchases (HT 40):
GOVDMNDS	Government consumption demands for composite commodities (HT 41):
GOVU	Utility from government consumption in r:
GHHDPRICE	Eq'n links domestic market and government consumption prices (HT 19):
GHHIPRICES	Eq'n links domestic market and government consumption prices (HT 22):
GCOMPRICE	Government consumption price for composite commodities (HT 42):
GHHLDAGRIMP	Government consumption demand for aggregate imports (HT 43):
GHHLDDOM	Government consumption demand for domestic goods (HT 44):
TGCRATIO	Change in ratio of government consumption tax payments to regional income:

5.2 Private consumption

Symbol	Description
PHHLDINDEX	Price index for private consumption expenditure:
PRIVATEU	Computation of utility from private consumption in r (HT 45):
UTILELASPRIV	Elasticity of expenditure wrt utility from private consumption:
PRIVDMNDS	Private consumption demands for composite commodities (HT 46):
TPDSHIFT	Permits uniform consumption tax change:
PHHDPRICE	Eq'n links domestic market and private consumption prices (HT 18):
TPMSHIFT	Permits uniform consumption tax change:
PHHIPRICES	Eq'n links domestic market and private consumption prices (HT 21):
TPCRATIO	Change in ratio of private consumption tax payments to regional income:
PCOMPRICE	Private consumption price for composite commodities (HT 47):
PHHLDDOM	Private consumption demand for domestic goods (HT 48):
PHHLDAGRIMP	Private consumption demand for aggregate imports (HT 49):

5.3 Firms

Symbol	Description
AOWORLD	Sector/region specific average rate of output augmenting tech change:
AVAWORLD	Sector/region specific average rate of value added augmenting tech change:
VADEMAND	Sector demands for primary factor composite:
AFWORLD	Sector/region specific average rate of intermediates augmenting tech change:
INTDEMAND	Industry demands for intermediate inputs, including cgds:
DMNDDPRICE	Eq'n links domestic market and firm prices (HT 20):
DMNDIPRICES	Eq'n links domestic market and firm prices (HT 23):
TIURATIO	Change in ratio of tax payments on intermediate goods to regional income:
ICOMPRICE	Industry price for composite commodities (HT 30):
INDIMP	Industry j demands for composite import i (HT 31):
INDDOM	Industry j demands for domestic good i (HT 32):
MPFACTPRICE	Eq'n links domestic and firm demand prices (HT 16):
SPFACTPRICE	Eq'n links domestic and firm demand prices (HT 17):
AFEWORLD	Sector/region specific average rate of prim. factor i augmenting tech change:
VAPRICE	Effective price of primary factor composite in each sector/region (HT 33):
TFURATIO	Change in ratio of tax payments on factor usage to regional income:
ENDWDEMAND	Demands for endowment commodities (HT 34):
OUTPUTPRICES	Eq'n links pre- and post-tax supply prices for all industries (HT 15):
TOUTRATIO	Change in ratio of output tax payments to regional income:
ZEROPROFITS	Industry zero pure profits condition (HT 6):

5.4 Investment, Global Bank, and Savings

Symbol	Description
KAPSVCES	Eq'n defines a variable for capital services (HT 52):
KAPRENTAL	Eq'n defines a variable for capital rental rate (HT 53):
CAPGOODS	Eq'n defines a variable for gross investment (HT 54):
PRCGOODS	Eq'n defines the price of cgds (HT 55):
KBEGINNING	Associates change in cap. services w/ change in cap. stock (HT 56):
KEND	Ending capital stock equals beginning stock plus net investment. (HT 10):
RORCURRENT	Current rate of return on capital in region r (HT 57):
ROREXPECTED	Expected rate of return depends on the current return and investment (HT 58):
BALDWIN	Change in investment levels relative to endowment stock:
RORGLOBAL	Either gross investment or expected rate of return in region r (HT 59):
GLOBALINV	Either expected global rate of return or global net investment (HT 11):
PRICGDS	Eq'n generates a price index for the aggregate global cgds composite (HT 60):
SAVEPRICE	Savings price:

5.5 International Trade

Symbol	Description
EXPRICES	Eq'n links agent's and world prices (HT 27):
TEXPRATIO	Change in ratio of export tax payments to regional income:
MKTPRICES	Eq'n links domestic and world prices (HT 24):
DPRICEIMP	Price for aggregate imports (HT 28):
PRICETGT	Eq'n defines target price ratio to be attained via the variable levy (HT 25):
IMPORTDEMAND	Regional demand for disaggregated imported commodities by source (HT 29):
TIMPRATIO	Change in ratio of import tax payments to regional income:

5.6 International Transport Services

Symbol	Description
QTRANS_MFSD	Bilateral demand for transport services:
TRANS_DEMAND	Global demand for margin m:
PTRANSPORT	Generate price index for composite transportation services:
TRANSCOSTINDEX	Generates flow-specific modal average cost of transport index (cf. HT7):
TRANSTECHANGE	Generates flow-specific average rate of technical change:
TRANSVCES	Generate demand for regional supply of global transportation service (HT 61):
FOBCIF	Eq'n links FOB and CIF prices for good i shipped from region r to s (HT 26'):

5.7 Regional Household

Symbol	Description
FACTORINCPRICES	Eq'n links pre- and post-tax endowment supply prices (HT 15):
TINCRATIO	Change in ratio of income tax payments to regional income:
ENDW_PRICE	Eq'n generates the composite price for sluggish endowments (HT 50):
ENDW_SUPPLY	Eq'n distributes the sluggish endowments across sectors (HT 51):
FACTORINCOME	Factor income at market prices net of depreciation:
DINDTAXRATIO	Change in ratio of indirect taxes to INCOME in r:
DTAXRATIO	Change in ratio of taxes to INCOME in r:
REGIONALINCOME	Regional income = sum of primary factor income and indirect tax receipts:
DPARAV	Average distribution parameter shift:
UTILITELASTIC	Elasticity of cost of utility wrt utility:
PRIVCONSEXP	Private consumption expenditure:
GOVCONSEXP	Government consumption expenditure:
SAVING	Saving:
PRICEINDEXREG	Price index for disposition of income by regional household:
UTILITY	Regional household utility:
DISTPARSUM	Sum of the distribution parameters:

5.8 Equilibrium Conditions

Symbol	Description
MKTCLDOM	Eq'n assures market clearing for domestic sales (HT 3):
MKTCLTRD_MARG	Eq'n assures market clearing for margins commodities (HT 1):
MKTCLTRD_NMRG	Eq'n assures market clearing for the non-margins
MKTCLIMP	Eq'n assures mkt clearing for imported goods entering each region (HT 2):
MKTCLENDWM	Eq'n assures mkt clearing for perfectly mobile endowments in each r (HT 4):
MKTCLENDWS	Eq'n assures mkt clearing for imperfectly mobile endowments in each r (HT 5):
WALRAS_S	Extra eq'n computes change in supply in the omitted market.:
WALRAS_D	Extra eq'n computes change in demand in the omitted market.:
WALRAS	Check Walras' Law. Value of "walraslack" should be zero. (HT 14):

6 Formulas

6.1 Preliminaries

6.1.1 Common coefficients:

Producer expenditure on i by j in r valued at agent's prices

$$\begin{aligned}\text{VFA}(i,j,r) &= \text{EVFA}(i,j,r) \qquad i \in E, j \in P \\ &= \text{VDFA}(i,j,s) + \text{VIFA}(i,j,s) \qquad i \in T, j \in P\end{aligned}$$

Value of commodity i output in region r at agent's prices:

$$\begin{aligned}\text{VOA}(i,r) &= \text{EVOA}(i,r) \qquad i \in E \\ &= \text{sum}(j, D, \text{VFA}(j,i,r)) \qquad i \in P\end{aligned}$$

Domestic sales of i in r at mkt prices (tradeables only):

$$\text{VDM}(i,r) = \text{VDPM}(i,r) + \text{VDGM}(i,r) + \text{sum}(j, P, \text{VDFM}(i,j,r)) \qquad i \in T$$

Value of commodity i output in region r at market prices:

$$\begin{aligned}\text{VOM}(i,r) &= \text{sum}(j, P, VFM(i,j,r)) \qquad i \in E \\ &= \text{VDM}(m,r) + \text{sum}(s, R, \text{VXMD}(m,r,s)) + \text{VST}(m,r) \qquad m \in M \\ &= \text{VDM}(i,r) + \text{sum}(s, R, \text{VXMD}(i,r,s)) \qquad i \in NM \\ &= \text{VOA}(i,r) \qquad i \in C\end{aligned}$$

Government consn expenditure on i in r at agent's prices:

$$\text{VGA}(i,s) = \text{VDGA}(i,s) + \text{VIGA}(i,s) \qquad i \in T$$

Government expenditure in region r:

$$\text{GOVEXP}(r) = \text{sum}(i, T, \text{VGA}(i,r))$$

Private hhld expenditure on i in r valued at agent's prices:

$$\text{VPA}(i,s) = \text{VDPA}(i,s) + \text{VIPA}(i,s) \qquad i \in T$$

Private consumption expenditure in region r:

$$\text{PRIVEXP}(r) = \text{sum}(i, T, \text{VPA}(i,r))$$

Level of expenditure, which equals NET income in region r:

$$\text{INCOME}(r) = \text{PRIVEXP}(r) + \text{GOVEXP}(r) + \text{SAVE}(r)$$

Tax on government consumption of domestic good i in region r:

$$\text{DGTAX}(i,r) = \text{VDGA}(i,r) - \text{VDGM}(i,r) \qquad i \in T$$

Tax on government consumption of imported good i in region r:

$$\text{IGTAX}(i,r) = \text{VIGA}(i,r) - \text{VIGM}(i,r) \qquad i \in T$$

Government consumption tax payments in r:

$$\text{TGC}(r) = \text{sum}(i, T, \text{DGTAX}(i,r) + \text{IGTAX}(i,r))$$

Tax on private consumption of domestic good i in region r:

$$\text{DPTAX}(i,r) = \text{VDPA}(i,r) - \text{VDPM}(i,r) \qquad i \in T$$

Tax on private consumption of imported good i in region r:

$$\text{IPTAX}(i,r) = \text{VIPA}(i,r) - \text{VIPM}(i,r) \qquad i \in T$$

Private consumption tax payments in r:

$$\text{TPC}(r) = \text{sum}(i, T, \text{DPTAX}(i, r) + \text{IPTAX}(i, r))$$

Tax on use of domestic intermediate good i by j in r:

$$\text{DFTAX}(i, j, r) = \text{VDFA}(i, j, r) - \text{VDFM}(i, j, r) \qquad i \in T, j \in P$$

Tax on use of imported intermediate good i by j in r:

$$\text{IFTAX}(i, j, r) = \text{VIFA}(i, j, r) - \text{VIFM}(i, j, r) \qquad i \in T, j \in P$$

Firms' tax payments on intermediate goods usage in r:

$$\text{TIU}(r) = \text{sum}(i, T, \text{sum}(j, P, \text{DFTAX}(i, j, r) + \text{IFTAX}(i, j, r)))$$

Tax on use of endowment good i by industry j in region r:

$$\text{ETAX}(i, j, r) = \text{VFA}(i, j, r) - \text{VFM}(i, j, r) \qquad i \in E, j \in P$$

Firms' tax payments on primary factor usage in r:

$$\text{TFU}(r) = \text{sum}(i, E, \text{sum}(j, P, \text{ETAX}(i, j, r)))$$

Output tax on good i in region r:

$$\text{PTAX}(i, r) = \text{VOM}(i, r) - \text{VOA}(i, r) \qquad i \in NS$$

Production tax payments in r:

$$\text{TOUT}(r) = \text{sum}(i, P, \text{PTAX}(i, r))$$

Tax on exports of good i from source r to destination s:

$$\text{XTAXD}(i, r, s) = \text{VXWD}(i, r, s) - \text{VXMD}(i, r, s) \qquad i \in T$$

Export tax payments in r:

$$\text{TEX}(r) = \text{sum}(i, T, \text{sum}(s, R, \text{XTAXD}(i, r, s)))$$

Tax on imports of good i from source r in destination s:

$$\text{MTAX}(i, r, s) = \text{VIMS}(i, r, s) - \text{VIWS}(i, r, s) \qquad i \in T$$

Import tax payments in r:

$$\text{TIM}(r) = \text{sum}(i, T, \text{sum}(s, R, \text{MTAX}(i, s, r)))$$

Regional GROSS investment in r (value of "cgds" output):

$$\text{REGINV}(r) = \text{sum}(k, C, \text{VOA}(k, r))$$

Regional NET investment in region r:

$$\text{NETINV}(r) = \text{sum}(k, C, \text{VOA}(k, r)) - \text{VDEP}(r)$$

Global expenditures on net investment:

$$\text{GLOBINV} = \text{sum}(r, R, \text{NETINV}(r))$$

Share of private hhld consumption devoted to good i in r:

$$\text{CONSHR}(i, r) = \text{VPA}(i, r) / \text{sum}(k, T, \text{VPA}(k, r)) \qquad i \in T$$

Elasticity of cost wrt utility from private consumption:

$$\text{UELASPRIV}(r) = \text{sum}(i, T, \text{CONSHR}(i, r) * \text{INCPAR}(i, r))$$

6.2 Modules

6.2.1 Government consumption

Share of imports for gov't hhld at agent's prices:

$$\text{GMSHR}(i,s) = \text{VIGA}(i,s)/\text{VGA}(i,s) \qquad i \in T$$

Expansion-parameter-weighted consumption share:

$$\text{XWCONSHR}(i,r) = \text{CONSHR}(i,r) * \text{INCPAR}(i,r)/\text{UELASPRIV}(r) \qquad i \in T$$

1 - sub. parameter in the CDE minimum expenditure function:

$$\text{ALPHA}(i,r) = 1 - \text{SUBPAR}(i,r) \qquad i \in T$$

Allen partial elst. of sub. between composite i and k in r:

$$\begin{aligned}\text{APE}(i,k,r) &= \text{ALPHA}(i,r) + \text{ALPHA}(k,r) \\ &\quad - \text{sum}(n,T,\text{CONSHR}(n,r) * \text{ALPHA}(n,r)) \qquad i,k \in T, i \neq k \\ &= 2.0 * \text{ALPHA}(i,r) - \text{sum}(n,T,\text{CONSHR}(n,r) * \text{ALPHA}(n,r)) \\ &\quad - \text{ALPHA}(i,r)/\text{CONSHR}(i,r) \qquad i \in T\end{aligned}$$

Income elast. of private hhld demand for i in r (HT F4):

$$\begin{aligned}\text{EY}(i,r) &= [1.0/\text{sum}(n,T,\text{CONSHR}(n,r) * \text{INCPAR}(n,r))] \\ &\quad * [\text{INCPAR}(i,r) * [1.0 - \text{ALPHA}(i,r)]] \\ &\quad + \text{sum}(n,T,\text{CONSHR}(n,r) * \text{INCPAR}(n,r) * \text{ALPHA}(n,r)) \\ &\quad + [\text{ALPHA}(i,r) - \text{sum}(n,T,\text{CONSHR}(n,r) * \text{ALPHA}(n,r))] \qquad i \in T\end{aligned}$$

Uncomp. elast. of private hhld demand for i wrt price of k in r (HT F5):

$$\begin{aligned}\text{EP}(i,k,r) &= 0 \\ &= [\text{APE}(i,k,r) - \text{EY}(i,r)] * \text{CONSHR}(k,r) \qquad i,k \in T(?)\end{aligned}$$

6.2.2 Composite Tradeables

Share of imports for priv hhld at agent's prices:

$$\text{PMSHR}(i,s) = \text{VIPA}(i,s)/\text{VPA}(i,s) \qquad i \in T$$

6.2.3 Firms

Share of firms' imports in dom. composite, agent's prices:

$$\text{FMSHR}(i,j,s) = \text{VIFA}(i,j,s)/\text{VFA}(i,j,s) \qquad i \in T, j \in P$$

