A Supplementary Paper to "Regional Effects of Trade Liberalization in Japan: A CGE Analysis Based on Inter-Regional IO Table"

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1. Terms of trade

Table 1 reports the change in terms of trade (the relative price of export and import) derived from the GTAP model.

Table 1: Terms of trade change derived from the GTAP model (%).

Goods	Change in TOT (%)
agr	-3.240
min	1.440
fue	1.640
foo	-2.060
tex	1.530
wap	2.210
woo	-0.070
ppp	1.430
chm	1.720
p_c	2.280
scp	1.460
fem	1.760
nfm	1.630
met	1.670
mac	1.710
tmc	1.980
oip	1.460
cns	1.390
egw	1.310
trd	0.810
fin	1.160
trn	1.450
ser	1.180

2. Model

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2.1. Notes

In this section, we provides the full description of the model used in Takeda and Ban (2008). The simulation programs written in GAMS are available from the authors upon request.

- We assume optimizing behavior for all activities (profit maximizing, cost minimizing, utility maximizing, expenditure minimizing).
- Because in the GAMS programs, all CES functions are written in calibrated share form, we express all CES functions in calibrated share form in the following explanation. For the details of calibrated share form of CES functions, see Rutherford (1998).
- The variable in parentheses on the right end of the equation indicates the variable which is determined or defined by the equation if the equation uses "=" and the slack variable if the equation uses "≥". For example, suppose that the equation is defined as follows:

  $$f(x, y, z) \geq 0 \qquad \{z\}$$

  This strictly means

  $$f(x, y, z) \geq 0 \qquad f(x, y, z)z = 0 \qquad z \geq 0$$

- Variables with hat indicate value at the benchmark equilibrium.
- In the GAMS programs, the same model is written both in normal MCP format (model_mcp.gms) and MPSGE format (model_mpsge.gms). In the following, we explain the formulation used in the former program.
- In the GAMS programs, endogenous variables are normalized so that they are unity at the benchmark equilibrium (although there are some exceptions). For example, level of output of sector i in region r (Y_{ir}) is normalized as $Y_{ir} = \overline{Y_{ir}} y_{ir}$ where $\overline{Y_{ir}}$ is the benchmark value of Y_{ir}. GAMS programs use y_{ir}, which is unity at the benchmark equilibrium, as an endogenous variable.

2.2. Specification of functions

Our model uses CES functions for utility and production functions etc. Figure 2 – Figure 6 represent structure of functions. Symbols like E_X indicate elasticity of substitution.