Value added in activity j in region r:

$$\text{VVA}(j,r) = \text{sum}(i,E,\text{VFA}(i,j,r)) \qquad j \in P$$

Zerodivide default for SVA:

$$\text{SVADEFAULT}(i) = \text{sum}(j,P,\text{sum}(r,R,\text{VFA}(i,j,r)))/\text{sum}(j,P,\text{sum}(r,R,\text{VVA}(j,r))) \qquad i \in E$$

Share of i in total value added in j in r:

$$\begin{aligned}\text{SVA}(i,j,r) &= \text{VFA}(i,j,r)/\text{VVA}(j,r) \qquad i \in E, j \in P, \text{VVA} \neq 0 \\ &= \text{SVADEFAULT}(i) \qquad i \in E, j \in P, \text{VVA} = 0\end{aligned}$$

Share of i in total costs of j in r:

$$\text{STC}(i,j,r) = \text{VFA}(i,j,r)/\text{sum}(k,D,\text{VFA}(k,j,r)) \qquad i \in D, j \in P$$

6.2.4 Investment, Global Bank, and Savings

Ratio of gross investment to end-of-period capital stock in r:

$$\text{INVKERATIO}(r) = \text{REGINV}(r)/[\text{VKB}(r) + \text{NETINV}(r)]$$

Ratio of GROSS/NET rates of return on capital in r:

$$\text{GRNETRATIO}(r) = \text{sum}(h, EC, \text{VOA}(h,r))/[\text{sum}(h, EC, \text{VOA}(h,r)) - \text{VDEP}(r)]$$

6.2.5 International Trade

Share of imports from r in import bill of s at mkt prices:

$$\text{MSHRS}(i,r,s) = \text{VIMS}(i,r,s)/\text{sum}(k, R, \text{VIMS}(i,k,s)) \qquad i \in T$$

6.2.6 International Transport Services

Aggregate value of svces in the shipment of i from r to s:

$$\text{VTFSD}(i,r,s) = \text{sum}(m, M, \text{VTMFSD}(m,i,r,s)) \qquad i \in T$$

International margin services usage, by type:

$$\text{VTMUSE}(m) = \text{sum}(i, T, \text{sum}(r, R, \text{sum}(s, R, \text{VTMFSD}(m,i,r,s)))) \qquad m \in M$$

International margin services provision:

$$\text{VTMPROV}(m) = \text{sum}(r, R, \text{VST}(m,r)) \qquad m \in M$$

International margin supply, by region:

$$\text{VTRPROV}(r) = \text{sum}(m, M, \text{VST}(m,r))$$

International margin supply:

$$\text{VT} = \text{sum}(m, M, \text{sum}(r, R, \text{VST}(m,r)))$$

Share of i,r,s usage in global demand for m:

$$\begin{aligned}\text{VTMUSESHR}(m,i,r,s) &= \text{VTFSD}(i,r,s)/\text{VT} \qquad m \in M, i \in T \\ &= \text{VTMFSD(m,i,r,s)}/\text{VTMUSE(m)} \qquad m \in Mi \in T, \text{VTMUSE(m)} \neq 0\end{aligned}$$

Share of region r in global supply of margin m:

$$\begin{aligned}\text{VTSUPPSHR}(m,r) &= \text{VTRPROV}(r)/\text{VT} \qquad m \in M \\ &= \text{VST}(m,r)/\text{VTMPROV}(m) \qquad m \in M, \text{VTMPROV}(m) \neq 0\end{aligned}$$

International margin services usage:

$$\text{VTUSE} = \text{sum}(m, M, \text{sum}(i, T, \text{sum}(r, R, \text{sum}(s, R, \text{VTMFSD}(m,i,r,s)))))$$

Share of margin m in cost of getting i from r to s:

$$\begin{aligned}\text{VTFSD\_}MSH(m,i,r,s) &= \text{VTMFSD}(m,i,r,s)/\text{VTFSD}(i,r,s) \qquad m \in M, i \in T, \text{VTFSD} > 0 \\ &= \text{VTMUSE}(m)/\text{VTUSE} \qquad m \in M, i \in T, \text{VTFSD} = 0\end{aligned}$$

Value of imports calculated as total cost of imports:

$$\text{VIWSCOST}(i,r,s) = \text{VXWD}(i,r,s) + \text{VTFSD}(i,r,s) \qquad i \in T$$

FOB share in VIW:

$$\text{FOBSHR}(i,r,s) = \text{VXWD}(i,r,s)/\text{VIWSCOST}(i,r,s) \qquad i \in T$$

Transport share in VIW:

$$\text{TRNSHR}(i,r,s) = \text{VTFSD}(i,r,s)/\text{VIWSCOST}(i,r,s) \qquad i \in T$$

6.2.7 Regional Household

Private expenditure share in regional income:

$$\text{XSHRPRIV}(r) = \text{PRIVEXP}(r)/\text{INCOME}(r)$$

Government expenditure share in regional income:

$$\text{XSHRGOV}(r) = \text{GOVEXP}(r)/\text{INCOME}(r)$$

Saving share in regional income:

$$\text{XSHRSAVE}(r) = \text{SAVE}(r)/\text{INCOME}(r)$$

Income tax payments in r:

$$\text{TINC}(r) = \text{sum}(i, E, \text{PTAX}(i, r))$$

?

$$\text{REVSHR}(i, j, r) = \text{VFM}(i, j, r)/\text{sum}(k, P, \text{VFM}(i, k, r)) \qquad i \in E, j \in P$$

Primary factor income in r net of depreciation:

$$\text{FY}(r) = \text{sum}(i, E, \text{VOM}(i, r)) - \text{VDEP}(r)$$

Indirect tax receipts in r:

$$\text{INDTAX}(r) = \text{TPC}(r) + \text{TGC}(r) + \text{TIU}(r) + \text{TFU}(r) + \text{TOUT}(r) + \text{TEX}(r) + \text{TIM}(r)$$

Elasticity of cost of utility wrt utility:

$$\text{UTILELAS}(r) = [\text{UELASPRIV}(r) * \text{XSHRPRIV}(r) + \text{XSHRGOV}(r) + \text{XSHRSAVE}(r)]/\text{DPARSUM}(r)$$

Private consumption distribution parameter:

$$\text{DPARPRIV}(r) = \text{UELASPRIV}(r) * \text{XSHRPRIV}(r)/\text{UTILELAS}(r)$$

Government consumption distribution parameter:

$$\text{DPARGOV}(r) = \text{XSHRGOV}(r)/\text{UTILELAS}(r)$$

Saving distribution parameter:

$$\text{DPARSAVE}(r) = \text{XSHRSAVE}(r)/\text{UTILELAS}(r)$$

Utility from private consumption:

$$\text{UTILPRIV}(r) = 1.0$$

Utility from government consumption:

$$\text{UTILGOV}(r) = 1.0$$

Utility from saving:

$$\text{UTILSAVE}(r) = 1.0$$

6.2.8 Equilibrium Conditions

Share of dom. prod. i used by sector j in r at mkt prices:

$$\text{SHRDFM}(i,j,r) = \text{VDFM}(i,j,r)/\text{VDM}(i,r) \qquad i \in T, j \in P$$

Share of domestic prod. of i used by private hhlds in r:

$$\text{SHRDPM}(i,r) = \text{VDPM}(i,r)/\text{VDM}(i,r) \qquad i \in T$$

Share of imports of i used by gov't hhlds in r:

$$\text{SHRDGM}(i,r) = \text{VDGM}(i,r)/\text{VDM}(i,r) \qquad i \in T$$

Share of domestic sales of i in r:

$$\text{SHRDM}(i,r) = \text{VDM}(i,r)/\text{VOM}(i,r) \qquad i \in T$$

Share of sales of m to global transport services in r:

$$\text{SHRST}(m,r) = \text{VST}(m,r)/\text{VOM}(m,r) \qquad m \in M$$

Share of export sales of i to s in r:

$$\text{SHRXMD}(i,r,s) = \text{VXMD}(i,r,s)/\text{VOM}(i,r) \qquad i \in T$$

Value of imports of commodity i in r at domestic market prices:

$$\text{VIM}(i,r) = \text{sum}(j,P,\text{VIFM}(i,j,r)) + \text{VIPM}(i,r) + \text{VIGM}(i,r) \qquad i \in T$$

Share of import i used by sector j in r

$$\text{SHRIFM}(i,j,r) = \text{VIFM}(i,j,r)/\text{VIM}(i,r) \qquad i \in T, j \in P$$

Share of import i used by private hhlds in r

$$\text{SHRIPM}(i,r) = \text{VIPM}(i,r)/\text{VIM}(i,r) \qquad i \in T$$

The share of import i used by gov't hhlds in r:

$$\text{SHRIGM}(i,r) = \text{VIGM}(i,r)/\text{VIM}(i,r) \qquad i \in T$$

Share of mobile endowment i used by sector j at mkt prices:

$$\text{SHREM}(i,j,r) = \text{VFM}(i,j,r)/\text{VOM}(i,r) \qquad i \in E, j \in P$$

7 Equations

7.1 Modules

7.1.1 Government consumption

GPRICEINDEX: definition of price index for aggregate gov't purchases (HT 40):

$$(O) \qquad \text{pgov}(r) = \text{sum}(i,T,[\text{VGA}(i,r)/\text{GOVEXP}(r)] * \text{pg}(i,r))$$

$$(L) \qquad \text{pgov}(r) = \text{pgov0}(r) \prod_{i \in T} \left[\frac{\text{pg}(i,r)}{\text{pg0}(i,r)}\right]^{\text{VGA}(i,r)/\text{GOVEXP}(r)} \qquad \{\text{pgov}(r)\}$$

The government purchases are aggregated through the CD function.

GOVDMNDS: government consumption demands for composite commodities (HT 41):

$$(O) \qquad \mathrm{qg}(i,r) - \mathrm{pop}(r) = \mathrm{ug}(r) - [\mathrm{pg}(i,r) - \mathrm{pgov}(r)] \qquad i \in T$$

$$(L) \qquad \mathrm{qg}(i,r) = \mathrm{qg0}(i,r)\frac{\mathrm{pop}(r)}{\mathrm{pop0}(r)}\frac{\mathrm{ug}(r)}{\mathrm{ug0}(r)}\left[\frac{\mathrm{pgov}(r)/\mathrm{pgov0}(r)}{\mathrm{pg}(i,r)/\mathrm{pg0}(i,r)}\right] \qquad \{\mathrm{qg}(i,r)\}_{i\in T}$$

GOVU: utility from government consumption in r:

$$(O) \qquad \mathrm{yg}(r) - \mathrm{pop}(r) = \mathrm{pgov}(r) + \mathrm{ug}(r)$$

$$(L) \qquad \mathrm{yg}(r)/\mathrm{pop}(r) = \mathrm{pgov}(r)\mathrm{ug}(r) \qquad \{\mathrm{ug}(r)\}$$

where $\mathrm{ug}(r)$ is the per capita utility and $\mathrm{yg}(r)$ is the aggregated utility.

GHHDPRICE: eq'n links domestic market and government consumption prices (HT 19):

$$(O) \qquad \mathrm{pgd}(i,r) = \mathrm{tgd}(i,r) + \mathrm{pm}(i,r) \qquad i \in T$$

$$(L) \qquad \mathrm{pgd}(i,r) = \mathrm{tgd}(i,r)\mathrm{pm}(i,r) \qquad \{\mathrm{pgd}(i,r)\}_{i\in T}$$

GHHIPRICES: eq'n links domestic market and government consumption prices (HT 22):

$$(O) \qquad \mathrm{pgm}(i,r) = \mathrm{tgm}(i,r) + \mathrm{pim}(i,r) \qquad i \in T$$

$$(L) \qquad \mathrm{pgm}(i,r) = \mathrm{tgm}(i,r)\mathrm{pim}(i,r) \qquad \{\mathrm{pgm}(i,r)\}_{i\in T}$$

GCOMPRICE: government consumption price for composite commodities (HT 42):

$$(O) \qquad \mathrm{pg}(i,s) = \mathrm{GMSHR}(i,s) * \mathrm{pgm}(i,s) + [1 - \mathrm{GMSHR}(i,s)] * pgd(i,s) \qquad i \in T$$

$$(L) \qquad \mathrm{pg}(i,s) = \mathrm{pg0}(i,s)\left[\mathrm{GMSHR}(i,s)\left[\frac{\mathrm{pgm}(i,s)}{\mathrm{pgm0}(i,s)}\right]^{1-\mathrm{ESUBD}(i)} + (1-\mathrm{GMSHR}(i,s))\left[\frac{\mathrm{pgd}(i,s)}{\mathrm{pgd0}(i,s)}\right]^{1-\mathrm{ESUBD}(i)}\right]^{1/(1-\mathrm{ESUBD}(i))} \qquad \{\mathrm{pg}(i,r)\}_{i\in T}$$

GHHLDAGRIMP: government consumption demand for aggregate imports (HT 43):

$$(O) \qquad \mathrm{qgm}(i,s) = \mathrm{qg}(i,s) + \mathrm{ESUBD}(i) * [\mathrm{pg}(i,s) - \mathrm{pgm}(i,s)] \qquad i \in T$$

$$(L) \qquad \mathrm{qgm}(i,s) = \mathrm{qgm0}(i,s)\frac{\mathrm{qg}(i,s)}{\mathrm{qg0}(i,s)}\left[\frac{\mathrm{pg}(i,s)/\mathrm{pg0}(i,s)}{\mathrm{pgm}(i,s)/\mathrm{pgm0}(i,s)}\right]^{\mathrm{ESUBD}(i)} \qquad \{\mathrm{qgm}(i,r)\}_{i\in T}$$

GHHLDDOM: government consumption demand for domestic goods (HT 44):

$$(O) \qquad \mathrm{qgd}(i,s) = \mathrm{qg}(i,s) + \mathrm{ESUBD}(i) * [\mathrm{pg}(i,s) - \mathrm{pgd}(i,s)] \qquad i \in T$$

$$(L) \qquad \mathrm{qgd}(i,s) = \mathrm{qgd0}(i,s)\frac{\mathrm{qg}(i,s)}{\mathrm{qg0}(i,s)}\left[\frac{\mathrm{pg}(i,s)/\mathrm{pg0}(i,s)}{\mathrm{pgd}(i,s)/\mathrm{pgd0}(i,s)}\right]^{\mathrm{ESUBD}(i)} \qquad \{\mathrm{qgd}(i,r)\}_{i\in T}$$

TGCRATIO: change in ratio of government consumption tax payments to regional income:

$$(O) \qquad 100.0 * \mathrm{INCOME}(r) * \mathrm{del\_taxrgc}(r) + \mathrm{TGC}(r) * \mathrm{y}(r)$$
$$= \mathrm{sum}(i, T, \mathrm{VDGA}(i,r) * \mathrm{tgd}(i,r) + \mathrm{DGTAX}(i,r) * [\mathrm{pm}(i,r) + \mathrm{qgd}(i,r)])$$
$$+ \mathrm{sum}(i, T, \mathrm{VIGA}(i,r) * \mathrm{tgm}(i,r) + \mathrm{IGTAX}(i,r) * [\mathrm{pim}(i,r) + \mathrm{qgm}(i,r)])$$

$$(L) \qquad \mathrm{del\_taxrgc}(r) = \sum_{i\in T}[(\mathrm{tgd}(i,r)-1)\mathrm{pm}(i,r)\mathrm{qgd}(i,r) + (\mathrm{tgm}(i,r)-1)\mathrm{pim}(i,r)\mathrm{qgm}(i,r)] \qquad \{\mathrm{del\_taxrgc}(r)\}$$

7.1.2 Private consumption

(A) PHHLDINDEX: price index for private consumption expenditure:

$$(O) \qquad \mathrm{ppriv}(r) = \mathrm{sum}(i, T, \mathrm{CONSHR}(i,r) * \mathrm{pp}(i,r))$$

PRIVATEU: computation of utility from private consumption in r (HT 45):

$$(O) \qquad \mathrm{yp}(r) - \mathrm{pop}(r) = \mathrm{ppriv}(r) + \mathrm{UELASPRIV}(r) * \mathrm{up}(r)$$

$$(L) \qquad \sum_{i \in T} \mathrm{B}(i,r)\mathrm{up}(r)^{\mathrm{SUBPAR}(i,r)\mathrm{INCPAR}(i,r)} \left[\frac{\mathrm{pp}(i,r)}{\mathrm{yp}(r)/\mathrm{pop}(r)}\right]^{\mathrm{SUBPAR}(i,r)} = 1 \qquad \{\mathrm{up}(r)\}$$

This equation determines private consumption utility for a representative household in region r, based on the per capita private expenditure function. (HT 45)

UTILELASPRIV: elasticity of expenditure wrt utility from private consumption:

$$(O) \qquad \mathrm{uepriv}(r) = \mathrm{sum}(i, T, \mathrm{XWCONSHR}(i,r) * [\mathrm{pp}(i,r) + \mathrm{qp}(i,r) - \mathrm{yp}(r)])$$

$$(L) \qquad \mathrm{uepriv}(r) = \sum_{i \in T} \mathrm{vconshr}(i,r)\mathrm{INCPAR}(i,r) \qquad \{\mathrm{uepriv}(r)\}$$

(B) E_VCONSHR: Share of private hhld consumption devoted to good i in r:

$$(L) \qquad \mathrm{vconshr}(i,r) = \mathrm{pp}(i,r)\mathrm{qp}(i,r)/\mathrm{yp}(r) \qquad \{\mathrm{vconshr}(r)\}$$

PRIVDMNDS: private consumption demands for composite commodities (HT 46):

$$(O) \qquad \mathrm{qp}(i,r) - \mathrm{pop}(r) = \mathrm{sum}(k, T, \mathrm{EP}(i,k,r) * \mathrm{pp}(k,r)) + \mathrm{EY}(i,r) * [\mathrm{yp}(r) - \mathrm{pop}(r)]$$

$$(L) \qquad \frac{\mathrm{qp}(i,r)}{\mathrm{pop}(r)} = \frac{\mathrm{BP}(i,r)\mathrm{up}(r)^{\mathrm{SUBPAR}(i,r)\mathrm{INCPAR}(i,r)}\mathrm{SUBPAR}(i,r)\left[\frac{\mathrm{pp}(i,r)}{\mathrm{yp}(r)/\mathrm{pop}(r)}\right]^{\mathrm{SUBPAR}(i,r)-1}}{\sum_{j \in T} \mathrm{BP}(j,r)\mathrm{up}(r)^{\mathrm{SUBPAR}(j,r)\mathrm{INCPAR}(j,r)}\mathrm{SUBPAR}(j,r)\left[\frac{\mathrm{pp}(j,r)}{\mathrm{yp}(r)/\mathrm{pop}(r)}\right]^{\mathrm{SUBPAR}(j,r)}} \qquad \{\mathrm{qp}(i,r)\}_{i \in T}$$

Private consumption demands for composite commodities. Demand system is on a per capita basis. Here, yp(r) - pop(r) is % change in per capita income. (HT 46)

7.1.3 Private consumption (level)

7.1.4 Private consumption (common)

TPDSHIFT: permits uniform consumption tax change:

$$(O) \qquad \mathrm{atpd}(i,r) = \mathrm{tpd}(i,r) + \mathrm{tp}(r) \qquad i \in T$$

$$(L) \qquad \mathrm{atpd}(i,r) = \mathrm{tpd}(i,r)\mathrm{tp}(r) \qquad \{\mathrm{atpd}(i,r)\}_{i \in T}$$

PHHDPRICE: eq'n links domestic market and private consumption prices (HT 18):

$$(O) \qquad \mathrm{ppd}(i,r) = \mathrm{atpd}(i,r) + \mathrm{pm}(i,r) \qquad i \in T$$

$$(L) \qquad \mathrm{ppd}(i,r) = \mathrm{atpd}(i,r)\mathrm{pm}(i,r) \qquad \{\mathrm{ppd}(i,r)\}_{i \in T}$$

TPMSHIFT: permits uniform consumption tax change:

$$(O) \qquad \mathrm{atpm}(i,r) = \mathrm{tpm}(i,r) + \mathrm{tp}(r) \qquad i \in T$$

$$(L) \qquad \mathrm{atpm}(i,r) = \mathrm{tpm}(i,r)\mathrm{tp}(r) \qquad \{\mathrm{atpm}(i,r)\}_{i \in T}$$

PHHIPRICES: eq'n links domestic market and private consumption prices (HT 21):

$$(O) \qquad \mathrm{ppm}(i,r) = \mathrm{atpm}(i,r) + \mathrm{pim}(i,r) \qquad i \in T$$

$$(L) \qquad \mathrm{ppm}(i,r) = \mathrm{atpm}(i,r)\mathrm{pim}(i,r) \qquad \{\mathrm{ppm}(i,r)\}_{i \in T}$$

TPCRATIO: change in ratio of private consumption tax payments to regional income:

$$(O) \quad 100.0 * \text{INCOME}(r) * \text{del\_taxrpc}(r) + \text{TPC}(r) * \text{y}(r) = \text{sum}(i, T, \text{VDPA}(i,r) * \text{atpd}(i,r) + \text{DPTAX}(i,r) * [\text{pm}(i,r) + \text{qpd}(i,r)]) + \text{sum}(i, T, \text{VIPA}(i,r) * \text{atpm}(i,r) + \text{IPTAX}(i,r) * [\text{pim}(i,r) + \text{qpm}(i,r)])$$

$$(L) \quad \text{del\_taxrpc}(r) = \sum_{i \in T} [(\text{atpd}(i,r) - 1)\text{pm}(i,r)\text{qpd}(i,r) + (\text{atpm}(i,r) - 1)\text{pim}(i,r)\text{qpm}(i,r)] \qquad \{\text{del\_taxrpc}(r)\}$$

PCOMPRICE: private consumption price for composite commodities (HT 47):

$$(O) \quad \text{pp}(i,s) = \text{PMSHR}(i,s) * \text{ppm}(i,s) + [1 - \text{PMSHR}(i,s)] * \text{ppd}(i,s) \qquad i \in T$$

$$(L) \quad \text{pp}(i,s) = \text{pp0}(i,s) \left[\text{PMSHR}(i,s) \left[\frac{\text{ppm}(i,s)}{\text{ppm0}(i,s)} \right]^{1-\text{ESUBD}(i)} + (1 - \text{PMSHR}(i,s)) \left[\frac{\text{ppd}(i,s)}{\text{ppd0}(i,s)} \right]^{1-\text{ESUBD}(i)} \right]^{1/(1-\text{ESUBD}(i))} \qquad \{\text{pp}(i,s)\}_{i \in T}$$

PHHLDDOM: private consumption demand for domestic goods (HT 48):

$$(O) \quad \text{qpd}(i,s) = \text{qp}(i,s) + \text{ESUBD}(i) * [\text{pp}(i,s) - \text{ppd}(i,s)] \qquad i \in T$$

$$(L) \quad \text{qpd}(i,s) = \text{qpd0}(i,s) \frac{\text{qp}(i,s)}{\text{qp0}(i,s)} \left[\frac{\text{pp}(i,s)/\text{pp0}(i,s)}{\text{ppd}(i,s)/\text{ppd0}(i,s)} \right]^{\text{ESUBD}(i)} \qquad \{\text{qpd}(i,s)\}_{i \in T}$$

PHHLDAGRIMP: private consumption demand for aggregate imports (HT 49):

$$(O) \quad \text{qpm}(i,s) = \text{qp}(i,s) + \text{ESUBD}(i) * [\text{pp}(i,s) - \text{ppm}(i,s)] \qquad i \in T$$

$$(L) \quad \text{qpm}(i,s) = \text{qpm0}(i,s) \frac{\text{qp}(i,s)}{\text{qp0}(i,s)} \left[\frac{\text{pp}(i,s)/\text{pp0}(i,s)}{\text{ppm}(i,s)/\text{ppm0}(i,s)} \right]^{\text{ESUBD}(i)} \qquad \{\text{qpm}(i,s)\}_{i \in T}$$

7.1.5 Firms

AOWORLD: sector/region specific average rate of output augmenting tech change:

$$(O) \quad \text{ao}(j,r) = \text{aosec}(j) + \text{aoreg}(r) + \text{aoall}(j,r) \qquad j \in P$$

$$(L) \quad \text{ao}(j,r) = \text{aosec}(j)\text{aoreg}(r)\text{aoall}(j,r) \qquad \{\text{ao}(i,s)\}_{j \in P}$$

AVAWORLD: sector/region specific average rate of value added augmenting tech change:

$$(O) \quad \text{ava}(j,r) = \text{avasec}(j) + \text{avareg}(r) + \text{avaall}(j,r) \qquad j \in P$$

$$(L) \quad \text{ava}(j,r) = \text{avasec}(j)\text{avareg}(r)\text{avaall}(j,r) \qquad \{\text{ava}(j,r)\}_{j \in P}$$

VADEMAND: sector demands for primary factor composite:

$$(O) \quad \text{qva}(j,r) = -\text{ava}(j,r) + \text{qo}(j,r) - \text{ao}(j,r) - \text{ESUBT}(j) * [\text{pva}(j,r) - \text{ava}(j,r) - \text{ps}(j,r) - \text{ao}(j,r)] \qquad j \in P$$

$$(L) \quad \text{qva}(j,r) = \frac{\text{qva0}(j,r)}{\text{ava}(j,r)} \frac{\text{qo}(j,r)}{\text{ao}(j,r)\text{qo0}(j,r)} \left[\frac{(\text{ps}(j,r)\text{ao}(j,r))/\text{ps0}(j,r)}{\text{pva}(j,r)/(\text{pva0}(j,r)\text{ava}(j,r))} \right]^{\text{ESUBT}(j)} \qquad \{\text{qva}(j,r)\}_{j \in P}$$

Sector demands for primary factor composite. This equation differs from HT 35 due to the presence of intermediate input substitution.

AFWORLD: sector/region specific average rate of intermediates augmenting tech change:

$$(O) \quad \text{af}(i,j,r) = \text{afcom}(i) + \text{afsec}(j) + \text{afreg}(r) + \text{afall}(i,j,r) \qquad i \in T, j \in P$$

$$(L) \quad \text{af}(i,j,r) = \text{afcom}(i)\text{afsec}(j)\text{afreg}(r)\text{afall}(i,j,r) \qquad \{\text{af}(i,j,r)\}_{i \in T, j \in P}$$

INTDEMAND: industry demands for intermediate inputs, including cgds:

$$(O) \qquad \mathrm{qf}(i,j,r) = -\mathrm{af}(i,j,r) + \mathrm{qo}(j,r) - \mathrm{ao}(j,r) \\ - \mathrm{ESUBT}(j) * [\mathrm{pf}(i,j,r) - \mathrm{af}(i,j,r) - \mathrm{ps}(j,r) - \mathrm{ao}(j,r)] \qquad i \in T, j \in P$$

$$(L) \qquad \mathrm{qf}(i,j,r) = \frac{\mathrm{qf0}(i,j,r)}{\mathrm{af}(i,j,r)} \frac{\mathrm{qo}(j,r)}{\mathrm{ao}(j,r)\mathrm{qo0}(j,r)} \left[\frac{(\mathrm{ps}(j,r)\mathrm{ao}(j,r)/\mathrm{ps0}(j,r))}{\mathrm{pf}(i,j,r)/(\mathrm{af}(i,j,r)\mathrm{pf0}(i,j,r))} \right]^{\mathrm{ESUBT}(j)} \qquad \{\mathrm{qf}(i,j,r)\}_{i \in T, j \in P}$$

Industry demands for intermediate inputs, including cgds. This equation differs from HT 36 due to the presence of intermediate input substitution.

DMNDDPRICE: eq'n links domestic market and firm prices (HT 20):

$$(O) \qquad \mathrm{pfd}(i,j,r) = \mathrm{tfd}(i,j,r) + \mathrm{pm}(i,r) \qquad i \in T, j \in P$$

$$(L) \qquad \mathrm{pfd}(i,j,r) = \mathrm{tfd}(i,j,r)\mathrm{pm}(i,r) \qquad \{\mathrm{pfd}(i,j,r)\}_{i \in T, j \in P}$$

This equation links domestic market and firm prices. It holds only for domestic goods and it captures the effect of commodity taxation of firms. (HT 20)

DMNDIPRICES: eq'n links domestic market and firm prices (HT 23):

$$(O) \qquad \mathrm{pfm}(i,j,r) = \mathrm{tfm}(i,j,r) + \mathrm{pim}(i,r) \qquad i \in T, j \in P$$

$$(L) \qquad \mathrm{pfm}(i,j,r) = \mathrm{tfm}(i,j,r)\mathrm{pim}(i,r) \qquad \{\mathrm{pfm}(i,j,r)\}_{i \in T, j \in P}$$

This equation links domestic market and firm prices. It holds only for imported goods and it captures the effect of commodity taxation of firms. (HT 23).

TIURATIO: change in ratio of tax payments on intermediate goods to regional income:

$$(O) \qquad 100.0 * \mathrm{INCOME}(r) * \mathrm{del\_taxriu}(r) + \mathrm{TIU}(r) * \mathrm{y}(r) \\ = \mathrm{sum}(i, T, \mathrm{sum}(j, P, \mathrm{VDFA}(i,j,r) * \mathrm{tfd}(i,j,r) + \mathrm{DFTAX}(i,j,r) * [\mathrm{pm}(i,r) + \mathrm{qfd}(i,j,r)])) \\ + \mathrm{sum}(i, T, \mathrm{sum}(j, P, \mathrm{VIFA}(i,j,r) * \mathrm{tfm}(i,j,r) + \mathrm{IFTAX}(i,j,r) * [\mathrm{pim}(i,r) + \mathrm{qfm}(i,j,r)]))$$

$$(L) \qquad \mathrm{del\_taxriu}(r) = \sum_{i \in T, j \in P} [(\mathrm{tfd}(i,j,r) - 1)\mathrm{pm}(i,r)\mathrm{qfd}(i,j,r) + (\mathrm{tfm}(i,j,r) - 1)\mathrm{pim}(i,r)\mathrm{qfm}(i,j,r)] \qquad \{\mathrm{del\_taxriu}(r)\}$$

ICOMPRICE: industry price for composite commodities (HT 30):

$$(O) \qquad \mathrm{pf}(i,j,r) = \mathrm{FMSHR}(i,j,r) * \mathrm{pfm}(i,j,r) + [1 - \mathrm{FMSHR}(i,j,r)] * \mathrm{pfd}(i,j,r) \qquad i \in T, j \in P$$

$$(L) \qquad \mathrm{pf}(i,j,r) = \mathrm{pf0}(i,j,r) \left[\mathrm{FMSHR}(i,j,r) \left[\frac{\mathrm{pfm}(i,j,r)}{\mathrm{pfm0}(i,j,r)} \right]^{1-\mathrm{ESUBD}(i)} \right. \\ \left. + (1 - \mathrm{FMSHR}(i,j,r)) \left[\frac{\mathrm{pfd}(i,j,r)}{\mathrm{pfd0}(i,j,r)} \right]^{1-\mathrm{ESUBD}(i)} \right]^{\frac{1}{1-\mathrm{ESUBD}(i)}} \qquad \{\mathrm{pf}(i,j,r)\}_{i \in T, j \in P}$$