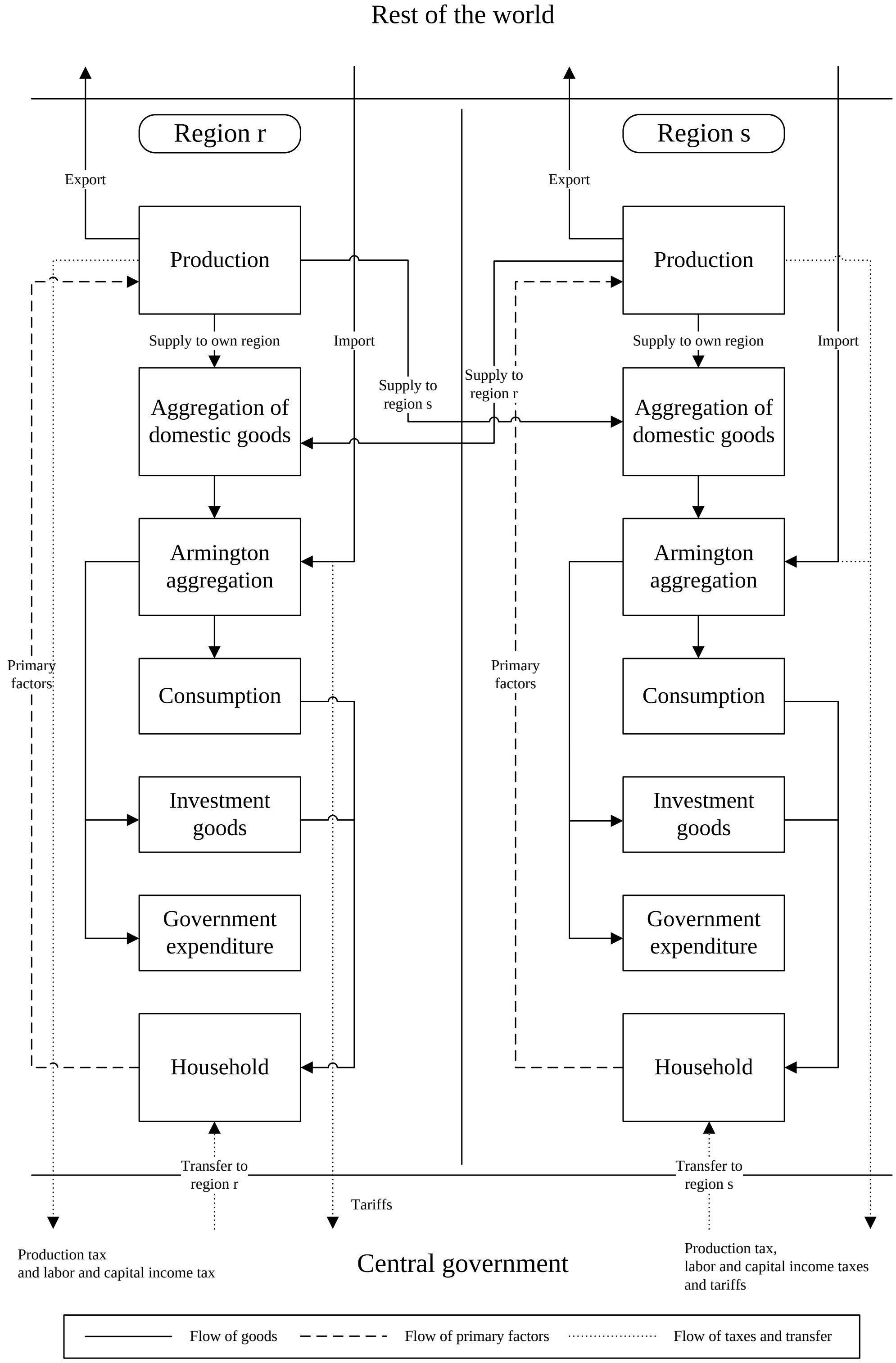

Figure 1. Flow of goods, primary factors, taxes and transfer

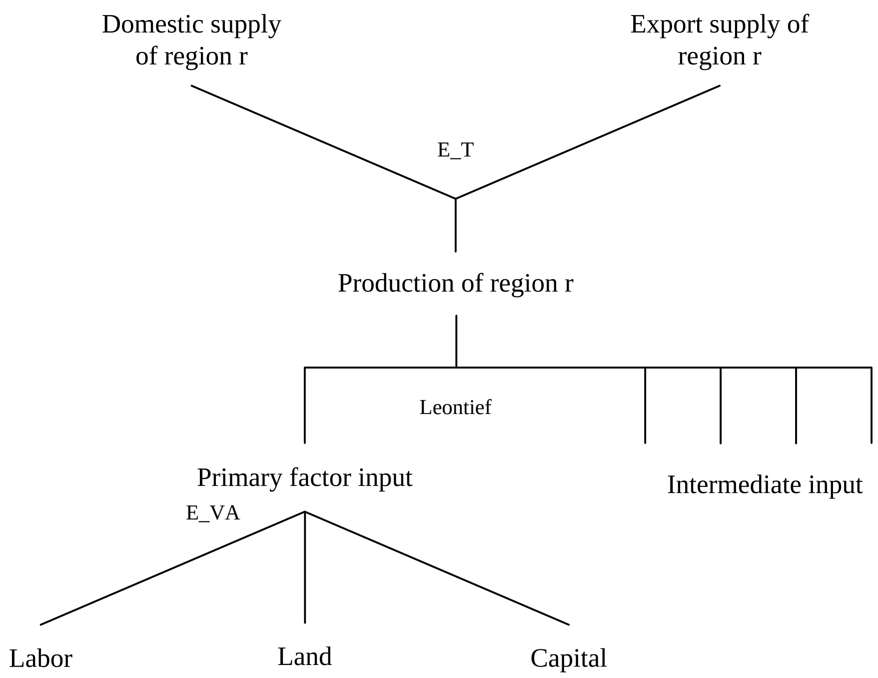

Figure 2. Production function

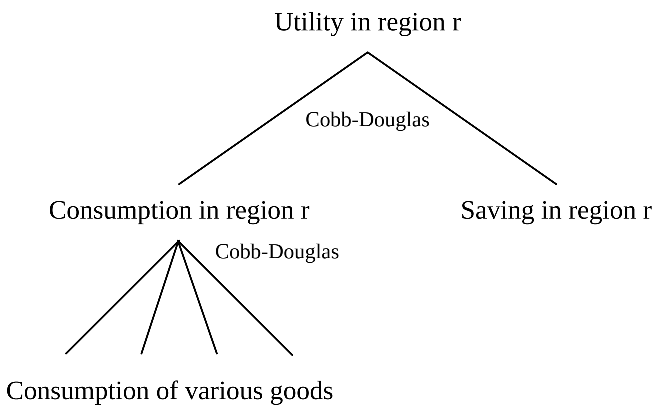

Figure 3. Utility function

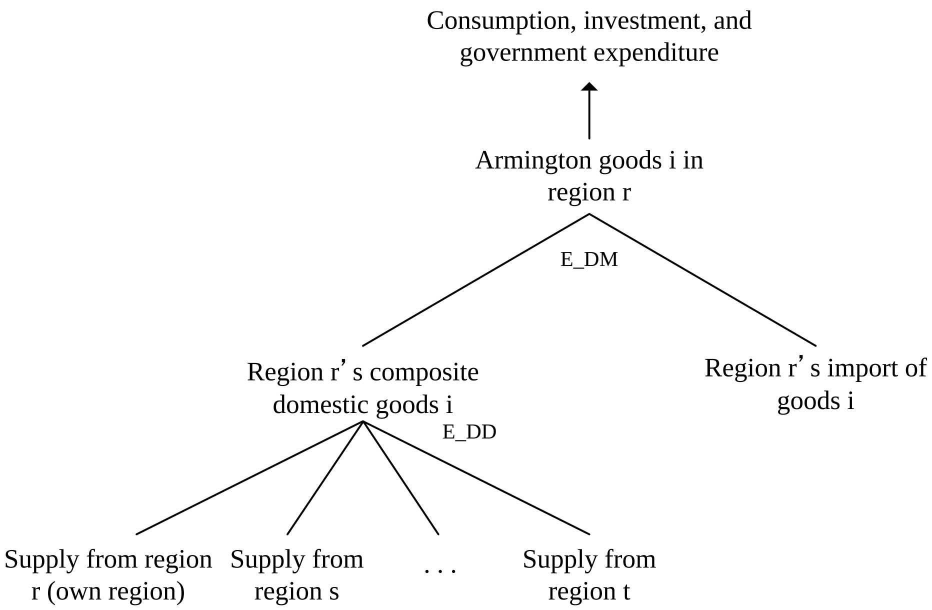

Figure 4. Armington aggregation function

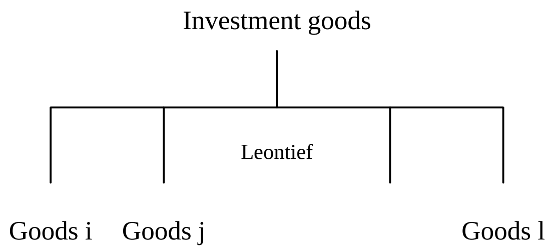

Figure 5. Production of investment goods

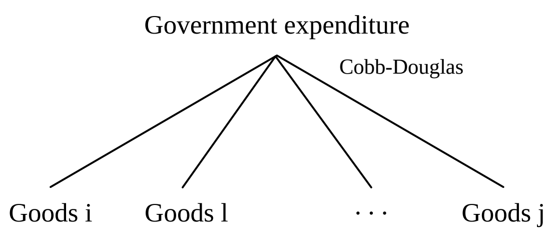

Figure 6. Government expenditure

2.3. Notations

First let us define notations used in the model description.