INDIMP: industry j demands for composite import i (HT 31):

$$(O) \qquad \mathrm{qfm}(i,j,s) = \mathrm{qf}(i,j,s) - \mathrm{ESUBD}(i) * [\mathrm{pfm}(i,j,s) - \mathrm{pf}(i,j,s)] \qquad i \in T, j \in P$$

$$(L) \qquad \mathrm{qfm}(i,j,s) = \mathrm{qfm0}(i,j,s) \frac{\mathrm{qf}(i,j,s)}{\mathrm{qf0}(i,j,s)} \left[\frac{\mathrm{pf}(i,j,s)/\mathrm{pf0}(i,j,s)}{\mathrm{pfm}(i,j,s)/\mathrm{pfm0}(i,j,s)} \right]^{\mathrm{ESUBD}(i)} \qquad \{\mathrm{qfm}(i,j,s)\}_{i \in T, j \in P}$$

INDDOM: industry j demands for domestic good i (HT 32):

$$(O) \qquad \mathrm{qfd}(i,j,s) = \mathrm{qf}(i,j,s) - \mathrm{ESUBD}(i) * [\mathrm{pfd}(i,j,s) - \mathrm{pf}(i,j,s)] \qquad i \in T, j \in P$$

$$(L) \qquad \mathrm{qfd}(i,j,s) = \mathrm{qfd0}(i,j,s) \frac{\mathrm{qf}(i,j,s)}{\mathrm{qf0}(i,j,s)} \left[\frac{\mathrm{pf}(i,j,s)/\mathrm{pf0}(i,j,s)}{\mathrm{pfd}(i,j,s)/\mathrm{pfd0}(i,j,s)} \right]^{\mathrm{ESUBD}(i)} \qquad \{\mathrm{qfd}(i,j,s)\}_{i \in T, j \in P}$$

MPFACTPRICE: eq'n links domestic and firm demand prices (HT 16):

$$(O) \qquad \text{pfe}(i,j,r) = \text{tf}(i,j,r) + \text{pm}(i,r) \qquad i \in EM, j \in P$$
$$(L) \qquad \text{pfe}(i,j,s) = \text{tf}(i,j,r)\text{pm}(i,r) \qquad \{\text{pfe}(i,j,r)\}_{i \in EM, j \in P}$$

SPFACTPRICE: eq'n links domestic and firm demand prices (HT 17):

$$(O) \qquad \text{pfe}(i,j,r) = \text{tf}(i,j,r) + \text{pmes}(i,j,r) \qquad i \in ES, j \in P$$
$$(L) \qquad \text{pfe}(i,j,r) = \text{tf}(i,j,r)\text{pmes}(i,j,r) \qquad \{\text{pfe}(i,j,r)\}_{i \in ES, j \in P}$$

AFEWORLD: sector/region specific average rate of prim. factor i augmenting tech change:

$$(O) \qquad \text{afe}(i,j,r) = \text{afecom}(i) + \text{afesec}(j) + \text{afereg}(r) + \text{afeall}(i,j,r) \qquad i \in E, j \in P$$
$$(L) \qquad \text{afe}(i,j,r) = \text{afecom}(i)\text{afesec}(j)\text{afereg}(r)\text{afeall}(i,j,r) \qquad \{\text{afe}(i,j,r)\}_{i \in E, j \in P}$$

VAPRICE: effective price of primary factor composite in each sector/region (HT 33):

$$(O) \qquad \text{pva}(j,r) = \text{sum}(k, E, \text{SVA}(k,j,r) * [\text{pfe}(k,j,r) - \text{afe}(k,j,r)]) \qquad j \in P$$
$$(L) \qquad \text{pva}(j,r) = \text{pva0}(j,r) \left[\sum_{k \in E} \text{SVA}(k,j,r) \left[\frac{\text{pfe}(k,j,r)}{\text{afe}(k,j,r)\text{pfe0}(k,j,r)} \right]^{1-\text{ESUBVA}(j)} \right]^{\frac{1}{1-\text{ESUBVA}(j)}} \qquad \{\text{pva}(j,r)\}_{j \in P}$$

TFURATIO: change in ratio of tax payments on factor usage to regional income:

$$\begin{aligned}(O) \qquad & 100.0 * \text{INCOME}(r) * \text{del\_taxrfu}(r) + \text{TFU}(r) * \text{y}(r) \\ & = \text{sum}(i, EM, \text{sum}(j, P, \text{VFA}(i,j,r) * \text{tf}(i,j,r) + \text{ETAX}(i,j,r) * [\text{pm}(i,r) + \text{qfe}(i,j,r)])) \\ & + \text{sum}(i, ES, \text{sum}(j, P, \text{VFA}(i,j,r) * \text{tf}(i,j,r) + \text{ETAX}(i,j,r) * [\text{pmes}(i,j,r) + \text{qfe}(i,j,r)]))\end{aligned}$$
$$(L) \qquad \text{del\_taxrfu}(r) = \sum_{i \in EM, j \in P} (\text{tf}(i,j,r) - 1)\text{pm}(i,r)\text{qfe}(i,j,r) + \sum_{i \in ES, j \in P} (\text{tf}(i,j,r) - 1)\text{pmes}(i,j,r)\text{qfe}(i,j,r) \qquad \{\text{del\_taxrfu}(r)\}$$

ENDWDEMAND: demands for endowment commodities (HT 34):

$$(O) \qquad \text{qfe}(i,j,r) = -\text{afe}(i,j,r) + \text{qva}(j,r) - \text{ESUBVA}(j) * [\text{pfe}(i,j,r) - \text{afe}(i,j,r) - \text{pva}(j,r)] \qquad i \in E, j \in P$$
$$(L) \qquad \text{qfe}(i,j,r) = \frac{\text{qfe0}(i,j,r)}{\text{afe}(i,j,r)} \frac{\text{qva}(j,r)}{\text{qva0}(j,r)} \left[\frac{\text{pva}(j,r)/\text{pva0}(j,r)}{\text{pfe}(i,j,r)/(\text{pfe0}(i,j,r)\text{afe}(i,j,r))} \right]^{\text{ESUBVA}(j)} \qquad \{\text{qfe}(i,j,r)\}_{i \in E, j \in P}$$

OUTPUTPRICES: eq'n links pre- and post-tax supply prices for all industries (HT 15):

$$(O) \qquad \text{ps}(i,r) = \text{to}(i,r) + \text{pm}(i,r) \qquad i \in P$$
$$(L) \qquad \text{ps}(i,r) = \text{to}(i,r)\text{pm}(i,r) \qquad \{\text{ps}(i,r)\}_{i \in P}$$

ThisThis equation links pre- and post-tax supply prices for all industries. This captures the effect of output taxes. TO(i,r) < 1 in the case of a tax. (HT 15)

TOUTRATIO: change in ratio of output tax payments to regional income:

$$\begin{aligned}(O) \qquad & 100.0 * \text{INCOME}(r) * \text{del\_taxrout}(r) + \text{TOUT}(r) * \text{y}(r) \\ & = \text{sum}(i, P, \text{VOA}(i,r) * [-\text{to}(i,r)] + \text{PTAX}(i,r) * [\text{pm}(i,r) + \text{qo}(i,r)])\end{aligned}$$
$$(L) \qquad \text{del\_taxrout}(r) = \sum_{i \in P} (1 - \text{to}(i,r))\text{pm}(i,r)\text{qo}(i,r) \qquad \{\text{del\_taxrout}(r)\}$$

ZEROPROFITS: industry zero pure profits condition (HT 6):

$$(O) \qquad \mathrm{ps}(j,r) + \mathrm{ao}(j,r) = \mathrm{sum}(i,E,\mathrm{STC}(i,j,r) * [\mathrm{pfe}(i,j,r) - \mathrm{afe}(i,j,r) - \mathrm{ava}(j,r)]) \\ + \mathrm{sum}(i,T,\mathrm{STC}(i,j,r) * [\mathrm{pf}(i,j,r) - \mathrm{af}(i,j,r)]) + \mathrm{profitslack}(j,r) \qquad j \in P$$

$$(L) \qquad \mathrm{ps}(j,r) = \frac{\mathrm{ps0}(j,r)}{\mathrm{ao}(j,r)} \left[\left(\sum_{i \in E} \mathrm{STC}(i,j,r) \right) \left[\frac{\mathrm{pva}(j,r)}{\mathrm{ava}(j,r)\mathrm{pva0}(j,r)} \right]^{1-\mathrm{ESUBT}(j)} \right. \\ \left. + \sum_{i \in T} \mathrm{STC}(i,j,r) \left[\frac{\mathrm{pf}(i,j,r)}{\mathrm{pf0}(i,j,r)\mathrm{af}(i,j,r)} \right]^{1-\mathrm{ESUBT}(j)} \right]^{1/(1-\mathrm{ESUBT}(j))} \\ + \mathrm{profitslack} \qquad \{\mathrm{qo}(j,r)\}_{j \in P}$$

Industry zero pure profits condition (HT 6). This condition permits us to determine the endogenous output level for each of the non-endowment sectors, excepting when profitslack is itself endogenous. The level of activity in the endowment sectors is exogenously determined.

7.1.6 Investment, Global Bank, and Savings

KAPSVCES: eq'n defines a variable for capital services (HT 52):

$$(O) \qquad \mathrm{ksvces}(r) = \mathrm{sum}(h,EC,[\mathrm{VOA}(h,r)/\mathrm{sum}(k,EC,\mathrm{VOA}(k,r))] * \mathrm{qo}(h,r))$$

$$(L) \qquad \mathrm{ksvces}(r) = \mathrm{ksvces0}(r) \sum_{h \in EC} \left[\frac{\mathrm{qo}(h,r)}{\mathrm{qo0}(h,r)} \right] \qquad \{\mathrm{ksvces}(r)\}$$

This equation defines a variable for capital services, for convenience. (There is really only one capital services item.) (HT 52)

KAPRENTAL: eq'n defines a variable for capital rental rate (HT 53):

$$(O) \qquad \mathrm{rental}(r) = \mathrm{sum}(h,EC,[\mathrm{VOA}(h,r)/\mathrm{sum}(k,EC,\mathrm{VOA}(k,r))] * \mathrm{ps}(h,r))$$

$$(L) \qquad \mathrm{rental}(r) = \mathrm{rental0}(r) \sum_{h \in EC} \frac{\mathrm{ps}(h,r)}{\mathrm{ps0}(h,r)} \qquad \{\mathrm{rental}(r)\}$$

CAPGOODS: eq'n defines a variable for gross investment (HT 54):

$$(O) \qquad \mathrm{qcgds}(r) = \mathrm{sum}(h,C,[\mathrm{VOA}(h,r)/\mathrm{REGINV}(r)] * \mathrm{qo}(h,r))$$

$$(L) \qquad \mathrm{qcgds}(r) = \mathrm{qcgds0}(r) \sum_{h \in C} \frac{\mathrm{qo}(h,r)}{\mathrm{qo0}(h,r)} \qquad \{\mathrm{qcgds}(r)\}$$

This equation defines a variable for gross investment, for convenience. There is really only one capital goods item.

PRCGOODS: eq'n defines the price of cgds (HT 55):

$$(O) \qquad \mathrm{pcgds}(r) = \mathrm{sum}(h,C,[\mathrm{VOA}(h,r)/\mathrm{REGINV}(r)] * \mathrm{ps}(h,r))$$

$$(L) \qquad \mathrm{pcgds}(r) = \mathrm{pcgds0}(r) \sum_{h \in C} \frac{\mathrm{ps}(h,r)}{\mathrm{ps0}(h,r)} \qquad \{\mathrm{pcgds}(r)\}$$

KBEGINNING: associates change in cap. services w/ change in cap. stock (HT 56):

$$(O) \qquad \mathrm{kb}(r) = \mathrm{ksvces}(r)$$

$$(L) \qquad \mathrm{kb}(r) = \mathrm{kb0}(r) \frac{\mathrm{ksvces}(r)}{\mathrm{ksvces0}(r)} \qquad \{\mathrm{kb}(r)\}$$

This equation associates any change in capital services during the period with a change in capital stock. Full capacity utilization is assumed.

KEND: Ending capital stock equals beginning stock plus net investment. (HT 10):

$$(O) \qquad \mathrm{ke}(r) = \mathrm{INVKERATIO}(r) * \mathrm{qcgds}(r) + [1.0 - \mathrm{INVKERATIO}(r)] * \mathrm{kb}(r)$$
$$(L) \qquad \mathrm{ke}(r) = (1 - \mathrm{depr}(r))\mathrm{kb}(r) + \mathrm{qcgds}(r) \qquad \{\mathrm{ke}(r)\}$$

RORCURRENT: current rate of return on capital in region r (HT 57):

$$(O) \qquad \mathrm{rorc}(r) = \mathrm{GRNETRATIO}(r) * [\mathrm{rental}(r) - \mathrm{pcgds}(r)]$$
$$(L) \qquad \mathrm{rorc}(r) = \mathrm{rorc0}(r)\frac{\mathrm{rental}(r)/\mathrm{pcgds}(r) - \mathrm{DEPR}(r)}{\mathrm{rental0}(r)/\mathrm{pcgds0}(r) - \mathrm{DEPR}(r)} \qquad \{\mathrm{rorc}(r)\}$$

ROREXPECTED: expected rate of return depends on the current return and investment (HT 58):

$$(O) \qquad \mathrm{rore}(r) = \mathrm{rorc}(r) - \mathrm{RORFLEX}(r) * [\mathrm{ke}(r) - \mathrm{kb}(r)]$$
$$(L) \qquad \mathrm{rore}(r) = \mathrm{rore0}(r)\frac{\mathrm{rorc}(r)}{\mathrm{rorc0}(r)}\left[\frac{\mathrm{ke}(r)/\mathrm{ke0}(r)}{\mathrm{kb}(r)/\mathrm{kb0}(r)}\right]^{-\mathrm{RORFLEX}(r)} \qquad \{\mathrm{rore}(r)\}$$

BALDWIN: change in investment levels relative to endowment stock:

$$(O) \qquad \mathrm{EXPAND}(i,r) = \mathrm{qcgds}(r) - \mathrm{qo}(i,r) \qquad i \in EC$$
$$(L) \qquad \mathrm{EXPAND}(i,r) = \mathrm{EXPAND0}(i,r)\frac{\mathrm{qcgds}(r)/\mathrm{qcgds0}(r)}{\mathrm{qo}(i,r)/\mathrm{qo0}(i,r)} \qquad \{\mathrm{EXPAND}(i,r)\}_{i \in EC}$$

RORGLOBAL: either gross investment or expected rate of return in region r (HT 59):

$$\begin{aligned}(O) \qquad & \mathrm{RORDELTA} * \mathrm{rore}(r) + [1 - \mathrm{RORDELTA}] \\ & * [[\mathrm{REGINV}(r)/\mathrm{NETINV}(r)] * \mathrm{qcgds}(r) - [\mathrm{VDEP}(r)/\mathrm{NETINV}(r)] * \mathrm{kb}(r)] \\ & = \mathrm{RORDELTA} * \mathrm{rorg} + [1 - \mathrm{RORDELTA}] * \mathrm{globalcgds} + \mathrm{cgdslack}(r)\end{aligned}$$

$$\begin{aligned}(L) \qquad & \mathrm{RORDELTA}\left[\mathrm{rorg0}\frac{\mathrm{rore}(r)}{\mathrm{rore0}(r)} - \mathrm{rorg}\right] \\ & + (1 - \mathrm{RORDELTA})\left[\mathrm{globalcgds} - \mathrm{globalcgds0}\frac{\mathrm{qcgds}(r) - \mathrm{depr}(r)\mathrm{kb}(r)}{\mathrm{qcgds0}(r) - \mathrm{depr}(r)\mathrm{kb0}(r)}\right] + \mathrm{cgdslack}(r) = 0 \qquad \{\mathrm{qcgds}(r)\}\end{aligned}$$

This equation determines either gross investment or the expected rate of return in each region, depending on the setting for the binary RORDELTA parameter.

GLOBALINV (*): either expected global rate of return or global net investment (HT 11):

$$\begin{aligned}(O) \qquad & \mathrm{RORDELTA} * \mathrm{globalcgds} + [1 - \mathrm{RORDELTA}] * \mathrm{rorg} \\ & = \mathrm{RORDELTA} * \mathrm{sum}(r, R, [\mathrm{REGINV}(r)/\mathrm{GLOBINV}] * \mathrm{qcgds}(r) - [\mathrm{VDEP}(r)/\mathrm{GLOBINV}] * \mathrm{kb}(r)) \\ & + [1 - \mathrm{RORDELTA}] * \mathrm{sum}(r, R, [\mathrm{NETINV}(r)/\mathrm{GLOBINV}] * \mathrm{rore}(r))\end{aligned}$$

$$\begin{aligned}(L) \qquad & \mathrm{RORDELTA}(\mathrm{pcgdswld} \times \mathrm{globalcgds} - \sum_{r \in R} \mathrm{pcgds}(r)\,[\mathrm{qcgds}(r) - \mathrm{depr}(r)\mathrm{kb}(r)]) \\ & + (1 - \mathrm{RORDELTA})\left[\mathrm{rorg} - \mathrm{rorg0}\prod_{r \in R}\left[\frac{\mathrm{rore}(r)}{\mathrm{rore0}(r)}\right]^{\mathrm{sh\_netinv\_globinv}(r)}\right] = 0 \qquad \{\mathrm{globalcgds}\}\end{aligned}$$

This equation computes either the change in global net investment (when RORDELTA = 1), or the change in the expected global rate of return on capital (when RORDELTA = 0).

(B) E_SH_NETINV_GLOBINV: Share of net investment:

$$(L) \qquad \mathrm{sh\_netinv\_globinv}(r) = \frac{1}{2}\frac{\mathrm{NETINV}(r)}{\mathrm{GLOBINV}} + \frac{1}{2}\frac{\mathrm{pcgds}(r)[\mathrm{qcgds}(r) - \mathrm{DEPR}(r)\mathrm{kb}(r)]}{\mathrm{pcgdswld} \times \mathrm{globalcgds}} \qquad \{\mathrm{sh\_netinv\_globinv}(r)\}$$

PRICGDS (*): eq'n generates a price index for the aggregate global cgds composite (HT 60):

$$(O) \qquad \text{pcgdswld} = \text{sum}(r, R, [\text{NETINV}(r)/\text{GLOBINV}] * \text{pcgds}(r))$$

$$(L) \qquad \text{pcgdswld} = \text{pcgdswld0} \prod_{r \in R} \left[\frac{\text{pcgds}(r)}{\text{pcgds0}(r)} \right]^{\text{sh\_netinv\_globinv}(r)} \qquad \{\text{pfactwld}\}$$

SAVEPRICE (*): savings price.

$$(O) \qquad \text{psave}(r) = \text{pcgds}(r) + \text{sum}(s, R, [[\text{NETINV}(s) - \text{SAVE}(s)]/\text{GLOBINV}] * \text{pcgds}(s)) + \text{psaveslack}(r)$$

$$(L) \qquad \text{psave}(r) = \text{psave0}(r) \frac{\text{pcgds}(r)}{\text{pcgds0}(r)} \frac{\text{psave\_adj}}{\text{psave\_adj0}} + \text{psaveslack}(r) \qquad \{\text{psave}(r)\}$$

In contrast to the GTAP book, the price of savings is now region-specific and it changes at the same rate as the price of regional investment, plus an adjustment factor which accounts for the fact that savings and investment are not equal at the regional level.

(B) E_PSAVE_ADJ:

$$(L) \qquad \text{psave\_adj} = \text{psave\_adj0} \prod_{s \in R} \left[\frac{\text{pcgds}(s)}{\text{pcgds0}(s)} \right]^{\text{sh\_netinv\_save\_globinv}(r)} \qquad \{\text{psave\_adj}\}$$

(B) E_SH_NETINV_SAVE_GLOBINV:

$$(L) \qquad \text{sh\_netinv\_save\_globinv}(r) = \frac{1}{2} \frac{\text{NETINV}(r) - \text{SAVE}(r)}{\text{GLOBINV}} + \frac{1}{2} \frac{\text{pcgds}(r)[\text{qcgds}(r) - \text{DEPR}(r)\text{kb}(r)] - \text{psave}(r)\text{qsave}(r)}{\text{pcgdswld} \times \text{globalcgds}} \qquad \{\text{sh\_netinv\_save\_globinv}(r)\}$$

7.1.7 International Trade

EXPRICES: eq'n links agent's and world prices (HT 27):

$$(O) \qquad \text{pfob}(i, r, s) = \text{pm}(i, r) - \text{tx}(i, r) - \text{txs}(i, r, s) \qquad i \in T$$

$$(L) \qquad \text{pfob}(i, r, s) = \frac{\text{pm}(i, r)}{\text{tx}(i, r)\text{txs}(i, r, s)} \qquad \{\text{pfob}(i, r, s)\}_{i \in T}$$

TEXPRATIO: change in ratio of export tax payments to regional income:

$$(O) \qquad 100.0 * \text{INCOME}(r) * \text{del\_taxrexp}(r) + \text{TEX}(r) * \text{y}(r) = \text{sum}(i, T, \text{sum}(s, R, \text{VXMD}(i, r, s) * [-\text{tx}(i, r) - \text{txs}(i, r, s)] + \text{XTAXD}(i, r, s) * [\text{pfob}(i, r, s) + \text{qxs}(i, r, s)])$$

$$(L) \qquad \text{del\_taxrexp}(r) = \sum_{i \in T, s \in R} [1 - \text{tx}(i, r)\text{txs}(i, r, s)]\text{pfob}(i, r, s)\text{qxs}(i, r, s) \qquad \{\text{del\_taxrexp}(r)\}$$

MKTPRICES: eq'n links domestic and world prices (HT 24):

$$(O) \qquad \text{pms}(i, r, s) = \text{tm}(i, s) + \text{tms}(i, r, s) + \text{pcif}(i, r, s)$$

$$(L) \qquad \text{pms}(i, r, s) = \text{tm}(i, s)\text{tms}(i, r, s)\text{pcif}(i, r, s) \qquad \{\text{pms}(i, r, s)\}$$

DPRICEIMP: price for aggregate imports (HT 28):

$$(O) \qquad \text{pim}(i, s) = \text{sum}(k, R, \text{MSHRS}(i, k, s) * [\text{pms}(i, k, s) - \text{ams}(i, k, s)]) \qquad i \in T$$

$$(L) \qquad \text{pim}(i, s) = \text{pim0}(i, s) \left[\sum_{k \in R} \text{MSHRS}(i, k, s) \left[\frac{\text{pms}(i, k, s)}{\text{pms0}(i, k, s)\text{ams}(i, k, s)} \right]^{1 - \text{ESUBM}(i)} \right]^{\frac{1}{1 - \text{ESUBM}(i)}} \qquad \{\text{pim}\}_{i \in T}$$

PRICETGT: eq'n defines target price ratio to be attained via the variable levy (HT 25):

$$(O) \qquad \text{pr}(i,s) = \text{pm}(i,s) - \text{pim}(i,s) \qquad i \in T$$

$$(L) \qquad \text{pr}(i,s) = \text{pr0}(i,s)\frac{\text{pm}(i,s)/\text{pm0}(i,s)}{\text{pim}(i,s)/\text{pim0}(i,s)} \qquad \{\text{pr}(i,s)\}_{i\in T}$$

This equation defines the target price ratio to be attained via the variable levy. This price ratio is the ratio of domestic to average imported goods' price. Note that the way this price ratio is defined, it includes intraregional imports as well. In most applications, regions will represent groups of individual countries. However, in the case of the EU, this is problematic, since recent versions of the database have incorporated intra-EU trade flows. Therefore, when aggregated to the EU level, the composite import price includes both intra-EU and outside imports. So some modification is needed to handle the EU case.

IMPORTDEMAND: regional demand for disaggregated imported commodities by source (HT 29):

$$(O) \qquad \text{qxs}(i,r,s) = -\text{ams}(i,r,s) + \text{qim}(i,s) - \text{ESUBM}(i) * [\text{pms}(i,r,s) - \text{ams}(i,r,s) - \text{pim}(i,s)] \qquad i \in T$$

$$(L) \qquad \text{qxs}(i,r,s) = \frac{\text{qxs0}(i,r,s)}{\text{ams}(i,r,s)}\frac{\text{qim}(i,s)}{\text{qim0}(i,s)}\left[\frac{\text{pim}(i,s)/\text{pim0}(i,s)}{\text{pms}(i,r,s)/(\text{pms0}(i,r,s)\text{ams}(i,r,s))}\right]^{\text{ESUBM}(i)} \qquad \{\text{qxs}(i,r,s)\}_{i\in T}$$

TIMPRATIO: change in ratio of import tax payments to regional income:

$$\begin{aligned}(O) \qquad & 100.0 * \text{INCOME}(r) * \text{del\_taxrimp}(r) + \text{TIM}(r) * \text{y}(r) \\ & = \text{sum}(i,T,\text{sum}(s,R,\text{VIMS}(i,s,r) * [\text{tm}(i,r) + \text{tms}(i,s,r)] \\ & + \text{MTAX}(i,s,r) * [\text{pcif}(i,s,r) + \text{qxs}(i,s,r)])\end{aligned}$$

$$(L) \qquad \text{del\_taxrimp}(r) = \sum_{i\in T, s\in R}[\text{tm}(i,r)\text{tms}(i,s,r) - 1]\text{pcif}(i,s,r)\text{qxs}(i,s,r) \qquad \{\text{del\_taxrimp}(r)\}$$

7.1.8 International Transport Services

QTRANS_MFSD: bilateral demand for transport services:

$$(O) \qquad \text{qtmfsd}(m,i,r,s) = \text{qxs}(i,r,s) - \text{atmfsd}(m,i,r,s) \qquad m \in M, i \in T$$

$$(L) \qquad \text{qtmfsd}(m,i,r,s) = \frac{\text{qxs}(i,r,s)}{\text{atmfsd}(m,i,r,s)} \qquad \{\text{qtmfsd}(m,i,r,s)\}_{m\in M, i\in T}$$

This equation computes the bilateral demand for international transportation services. It reflects the fact that the demand for services along any particular route is proportional to the quantity of merchandise shipped [i.e., QXS(i,r,s)]. It is here that we introduce the potential for input-augmenting tech change, atmfsd(m,i,r,s), which is commodity- and route-specific. Thus, in the levels: ATMFSD(m,i,r,s) * QTMFSD(m,i,r,s) = QXS(i,r,s) where QTMFSD is the amount of composite margins services m used along this route. Technological improvements are reflected by atmfsd(i,r,s) ¿ 0, and these reduce the margins services required for this i,r,s triplet. Tech. change also dampens the cost of shipping, thereby lowering the CIF price implied by a given FOB value (see 6-2).

TRANS_DEMAND: global demand for margin m:

$$(O) \qquad \text{qtm}(m) = \text{sum}(i,T,\text{sum}(r,R,\text{sum}(s,R,\text{VTMUSESHR}(m,i,r,s) * \text{qtmfsd}(m,i,r,s)))) \qquad m \in M$$

$$(L) \qquad \text{qtm}(m) = \sum_{i\in T, s, r\in R}\text{qtmfsd}(m,i,r,s) \qquad \{\text{qtm}(m)\}_{m\in M}$$

PTRANSPORT: generate price index for composite transportation services:

$$(O) \qquad \text{pt}(m) = \text{sum}(r,R,\text{VTSUPPSHR}(m,r) * \text{pm}(m,r)) \qquad m \in M$$

$$(L) \qquad \text{pt}(m) = \text{pt0}(m)\prod_{r\in R}\left[\frac{\text{pm}(m,r)}{\text{pm0}(m,r)}\right]^{\text{VTSUPPSHR}(m,r)} \qquad \{\text{pt}(m)\}_{m\in M}$$

This equation generates a price index for transportation services based on zero profits. NOTE: (1) Sales to international transportation are not subject to export tax. This is why we base the costs to the transport sector on market prices of the goods sold to international transportation. (2) We assume that the supply shares for margin services are uniform across freight, source of freight, and destination. (cf. HT 7).

TRANSCOSTINDEX: generates flow-specific modal average cost of transport index (cf. HT7):

$$(O) \qquad \text{ptrans}(i,r,s) = \text{sum}(m, M, \text{VTFSD\_MSH}(m,i,r,s) * [\text{pt}(m) - \text{atmfsd}(m,i,r,s)]) \qquad i \in T$$

$$(L) \qquad \text{ptrans}(i,r,s) = \text{ptrans0}(i,r,s) \sum_{m \in M} \left[\frac{\text{pt}(m)}{\text{pt0}(m)\text{atmfsd}(m,i,r,s)} \right] \qquad \{\text{ptrans}(i,r,s)\}_{i \in T}$$

TRANSTECHANGE: generates flow-specific average rate of technical change:

$$(O) \qquad \text{atmfsd}(m,i,r,s) = \text{atm}(m) + \text{atf}(i) + \text{ats}(r) + \text{atd}(s) + \text{atall}(m,i,r,s) \qquad m \in M, i \in T$$

$$(L) \qquad \text{atmfsd}(m,i,r,s) = \text{atm}(m)\text{atf}(i)\text{ats}(r)\text{atd}(s)\text{atall}(m,i,r,s) \qquad \{\text{atmfsd}(m,i,r,s)\}_{m \in M, i \in T}$$

TRANSVCES: generate demand for regional supply of global transportation service (HT 61):

$$(O) \qquad \text{qst}(m,r) = \text{qtm}(m) + [\text{pt}(m) - \text{pm}(m,r)] \qquad m \in M$$

$$(L) \qquad \text{qst}(m,r) = \text{qst0}(m,r) \frac{\text{qtm}(m)}{\text{qtm0}(m)} \frac{\text{pt}(m)/\text{pt0}(m)}{\text{pm}(m,r)/\text{pm0}(m,r)} \qquad \{\text{qst}(m,r)\}_{m \in M}$$

This equation generates the international transport sector's derived demand for regional supplies of transportation services. It reflects a unitary elasticity of substitution between transportation services inputs from different regions.

FOBCIF: eq'n links FOB and CIF prices for good i shipped from region r to s (HT 26'):

$$(O) \qquad \text{pcif}(i,r,s) = \text{FOBSHR}(i,r,s) * \text{pfob}(i,r,s) + \text{TRNSHR}(i,r,s) * \text{ptrans}(i,r,s) \qquad i \in T$$

$$(L) \qquad \text{pcif}(i,r,s) = \text{pcif0}(i,r,s) \left[\text{FOBSHR}(i,r,s) \left[\frac{\text{pfob}(i,r,s)}{\text{pfob0}(i,r,s)} \right] + (1 - \text{FOBSHR}(i,r,s)) \left[\frac{\text{ptrans}(i,r,s)}{\text{ptrans0}(i,r,s)} \right] \right] \qquad \{\text{pcif}(i,r,s)\}_{i \in T}$$

This equation links export and import prices for each commodity/route triplet. Note that technical change is embodied in ptrans(i,r,s) which is now a cost index, as opposed to (HT 26') where it represented the price of margins services.

7.1.9 Regional Household

FACTORINCPRICES: eq'n links pre- and post-tax endowment supply prices (HT 15):

$$(O) \qquad \text{ps}(i,r) = \text{to}(i,r) + \text{pm}(i,r) \qquad i \in E$$

$$(L) \qquad \text{ps}(i,r) = \text{to}(i,r)\text{pm}(i,r) \qquad \{\text{ps}(i,r)\}_{i \in E}$$

TINCRATIO: change in ratio of income tax payments to regional income:

$$(O) \qquad 100.0 * \text{INCOME}(r) * \text{del\_taxrinc}(r) + \text{TINC}(r) * \text{y}(r)$$
$$= \text{sum}(i, E, \text{VOA}(i,r) * [-\text{to}(i,r)] + \text{PTAX}(i,r) * [\text{pm}(i,r) + \text{qo}(i,r)])$$

$$(L) \qquad \text{del\_taxrinc}(r) = \sum_{i \in E} (1 - \text{to}(i,r))\text{pm}(i,r)\text{qo}(i,r) \qquad \{\text{del\_taxrinc}(r)\}$$

ENDW_PRICE: eq'n generates the composite price for sluggish endowments (HT 50):

$$(O) \qquad \text{pm}(i,r) = \text{sum}(k, P, \text{REVSHR}(i,k,r) * \text{pmes}(i,k,r)) \qquad i \in ES$$

$$(L) \qquad \text{pm}(i,r) = \text{pm0}(i,r) \left[\sum_{k \in P} \text{REVSHR}(i,k,r) \left[\frac{\text{pmes}(i,k,r)}{\text{pmes0}(i,k,r)} \right]^{1-\text{ETRAE}(i)} \right]^{1/(1-\text{ETRAE}(i))} \qquad \{\text{pm}(i,r)\}_{i \in ES}$$

ENDW_SUPPLY: eq'n distributes the sluggish endowments across sectors (HT 51):

$$(O) \qquad \text{qoes}(i,j,r) = \text{qo}(i,r) - \text{endwslack}(i,r) + \text{ETRAE}(i) * [\text{pm}(i,r) - \text{pmes}(i,j,r)] \qquad i \in ES, j \in P$$

$$(L) \qquad \text{qoes}(i,j,r) = \text{qoes0}(i,j,r) \frac{\text{qo}(i,r)}{\text{qo0}(i,r)} \left[\frac{\text{pm}(i,r)/\text{pm0}(i,r)}{\text{pmes}(i,j,r)/\text{pmes0}(i,j,r)} \right]^{\text{ETRAE}(i)} - \text{endwslack}(i,r) \qquad \{\text{qoes}(i,j,r)\}_{i \in ES, j \in P}$$

FACTORINCOME: factor income at market prices net of depreciation:

$$(O) \quad \text{FY}(r) * \text{fincome}(r) = \text{sum}(i, E, \text{VOM}(i,r) * [\text{pm}(i,r) + \text{qo}(i,r)]) - \text{VDEP}(r) * [\text{pcgds}(r) + \text{kb}(r)]$$

$$(L) \quad \text{fincome}(r) = \sum_{i \in E} \text{pm}(i,r)\text{qo}(i,r) - \text{depr}(r)\text{pcgds}(r)\text{kb}(r) \qquad \{\text{fincome}(r)\}$$

DINDTAXRATIO: change in ratio of indirect taxes to INCOME in r:

$$\begin{aligned}(O) \quad \text{del\_indtaxr}(r) &= \text{del\_taxrpc}(r) + \text{del\_taxrgc}(r) + \text{del\_taxriu}(r) \\ &\quad + \text{del\_taxrfu}(r) + \text{del\_taxrout}(r) + \text{del\_taxrexp}(r) + \textit{del\_taxrimp}(r) \\ (L) \quad \text{del\_indtaxr}(r) &= \text{del\_taxrpc}(r) + \text{del\_taxrgc}(r) + \text{del\_taxriu}(r) + \text{del\_taxrfu}(r) \\ &\quad + \text{del\_taxrout}(r) + \text{del\_taxrexp}(r) + \text{del\_taxrimp}(r) \qquad \{\text{del\_indtaxr}(r)\}\end{aligned}$$

DTAXRATIO: change in ratio of taxes to INCOME in r:

$$\begin{aligned}(O) \quad \text{del\_ttaxr}(r) &= \text{del\_taxrpc}(r) + \text{del\_taxrgc}(r) + \text{del\_taxriu}(r) + \text{del\_taxrfu}(r) \\ &\quad + \text{del\_taxrout}(r) + \text{del\_taxrexp}(r) + \text{del\_taxrimp}(r) + \text{del\_taxrinc}(r) \\ (L) \quad \text{del\_ttaxr}(r) &= \text{del\_taxrpc}(r) + \text{del\_taxrgc}(r) + \text{del\_taxriu}(r) + \text{del\_taxrfu}(r) \\ &\quad + \text{del\_taxrout}(r) + \text{del\_taxrexp}(r) + \text{del\_taxrimp}(r) + \text{del\_taxrinc}(r) \qquad \{\text{del\_ttaxr}(r)\}\end{aligned}$$

This variable can be swapped with the commodity- and source-generic consumption tax shift, tp(r), in order to generate a tax replacement scenario, whereby taxes remain a constant share of national income.

REGIONALINCOME: regional income = sum of primary factor income and indirect tax receipts:

$$\begin{aligned}(O) \quad \text{INCOME}(r) * \text{y}(r) &= \text{FY}(r) * \text{fincome}(r) + 100.0 * \text{INCOME}(r) * \text{del\_indtaxr}(r) \\ &\quad + \text{INDTAX}(r) * \text{y}(r) + \text{INCOME}(r) * \text{incomeslack}(r) \\ (L) \quad \text{y}(r) &= \text{fincome}(r) + \text{del\_indtaxr}(r) + \text{incomeslack}(r) \qquad \{\text{y}(r)\}\end{aligned}$$

This equation computes regional income as the sum of primary factor payment and indirect tax receipts. The first term computes the change in endowment income, net of depreciation. The subsequent terms compute the change in indirect tax receipts for various transactions taxes.

(A) DPARAV: average distribution parameter shift:

$$(O) \quad \text{dpav}(r) = \text{XSHRPRIV}(r) * \text{dppriv}(r) + \text{XSHRGOV}(r) * \text{dpgov}(r) + \text{XSHRSAVE}(r) * \text{dpsave}(r)$$

UTILITELASTIC: elasticity of cost of utility wrt utility:

$$\begin{aligned}(O) \quad \text{uelas}(r) &= \text{XSHRPRIV}(r) * \text{uepriv}(r) - \text{dpav}(r) \\ (L) \quad \text{uelas}(r) &= \frac{1}{\text{dppriv}(r)/\text{uepriv}(r) + \text{dpgov}(r) + \text{dpsave}(r)} \qquad \{\text{uelas}(r)\}\end{aligned}$$

PRIVCONSEXP: private consumption expenditure:

$$\begin{aligned}(O) \quad \text{yp}(r) - \text{y}(r) &= -[\text{uepriv}(r) - \text{uelas}(r)] + \textit{dppriv}(r) \\ (L) \quad \frac{\text{yp}(r)}{\text{y}(r)} &= \frac{\text{uelas}(r)}{\text{uepriv}(r)}\text{dppriv}(r) \qquad \{\text{yp}(r)\}\end{aligned}$$

GOVCONSEXP: government consumption expenditure:

$$\begin{aligned}(O) \quad \text{yg}(r) - \text{y}(r) &= \text{uelas}(r) + \text{dpgov}(r) \\ (L) \quad \frac{\text{yg}(r)}{\text{y}(r)} &= \text{uelas}(r)\text{dpgov}(r) \qquad \{\text{yg}(r)\}\end{aligned}$$

SAVING: saving:

$$(O) \qquad \text{psave}(r) + \text{qsave}(r) - \text{y}(r) = \text{uelas}(r) + \text{dpsave}(r)$$

$$(L) \qquad \frac{\text{psave}(r)\text{qsave}(r)}{\text{y}(r)} = \text{uelas}(r)\text{dpsave}(r) \qquad \{\text{qsave}(r)\}$$

(A) PRICEINDEXREG: price index for disposition of income by regional household:

$$(O) \qquad \text{p}(r) = \text{XSHRPRIV}(r) * \text{ppriv}(r) + \text{XSHRGOV}(r) * \text{pgov}(r) + \text{XSHRSAVE}(r) * \text{psave}(r)$$

UTILITY: regional household utility:

$$\begin{aligned}(O) \qquad \text{u}(r) = {} & \text{au}(r) + \text{DPARPRIV}(r) * \text{loge}(\text{UTILPRIV}(r)) * \text{dppriv}(r) \\ & + \text{DPARGOV}(r) * \text{loge}(\text{UTILGOV}(r)) * \text{dpgov}(r) \\ & + \text{DPARSAVE}(r) * \text{loge}(\text{UTILSAVE}(r)) * \text{dpsave}(r) \\ & + [1.0/\text{UTILELAS}(r)] * [\text{y}(r) - \text{pop}(r) - \text{p}(r)]\end{aligned}$$

$$(L) \qquad \text{u}(r) = \text{u0}(r)\frac{\text{au}(r)}{\text{au0}(r)}\left[\frac{\text{up}(r)}{\text{up0}(r)}\right]^{\text{dppriv}(r)}\left[\frac{\text{ug}(r)}{\text{ug0}(r)}\right]^{\text{dpgov}(r)}\left[\frac{\text{qsave}(r)}{\text{qsave0}(r)}\right]^{\text{dpsave}(r)} \qquad \{\text{u}(r)\}$$

(A) DISTPARSUM: sum of the distribution parameters:

$$\begin{aligned}(O) \qquad \text{DPARSUM}(r) * \text{dpsum}(r) = {} & \text{DPARPRIV}(r) * \text{dppriv}(r) + \text{DPARGOV}(r) * \text{dpgov}(r) \\ & + \text{DPARSAVE}(r) * \text{dpsave}(r)\end{aligned}$$

7.1.10 Equilibrium Conditions

MKTCLDOM: eq'n assures market clearing for domestic sales (HT 3):

$$\begin{aligned}(O) \qquad \text{qds}(i,r) = {} & \text{sum}(j, P, \text{SHRDFM}(i,j,r) * \text{qfd}(i,j,r)) + \text{SHRDPM}(i,r) * \text{qpd}(i,r) \\ & + \text{SHRDGM}(i,r) * \text{qgd}(i,r) \qquad i \in T\end{aligned}$$

$$(L) \qquad \text{qds}(i,r) = \sum_{j \in P} \text{qfd}(i,j,r) + \text{qpd}(i,r) + \text{qgd}(i,r) \qquad \{\text{qds}(i,r)\}_{i \in T}$$

MKTCLTRD_MARG: eq'n assures market clearing for margins commodities (HT 1):

$$\begin{aligned}(O) \qquad \text{qo}(m,r) = {} & \text{SHRDM}(m,r) * \text{qds}(m,r) + \text{SHRST}(m,r) * \text{qst}(m,r) \\ & + \text{sum}(s, R, \text{SHRXMD}(m,r,s) * \text{qxs}(m,r,s)) + \text{tradslack}(m,r) \qquad m \in M\end{aligned}$$

$$(L) \qquad \text{qo}(m,r) = \text{qds}(m,r) + \text{qst}(m,r) + \sum_{s \in R} \text{qxs}(m,r,s) + \text{tradslack}(m,r) \qquad \{\text{pm}(m,r)\}_{m \in M}$$

MKTCLTRD_NMRG: eq'n assures market clearing for the non-margins commodities (HT 1):

$$\begin{aligned}(O) \qquad \text{qo}(i,r) = {} & \text{SHRDM}(i,r) * \text{qds}(i,r) \\ & + \text{sum}(s, R, \text{SHRXMD}(i,r,s) * \text{qxs}(i,r,s)) + \text{tradslack}(i,r) \qquad i \in NM\end{aligned}$$

$$(L) \qquad \text{qo}(i,r) = \text{qds}(i,r) + \sum_{s \in R} \text{qxs}(i,r,s) + \text{tradslack}(i,r) \qquad \{\text{pm}(i,r)\}_{i \in NM}$$

MKTCLIMP: eq'n assures mkt clearing for imported goods entering each region (HT 2):

$$\begin{aligned}(O) \qquad \text{qim}(i,r) = {} & \text{sum}(j, P, \text{SHRIFM}(i,j,r) * \text{qfm}(i,j,r)) \\ & + \text{SHRIPM}(i,r) * \text{qpm}(i,r) + \text{SHRIGM}(i,r) * \text{qgm}(i,r) \qquad i \in T\end{aligned}$$

$$(L) \qquad \text{qim}(i,r) = \sum_{j \in P} \text{qfm}(i,j,r) + \text{qpm}(i,r) + \text{qgm}(i,r) \qquad \{\text{qim}(i,r)\}_{i \in T}$$

MKTCLENDWM: eq'n assures mkt clearing for perfectly mobile endowments in each r (HT 4):

$$(O) \qquad \text{qo}(i,r) = \text{sum}(j,P,\text{SHREM}(i,j,r) * \text{qfe}(i,j,r)) + \text{endwslack}(i,r) \qquad i \in EM$$
$$(L) \qquad \text{qo}(i,r) = \sum_{j \in P} \text{qfe}(i,j,r) + \text{endwslack}(i,r) \qquad \{\text{pm}(i,r)\}_{i \in EM}$$

This equation assures market clearing for perfectly mobile endowments (HT 4)

MKTCLENDWS: eq'n assures mkt clearing for imperfectly mobile endowments in each r (HT 5):

$$(O) \qquad \text{qoes}(i,j,r) = \text{qfe}(i,j,r) \qquad i \in ES, j \in P$$
$$(L) \qquad \text{qoes}(i,j,r) = \text{qfe}(i,j,r) \qquad \{\text{pm}(i,r)\}_{i \in ES}$$

This equation assures market clearing for sluggish endowments (HT 5)

WALRAS_S: Extra eq'n computes change in supply in the omitted market.:

$$(O) \qquad \text{walras\_sup} = \text{pcgdswld} + \text{globalcgds}$$
$$(L) \qquad \text{walras\_sup} = \text{pcgdswld} \times \text{globalcgds} \qquad \{\text{walras\_sup}\}$$

This is an extra equation which simply computes change in supply in the omitted market. (modified from HT 12 to reflect the value as opposed to quantity change, in light of the new treatment of psave(r))

WALRAS_D: Extra eq'n computes change in demand in the omitted market.:

$$(O) \qquad \text{GLOBINV} * \text{walras\_dem} = \text{sum}(r,R,\text{SAVE}(r) * [\text{psave}(r) + \text{qsave}(r)])$$
$$(L) \qquad \text{walras\_dem} = \sum_{r \in R} \text{psave}(r)\text{qsave}(r) \qquad \{\text{walras\_dem}\}$$

This is an extra equation which simply computes change in demand in the omitted market. (modified from HT 13 to reflect the value as opposed to quantity change)

WALRAS: Check Walras' Law. Value of "walraslack" should be zero. (HT 14):

$$(O) \qquad \text{walras\_sup} = \text{walras\_dem} + \text{walraslack}$$
$$(L) \qquad \text{walras\_sup} = \text{walras\_dem} + \text{walraslack} \qquad \{\text{rorg}\}$$

This equation checks Walras' Law. The value of walraslack should be zero in any GE simulation. (HT 14)

8 Appendices

1. Summary Indices
2. Equivalent Variation
3. Welfare Decomposition
4. Terms of Trade Decomposition

8.1 Summary Indices

The following equations calculate many useful summary statistics. They do not generally affect the equilibrium structure of the model, although they do include the equation for the usual numeraire variable, "pfactwld". Some are documented in Hertel and Tsigas, many are new.

1. Appendix-Specific Variables and Coefficients
2. Factor Price Indices
3. Regional Terms of Trade
4. GDP Indices (Value, Price and Quantity)
5. Trade Balance Indices

Specific variables and coefficients: Only used in this Summary Indices

Variables:

Symbol	Description
vxwfob(i,s)$_{i\in T}$	value of merchandise regional exports, by commodity, FOB
viwcif(i,s)$_{i\in T}$	value of merchandise regional imports, by commodity, CIF
vxwreg(r)	value of merchandise exports, by region
viwreg(r)	value of merchandise imports, by region, at world prices

Coefficients:

Symbol	Description
VXW(i,r)$_{i\in T}$	value of exports by comm. i and region r at FOB prices
VXWREGION(r)	value of exports by region r at FOB prices
VIW(i,s)$_{i\in T}$	value of commodity imports i into s at CIF prices
VIWREGION(r)	value of commodity imports by region r at CIF prices

A-1. Factor Price Indices

Variables:

Symbol	Description
pfactreal(i,r)$_{i\in E}$	ratio of return to primary factor i to CPI in r
pfactreal(i,r)$_{i\in E}$	ratio of return to primary factor i to CPI in r
pfactor(r)	market price index of primary factors, by region
pfactwld	world price index of primary factors

Coefficients:

Symbol	Description
VENDWREG(r)	value of primary factors, at mkt prices, by region
VENDWWLD	value of primary factors, at mkt prices, worldwide

A-2. Regional Terms of Trade:

Variables:

Symbol	Description
psw(r)	index of prices received for tradeables produced in r
pdw(r)	index of prices paid for tradeables used in region r
tot(r)	terms of trade for region r: tot(r) = psw(r) - pdw(r)

A-4. GDP Indices (Value, Price and Quantity)

Variables:

Symbol	Description
vgdp(r)	change in value of GDP
pgdp(r)	GDP price index
qgdp(r)	GDP quantity index
compvalad(i,r)$_{i\in P}$	composition of value added for good i and region r

Coefficients:

Symbol	Description
GDP(r)	Gross Domestic Product in region r

A-5. Aggregate Trade Indices (Value, Price and Quantity)

Variables:

Symbol	Description
qxw(i,r)$_{i\in T}$	aggregate exports of i from region r, FOB weights

A-6. Trade Balance Indices

Variables:

Symbol	Description
DTBALi(i,r)$_{i \in T}$	change in trade balance by i and by r, $US million
DTBAL(r)	change in trade balance X - M, $US million
DTBALR(r)	change in ratio of trade balance to regional income

Coefficients:

Symbol	Description
TBAL(r)	trade balance for region r

8.1.1 Formula

Value of exports by comm. i and region r at FOB prices

$$\text{VXW}(m,r) = \text{sum}(s, R, \text{VXWD}(m,r,s)) + \text{VST}(m,r) \qquad m \in M$$

Formula (all,i,NM)(all,r,REG)

$$\text{VXW}(i,r) = \text{sum}(s, R, \text{VXWD}(i,r,s)) \qquad i \in NM$$

Value of exports by region r at FOB prices

$$\text{VXWREGION}(r) = \text{sum}(i, T, \text{VXW}(i,r))$$

value of commodity imports i into s at CIF prices

$$\text{VIW}(i,s) = \text{sum}(r, R, \text{VIWS}(i,r,s)) \qquad i \in T$$

value of commodity imports by region r at CIF prices

$$\text{VIWREGION}(r) = \text{sum}(i, T, \text{VIW}(i,r))$$

value of primary factors, at mkt prices, by region

$$\text{VENDWREG}(r) = \text{sum}(i, E, \text{VOM}(i,r))$$

value of primary factors, at mkt prices, worldwide

$$\text{VENDWWLD} = \text{sum}(r, R, \text{VENDWREG}(r))$$

Gross Domestic Product in region r

$$\begin{aligned}\text{GDP}(s) = &\ \text{sum}(i, T, \text{VPA}(i,s)) \\ &+ \text{sum}(i, T, \text{VGA}(i,s)) \\ &+ \text{sum}(k, C, \text{VOA}(k,s)) \\ &+ \text{sum}(i, T, \text{sum}(r, R, \text{VXWD}(i,s,r))) \\ &+ \text{sum}(m, M, \text{VST}(m,s)) \\ &- \text{sum}(i, T, \text{sum}(r, R, \text{VIWS}(i,r,s)))\end{aligned}$$

Gross Domestic Product in region r. Trade is valued at FOB and CIF prices.
trade balance for region r

$$\text{TBAL}(r) = \text{VXWREGION}(r) - \text{VIWREGION}(r)$$

8.1.2 Equations

A-1. Factor Price Indices

REALRETURN: eq'n defines the real rate of return to primary factor i in region r:

$$(P) \qquad \text{pfactreal}(i,s) = \text{pm}(i,s) - \text{ppriv}(s) \qquad i \in E$$

$$(L) \qquad \text{pfactreal}(i,s) = \text{pfactreal0}(i,s)\frac{\text{pm}(i,s)/\text{pm0}(i,s)}{\text{ppriv}(s)/\text{ppriv0}(s)} \qquad \{\text{pfactreal}(i,s)\}_{i \in E}$$

This equation defines the real rate of return to primary factor i in region r (new).

PRIMFACTPR (*): computes % change in price index of primary factors, by region:

$$(O) \qquad \text{VENDWREG}(r) * \text{pfactor}(r) = \text{sum}(i, E, \text{VOM}(i,r) * \text{pm}(i,r)) \qquad i \in E$$

$$(L) \qquad \text{pfactor}(r) = \text{pfactor0}(r) \prod_{i \in E} \left[\frac{\text{pm}(i,r)}{\text{pm0}(i,r)} \right]^{\text{sh\_vom\_vendwreg}(i,r)} \qquad \{\text{pfactor}(r)\}$$

(B) E_SH_VOM_VENDWREG: Share of endowment commodities in region r:

$$(L) \qquad \text{sh\_vom\_vendwreg}(i,r) = \frac{1}{2} \frac{\text{VOM}(i,r)}{\text{VENDWREG}(r)} + \frac{1}{2} \frac{\text{pm}(i,r) * \text{qo}(i,r)}{\sum_{j \in E} \text{pm}(j,r) * \text{qo}(j,r)}$$
$$\{\text{sh\_vom\_vendwreg}(i,r)\}$$

PRIMFACTPRWLD (*): computes % change in global price index of primary factors:

$$(O) \qquad \text{VENDWWLD} * \text{pfactwld} = \text{sum}(r, R, \text{VENDWREG}(r) * \text{pfactor}(r))$$

$$(L) \qquad \text{pfactwld} = \text{pfactwld0} \prod_{r \in R} \left[\frac{\text{pfactor}(r)}{\text{pfactor0}(r)} \right]^{\text{sh\_vendwreg\_vendwwld}(r)} \qquad \{\text{pfactwld}\}$$

(B) E_SH_VENDWREG_VENDWWLD: Share of endowment commodities in the world:

$$(L) \qquad \text{sh\_vendwreg\_vendwwld}(r) = \frac{1}{2} \frac{\text{VENDWREG}(r)}{\text{VENDWWLD}} + \frac{1}{2} \frac{\sum_{i \in E} \text{pm}(i,r) * \text{qo}(i,r)}{\sum_{j \in E, s \in R} \text{pm}(j,s) * \text{qo}(j,s)}$$
$$\{\text{sh\_vendwreg\_vendwwld}(r)\}$$

A-2. Regional Terms of Trade

REGSUPRICE: estimate change in index of prices received for tradeables i produced in r:

$$(P) \qquad \text{VXWREGION}(r) * \text{psw}(r) = \text{sum}(i, T, \text{sum}(s, R, \text{VXWD}(i,r,s) * \text{pfob}(i,r,s))) + \text{sum}(m, M, \text{VST}(m,r) * \text{pm}(m,r))$$

$$(L) \qquad \text{psw}(r) = \text{psw0}(r) \prod_{i \in T, s \in R} \left[\frac{\text{pfob}(i,r,s)}{\text{pfob0}(i,r,s)} \right]^{\text{sh\_vxwd\_vxwregion}(i,r,s)} \prod_{m \in M} \left[\frac{\text{pm}(m,r)}{\text{pm0}(m,r)} \right]^{\text{sh\_vst\_vxwregion}(i,r)} \qquad \{\text{psw}(r)\}$$

This equation estimates the change in the index of prices received for tradeable products produced in r. (modified from HT 64 to eliminate the investment component)

(B) E_V_VXWREGION:

$$(L) \qquad \text{v\_vxwregion}(r) = \sum_{i \in T, s \in R} \text{pfob}(i,r,s) \text{qxs}(i,r,s) + \sum_{i \in M} \text{pm}(i,r) \text{qst}(i,r)$$
$$\{\text{v\_vxwregion}(r)\}$$

(B) E_SH_VXWD_VXWREGION:

$$(L) \qquad \text{sh\_vxwd\_vxwregion}(i,r,s) = \frac{1}{2} \frac{\text{VXWD}(i,r,s)}{\text{VXWREGION}(r)} + \frac{1}{2} \frac{\text{pfob}(i,r,s) \text{qxs}(i,r,s)}{\text{v\_vxwregion}(r)}$$
$$\{\text{sh\_vxwd\_vxwregion}(i,r,s)\}_{i \in T}$$

(B) E_SH_VST_VXWREGION:

$$(L) \qquad \text{sh\_vst\_vxwregion}(i,r) = \frac{1}{2} \frac{\text{VST}(i,r)}{\text{VXWREGION}(r)} + \frac{1}{2} \frac{\text{pm}(i,r) \text{qst}(i,r)}{\text{v\_vxwregion}(r)}$$
$$\{\text{sh\_vst\_vxwregion}(i,r)\}_{i \in M}$$

REGDEMPRICE: estimate change in index of prices paid for tradeable products used in r:

$$(P) \qquad \text{VIWREGION}(r) * \text{pdw}(r) = \text{sum}(i, T, \text{sum}(k, R, \text{VIWS}(i,k,r) * \text{pcif}(i,k,r)))$$

$$(L) \qquad \text{pdw}(r) = \text{pdw0}(r) \prod_{i \in T, k \in R} \left[\frac{\text{pcif}(i,k,r)}{\text{pcif0}(i,k,r)} \right]^{\text{sh\_viws\_viwregion}(i,k,r)} \qquad \{\text{pdw}(r)\}$$

This equation estimates the change in the index of prices paid for tradeable products used in r. (modified from HT 65 to eliminate savings)

(B) E_V_VIWREGION:

$$(L) \qquad \text{v\_viwregion}(r) = \sum_{i \in T, s \in R} \text{pcif}(i,s,r)\text{qxs}(i,s,r) \qquad \{\text{v\_viwregion}(r)\}$$

(B) E_SH_VIWS_VIWREGION:

$$(L) \qquad \text{sh\_viws\_viwregion}(i,r,s) = \frac{1}{2}\frac{\text{VIWS}(i,r,s)}{\text{VIWREGION}(s)} + \frac{1}{2}\frac{\text{pcif}(i,r,s)\text{qxs}(i,r,s)}{\text{v\_viwregion}(s)} \qquad \{\text{sh\_viws\_viwregion}(i,r,s)\}_{i \in T}$$

TOTeq: terms of trade equation computed as difference in psw and pdw (HT 66):

$$(P) \qquad \text{tot}(r) = \text{psw}(r) - \text{pdw}(r)$$

$$(L) \qquad \text{tot}(r) = \text{tot0}(r)\frac{\text{psw}(r)/\text{psw0}(r)}{\text{pdw}(r)/\text{pdw0}(r)} \qquad \{\text{tot}(r)\}$$

A-4. GDP Indices (Value, Price and Quantity)

VGDP_r: change in value of GDP (HT 70):

$$\begin{aligned}(P) \qquad \text{GDP}(r) * \text{vgdp}(r) &= \text{sum}(i, T, \text{VGA}(i,r) * [\text{qg}(i,r) + \text{pg}(i,r)]) \\ &+ \text{sum}(i, T, \text{VPA}(i,r) * [\text{qp}(i,r) + \text{pp}(i,r)]) \\ &+ \text{REGINV}(r) * [\text{qcgds}(r) + \text{pcgds}(r)] \\ &+ \text{sum}(i, T, \text{sum}(s, R, \text{VXWD}(i,r,s) * [\text{qxs}(i,r,s) + \text{pfob}(i,r,s)])) \\ &+ \text{sum}(m, M, \text{VST}(m,r) * [\text{qst}(m,r) + \text{pm}(m,r)]) \\ &- \text{sum}(i, T, \text{sum}(s, R, \text{VIWS}(i,s,r) * [\text{qxs}(i,s,r) + \text{pcif}(i,s,r)]))\end{aligned}$$

$$\begin{aligned}(L) \qquad \text{vgdp}(r) &= \sum_{i \in T} \text{pg}(i,r)\text{qg}(i,r) + \sum_{i \in T} \text{pp}(i,r)\text{qp}(i,r) + \text{pcgds}(r)\text{qcgds}(r) \\ &+ \sum_{i \in T, s \in R} \text{pfob}(i,r,s)\text{qxs}(i,r,s) + \sum_{m \in M} \text{pm}(m,r)\text{qst}(m,r) \\ &- \sum_{i \in T, s \in R} \text{pcif}(i,s,r)\text{qxs}(i,s,r) \qquad \{\text{vgdp}(r)\}\end{aligned}$$

PGDP_r: GDP price index (HT 71):

$$\begin{aligned}(P) \qquad \text{GDP}(r) * \text{pgdp}(r) &= \text{sum}(i, T, \text{VGA}(i,r) * \text{pg}(i,r)) \\ &+ \text{sum}(i, T, \text{VPA}(i,r) * \text{pp}(i,r)) \\ &+ \text{REGINV}(r) * \text{pcgds}(r) \\ &+ \text{sum}(i, T, \text{sum}(s, R, \text{VXWD}(i,r,s) * \text{pfob}(i,r,s))) \\ &+ \text{sum}(m, M, \text{VST}(m,r) * \text{pm}(m,r)) \\ &- \text{sum}(i, T, \text{sum}(s, R, \text{VIWS}(i,s,r) * \text{pcif}(i,s,r)))\end{aligned}$$

$$(L) \qquad \text{pgdp}(r) = \text{pgdp0}(r) \times \prod_{i \in T} \left[\frac{\text{pg}(i,r)}{\text{pg0}(i,r)} \right]^{\text{sh\_vga\_gdp}(i,r)} \times \prod_{i \in T} \left[\frac{\text{pp}(i,r)}{\text{pp0}(i,r)} \right]^{\text{sh\_vpa\_gdp}(i,r)}$$
$$\times \left[\frac{\text{pcgds}(r)}{\text{pcgds0}(r)} \right]^{\text{sh\_reginv\_gdp}(r)} \times \prod_{i \in T, s \in R} \left[\frac{\text{pfob}(i,r,s)}{\text{pfob0}(i,r,s)} \right]^{\text{sh\_vxwd\_gdp}(i,r,s)}$$
$$\times \prod_{m \in M} \left[\frac{\text{pm}(m,r)}{\text{pm0}(m,r)} \right]^{\text{sh\_vst\_gdp}(m,r)} \times \prod_{i \in T, s \in R} \left[\frac{\text{pcif}(i,s,r)}{\text{pcif0}(i,s,r)} \right]^{-\text{sh\_viws\_gdp}(i,s,r)} \qquad \{\text{pgdp}(r)\}$$

(B) E_SH_VGA_GDP:

$$(L) \qquad \text{sh\_vga\_gdp}(i,r) = \frac{1}{2} \frac{\text{VGA}(i,r)}{\text{GDP}(r)} + \frac{1}{2} \frac{\text{pg}(i,r)\text{qg}(i,r)}{\text{vgdp}(r)} \qquad \{\text{sh\_vga\_gdp}(i,r)\}_{i \in T}$$

(B) E_SH_VPA_GDP:

$$(L) \qquad \text{sh\_vpa\_gdp}(i,r) = \frac{1}{2} \frac{\text{VPA}(i,r)}{\text{GDP}(r)} + \frac{1}{2} \frac{\text{pp}(i,r)\text{qp}(i,r)}{\text{vgdp}(r)} \qquad \{\text{sh\_vpa\_gdp}(i,r)\}_{i \in T}$$

(B) E_SH_REGINV_GDP:

$$(L) \qquad \text{sh\_reginv\_gdp}(r) = \frac{1}{2} \frac{\text{REGINV}(r)}{\text{GDP}(r)} + \frac{1}{2} \frac{\text{pcgds}(r)\text{qcgds}(r)}{\text{vgdp}(r)} \qquad \{\text{sh\_reginv\_gdp}(r)\}$$

(B) E_SH_VXWD_GDP:

$$(L) \qquad \text{sh\_vxwd\_gdp}(i,r,s) = \frac{1}{2} \frac{\text{VXWD}(i,r,s)}{\text{GDP}(r)} + \frac{1}{2} \frac{\text{pfob}(i,r,s)\text{qxs}(i,r,s)}{\text{vgdp}(r)} \qquad \{\text{sh\_vxwd\_gdp}(i,r,s)\}_{i \in T}$$

(B) E_SH_VST_GDP:

$$(L) \qquad \text{sh\_vst\_gdp}(i,r) = \frac{1}{2} \frac{\text{VST}(i,r)}{\text{GDP}(r)} + \frac{1}{2} \frac{\text{pm}(i,r)\text{qst}(i,r)}{\text{vgdp}(r)} \qquad \{\text{sh\_vst\_gdp}(i,r)\}_{i \in M}$$

(B) E_SH_VIWS_GDP:

$$(L) \qquad \text{sh\_viws\_gdp}(i,s,r) = \frac{1}{2} \frac{\text{VIWS}(i,s,r)}{\text{GDP}(r)} + \frac{1}{2} \frac{\text{pcif}(i,s,r)\text{qxs}(i,s,r)}{\text{vgdp}(r)} \qquad \{\text{sh\_viws\_gdp}(i,s,r)\}_{i \in T}$$

QGDP_r: GDP quantity index:

$$\begin{aligned}(P) \qquad \text{GDP}(r) * \text{qgdp}(r) = {} & \text{sum}(i, T, \text{VGA}(i,r) * \text{qg}(i,r)) \\ & + \text{sum}(i, T, \text{VPA}(i,r) * \text{qp}(i,r)) \\ & + \text{REGINV}(r) * \text{qcgds}(r) \\ & + \text{sum}(i, T, \text{sum}(s, R, \text{VXWD}(i,r,s) * \text{qxs}(i,r,s))) \\ & + \text{sum}(m, M, \text{VST}(m,r) * \text{qst}(m,r)) \\ & - \text{sum}(i, T, \text{sum}(s, R, \text{VIWS}(i,s,r) * \text{qxs}(i,s,r)))\end{aligned}$$

$$(L) \qquad \text{qgdp}(r) = \text{vgdp}(r) / \text{pgdp}(r) \qquad \{\text{qgdp}(r)\}$$

COMPVALADEQ: track change in composition of value added:

$$(P) \qquad \text{compvalad}(i,r) = \text{qo}(i,r) - \text{qgdp}(r) \qquad i \in P$$

$$(L) \qquad \text{compvalad}(i,r) = \text{compvalad0}(i,r) \frac{\text{qo}(i,r)/\text{qo0}(i,r)}{\text{qgdp}(r)/\text{qgdp0}(r)} \qquad \{\text{compvalad}(i,r)\}_{i \in P}$$

A-5. Aggregate Trade Indices (Value, Price and Quantity)

VREGEX_ir_MARG: the change in FOB value of exports of m from r:

$$(P) \quad \text{VXW}(m,r) * \text{vxwfob}(m,r) = \text{sum}(s,R,\text{VXWD}(m,r,s) * [\text{qxs}(m,r,s) + \text{pfob}(m,r,s)]) + \text{VST}(m,r) * [\text{qst}(m,r) + \text{pm}(m,r)] \qquad m \in M$$

$$(L) \quad \text{vxwfob}(m,r) = \sum_{s \in R} \text{pfob}(m,r,s)\text{qxs}(m,r,s) + \text{pm}(m,r)\text{qst}(m,r) \qquad \{\text{vxwfob}(m,r)\}_{m \in M}$$

VREGEX_ir_NMRG: the change in FOB value of exports of commodity i from r (HT 73):

$$(P) \quad \text{VXW}(i,r) * \text{vxwfob}(i,r) = \text{sum}(s,R,\text{VXWD}(i,r,s) * [\text{qxs}(i,r,s) + \text{pfob}(i,r,s)]) \qquad i \in NM$$

$$(L) \quad \text{vxwfob}(i,r) = \sum_{s \in R} \text{pfob}(i,r,s)\text{qxs}(i,r,s) \qquad \{\text{vxwfob}(i,r)\}_{i \in NM}$$

VREGEX_r: computes % change in value of merchandise exports, by region (HT 75):

$$(P) \quad \text{VXWREGION}(r) * \text{vxwreg}(r) = \text{sum}(i,T,\text{VXW}(i,r) * \text{vxwfob}(i,r))$$

$$(L) \quad \text{vxwreg}(r) = \sum_{i \in T} \text{vxwfob}(i,r) \qquad \{\text{vxwreg}(r)\}$$

VREGIM_is: the change in CIF value of imports of commodity i into s (HT 74):

$$(P) \quad \text{VIW}(i,s) * \text{viwcif}(i,s) = \text{sum}(r,R,\text{VIWS}(i,r,s) * [\text{pcif}(i,r,s) + \text{qxs}(i,r,s)]) \qquad i \in T$$

$$(L) \quad \text{viwcif}(i,s) = \sum_{r \in R} \text{pcif}(i,r,s)\text{qxs}(i,r,s) \qquad \{\text{viwcif}(i,r)\}_{i \in T}$$

VREGIM_s: computes % change in value of imports, CIF basis, by region (HT 76):

$$(P) \quad \text{VIWREGION}(s) * \text{viwreg}(s) = \text{sum}(i,T,\text{VIW}(i,s) * \text{viwcif}(i,s))$$

$$(L) \quad \text{viwreg}(s) = \sum_{i \in T} \text{viwcif}(i,s) \qquad \{\text{viwreg}(s)\}$$

QREGEX_ir_MARG: change in volume of exports of margin commodity m from r:

$$(P) \quad \text{VXW}(m,r) * \text{qxw}(m,r) = \text{sum}(s,R,\text{VXWD}(m,r,s) * \text{qxs}(m,r,s)) + \text{VST}(m,r) * \text{qst}(m,r) \qquad m \in M$$

$$(L) \quad \text{qxw}(m,r) = \prod_{s \in R} \text{qxs}(m,r,s)^{\text{sh\_vxwd\_vxw(m,r,s)}} \text{qst}(m,r)^{\text{sh\_vst\_vxw(m,r)}} \qquad \{\text{qxw}(m,r,s)\}_{m \in M}$$

QREGEX_ir_NMRG: change in volume of exports of non-margin commodity i from r:

$$(P) \quad \text{VXW}(i,r) * \text{qxw}(i,r) = \text{sum}(s,R,\text{VXWD}(i,r,s) * \text{qxs}(i,r,s)) \qquad i \in NM$$

$$(L) \quad \text{qxw}(i,r) = \prod_{s \in R} \text{qxs}(i,r,s)^{\text{sh\_vxwd\_vxw(i,r,s)}} \qquad \{\text{qxw}(i,r,s)\}_{i \in NM}$$

E_V_VXW:

$$(L) \quad \text{v\_vxw(i,r)} = \sum_{s \in R} \text{pfob}(i,r,s)\text{qxs}(i,r,s) + \text{pm}(i,r)\text{qst}(i,r) \qquad \{\text{v\_vxw(i,r)}\}_{i \in T}$$

E_SH_VXWD_VXW:

$$(L) \quad \text{sh\_vxwd\_vxw(i,r,s)} = \frac{1}{2}\frac{\text{VXWD}(i,r,s)}{\text{VXW}(i,r)} + \frac{1}{2}\frac{\text{pfob}(i,r,s)\text{qxs}(i,r,s)}{\text{v\_vxw}(i,r)} \qquad \{\text{sh\_vxwd\_vxw(i,r,s)}\}$$

E_SH_VST_VXW:

$$(L) \quad \text{sh\_vst\_vxw(m,r)} = \frac{1}{2}\frac{\text{VST}(m,r)}{\text{VXW}(m,r)} + \frac{1}{2}\frac{\text{pm}(m,r)\text{qst}(m,r)}{\text{v\_vxw}(m,r)} \qquad \{\text{sh\_vst\_vxw(m,r)}\}_{m \in M}$$

A-6: Trade Balance Indices:

TRADEBAL_i: computes change in trade balance by commodity and by region (HT 97):

$$(P) \qquad \text{DTBALi}(i,r) = [\text{VXW}(i,r)/100] * \text{vxwfob}(i,r) - [\text{VIW}(i,r)/100] * \text{viwcif}(i,r) \qquad i \in T$$
$$(L) \qquad \text{DTBALi}(i,r) = \text{vxwfob}(i,r) - \text{viwcif}(i,r) \qquad \{\text{DTBALi}(i,r)\}_{i \in T}$$

TRADEBALANCE: computes change in trade balance (X - M), by region (HT 98):

$$(P) \qquad \text{DTBAL}(r) = [\text{VXWREGION}(r)/100] * \text{vxwreg}(r) - [\text{VIWREGION}(r)/100] * \text{viwreg}(r)$$
$$(L) \qquad \text{DTBAL}(r) = \text{vxwreg}(r) - \text{viwreg}(r) \qquad \{\text{DTBALi}(i,r)\}_{i \in T}$$

DTBALRATIO: change in ratio of trade balance to regional income:

$$(P) \qquad 100 * \text{INCOME}(r) * \text{DTBALR}(r) = 100 * \text{DTBAL}(r) - \text{TBAL}(r) * \text{y}(r)$$
$$(L) \qquad \text{DTBALR}(r) = \frac{\text{DTBAL}(r)}{y(r)} \qquad \{\text{DTBALR}(r)\}$$

8.2 Equivalent Variation

(B) GOVU_EV: utility from government consumption in r:

$$(L) \qquad \text{yg\_ev}(r)/\text{pop}(r) = \text{pgov0}(r)\text{ug\_ev}(r) \qquad \{\text{ug\_ev}(r)\}$$

where $\text{ug}(r)$ is the per capita utility and $\text{yg}(r)$ is the aggregated utility.

(B) GOVDMNDS_EV: government consumption demands for composite commodities (HT 41):

$$(L) \qquad \text{qg\_ev}(i,r) = \text{qg0}(i,r)\frac{\text{pop}(r)}{\text{pop0}(r)}\frac{\text{ug\_ev}(r)}{\text{ug0}(r)} \qquad \{\text{qg\_ev}(r)\}$$

(B) PRIVATEU_EV: computation of utility from private consumption in r (HT 45):

$$(L) \qquad \sum_{i \in T} \text{B}(i,r)\text{up\_ev}(r)^{\text{SUBPAR}(i,r)\text{INCPAR}(i,r)} \left[\frac{\text{pp0}(i,r)}{\text{yp\_ev}(r)/\text{pop}(r)}\right]^{\text{SUBPAR}(i,r)} = 1 \qquad \{\text{up\_ev}(r)\}$$

This equation determines private consumption utility for a representative household in region r, based on the per capita private expenditure function. (HT 45)

(B) UTILELASPRIV_EV: elasticity of expenditure wrt utility from private consumption:

$$(L) \qquad \text{uepriv\_ev}(r) = \sum_{i \in T} \text{vconshr\_ev}(i,r)\text{INCPAR}(i,r) \qquad \{\text{uepriv\_ev}(r)\}$$

(B) E_VCONSHR_EV: Share of private hhld consumption devoted to good i in r:

$$(L) \qquad \text{vconshr\_ev}(i,r) = \text{pp0}(i,r)\text{qp\_ev}(i,r)/\text{yp\_ev}(r) \qquad \{\text{vconshr\_ev}(i,r)\}$$

(B) PRIVDMNDS_EV: private consumption demands for composite commodities (HT 46):

$$(L) \qquad \frac{\text{qp\_ev}(i,r)}{\text{pop}(r)} = \frac{\text{BP}(i,r)\text{up\_ev}(r)^{\text{SUBPAR}(i,r)\text{INCPAR}(i,r)}\text{SUBPAR}(i,r)\left[\frac{\text{pp0}(i,r)}{\text{yp\_ev}(r)/\text{pop}(r)}\right]^{\text{SUBPAR}(i,r)-1}}{\sum_{j \in T}\text{BP}(j,r)\text{up\_ev}(r)^{\text{SUBPAR}(j,r)\text{INCPAR}(j,r)}\text{SUBPAR}(j,r)\left[\frac{\text{pp0}(j,r)}{\text{yp\_ev}(r)/\text{pop}(r)}\right]^{\text{SUBPAR}(j,r)}} \qquad \{\text{qp\_ev}(i,r)\}$$

Private consumption demands for composite commodities. Demand system is on a per capita basis. Here, yp(r) - pop(r) is % change in per capita income. (HT 46)

(B) UTILITELASTIC_EV: elasticity of cost of utility wrt utility:

$$(L) \qquad \text{uelas\_ev}(r) = \frac{1}{\text{dppriv}(r)/\text{uepriv\_ev}(r) + \text{dpgov}(r) + \text{dpsave}(r)} \qquad \{\text{uelas\_ev}(r)\}$$

(B) PRIVCONSEXP_EV: private consumption expenditure:

$$(L) \qquad \frac{\text{yp\_ev}(r)}{\text{y\_ev}(r)} = \frac{\text{uelas\_ev}(r)}{\text{uepriv\_ev}(r)}\text{dppriv}(r) \qquad \{\text{yp\_ev}(r)\}$$

(B) GOVCONSEXP_EV: government consumption expenditure:

$$(L) \qquad \frac{\text{yg\_ev}(r)}{\text{y\_ev}(r)} = \text{uelas\_ev}(r)\text{dpgov}(r) \qquad \{\text{yg\_ev}(r)\}$$

(B) SAVING_EV: saving:

$$(L) \qquad \frac{\text{psave0}(r)\text{qsave\_ev}(r)}{\text{y\_ev}(r)} = \text{uelas\_ev}(r)\text{dpsave}(r) \qquad \{\text{qsave\_ev}(r)\}$$

(B) UTILITY_EV: regional household utility:

$$(L) \qquad \text{u\_ev}(r) = \text{u0}(r)\frac{\text{au}(r)}{\text{au0}(r)}\left[\frac{\text{up\_ev}(r)}{\text{up0}(r)}\right]^{\text{dppriv}(r)}\left[\frac{\text{ug\_ev}(r)}{\text{ug0}(r)}\right]^{\text{dpgov}(r)}\left[\frac{\text{qsave\_ev}(r)}{\text{qsave0}(r)}\right]^{\text{dpsave}(r)} \qquad \{\text{y\_ev}(r)\}$$

(B) EV:

$$(L) \qquad \text{EV}(r) = y\text{\_ev}(r) - y(r) \qquad \{\text{EV}(r)\}$$

References

Burniaux, Jean-Marc and Truong P. Truong (2002) "GTAP-E: An Energy-Environmental Version of the GTAP Model." GTAP Technical Paper No. 16, Revised January 2002.

Hertel, Thomas W. ed. (1997) *Global Trade Analysis: Modeling and Applications*, New York: Cambridge University Press.

McDougall, Robert (2003) "A New Regional Household Demand System for GTAP." GTAP Technical Paper No. 20, September.

Rutherford, Thomas F. (2006) "GTAP6inGAMS," January.
(available at: `http://www.mpsge.org/`).