Definition of set

Symbol	Description
i, j	Index of sectors and goods
r, s	Index of regions in Japan
AGR	Set for sector AGR (agriculture, fishery and forestry)

Activity level

Symbol	Description
Y_{ir}	Level of output of sector i in region r
A_{ir}	Armington aggregation of goods i in regions r
AD_{ir}	Aggregation of domestic goods in region r
M_{ir}	Import of goods i in region r
X_{ir}	Export of goods i in region r
C_r	Consumption of region r
U_r	Utility of region r
INV_r	Investment goods in region r
G_r	Government expenditure in region r

Unit cost and price index

Symbol	Description
c_{ir}^{Y}	Unit cost of production
c_{ir}^{A}	Unit cost of Armington aggregation
c_{ir}^{AD}	Unit cost of aggregation of domestic goods
c_r^{C}	Unit cost of consumption
c_r^{U}	Unit cost of utility
c_r^{INV}	Unit cost of investment goods
c_r^{G}	Unit cost of government expenditure
p_{ir}^{Y}	Price index of output of sector i
p_{ir}^{VA}	Price index of value added (primary factors)
p_{ir}^{D}	Price of domestic supply
p_{ir}^{A}	Price index of Armington goods
p_{ir}^{AD}	Price index of composite domestic goods
p_{ir}^{M}	Price of import (including tariffs)
p_{ir}^{X}	Price of export

p_{ir}^{C}	Price index of consumption
p_{ir}^{U}	Price index of utility
p_{r}^{INV}	Price index of investment goods
p_{r}^{G}	Price index of government expenditure
p_{r}^{L}	Price of labor of region r
p_{r}^{K}	Price of capital of region r
p_{ir}^{LND}	Price of land in sector i of region r ($i \in \mathrm{AGR}$)
p^{FX}	Price of foreign exchange

Demand and supply functions

Symbol	Description
a_{fir}^{YX}	Unit export supply
a_{fir}^{YD}	Unit domestic supply
a_{ir}^{L}	Unit demand for labor
a_{ir}^{K}	Unit demand for capital
a_{ir}^{LND}	Unit demand for land ($i \in \mathrm{AGR}$)
a_{ir}^{M}	Unit demand for import goods
a_{ir}^{AD}	Unit demand for composite domestic goods
a_{isr}^{D}	Unit demand of region r for domestic goods produced in region s
a_{ir}^{CC}	Unit consumption demand
a_{r}^{C}	Unit demand for composite consumption
a_{r}^{S}	Unit demand for saving
a_{ir}^{INV}	Unit demand in investment
a_{ir}^{GOV}	Unit demand in government expenditure

Share parameters (exogenous variables)

Parameters that represent input shares at the benchmark equilibrium.

Symbol	Description
θ_{ir}^{X}	Share of export in total supply
$\theta_{ir}^{\mathrm{VA}}$	Share of VA in production cost
$\theta_{ir}^{\mathrm{VAL}}$	Share of labor in VA
$\theta_{ir}^{\mathrm{VAK}}$	Share of capital in VA
$\theta_{ir}^{\mathrm{VAN}}$	Share of land in VA
θ_{jir}^{I}	Share of intermediate inputs
$\theta_{ir}^{\mathrm{AD}}$	Share of domestic products in Armington aggregation
θ_{ir}^{M}	Share of imported goods in Armington aggregation
θ_{isr}^{D}	Share of goods produced by regions s in demand of region s
θ_{ir}^{C}	Share of goods i in consumption
$\theta_{C,r}^{U}$	Share of consumption in utility

$\theta_{S,r}^{U}$	Share of saving in utility
θ_{ir}^{INV}	Share of goods i in investment
θ_{ir}^{G}	Share of goods i in government expenditure
θ_{r}^{T}	Share of transfer to region r in total transfer

Elasticity of substitution (exogenous variables)

Symbol	Description
σ_i^{DM}	Armington elasticity (domestic vs import)
σ_i^{DD}	EOS between goods from different domestic regions
σ_i^{VA}	EOS between capital and labor (and land)
σ^{C}	EOS in consumption (=1)
σ^{U}	EOS between consumption and saving (=1)
σ^{G}	EOS in government expenditure (=1)
σ^{INV}	EOS in investment (=0)
η_i	EOT between domestic and export supply

Tax rates (exogenous variables)

Symbol	Description
t_r^{K}	Tax rate on capital in region r
t_r^{L}	Tax rate on labor in region r
t_{ir}^{M}	Tariff rate + rate of commodity tax on imports
t_{ir}^{Y}	Tax rate on production

Other variables and parameters

Symbol	Description
M_r^{H}	Income of representative household in region r
M^{G}	Income of the central government
$\bar{E}_r^{L}$	Endowment of labor in region r (exogenous)
$\bar{E}_r^{K}$	Endowment of capital in region r (exogenous)
$\bar{E}_{ir}^{\text{LND}}$	Endowment of land in region r (exogenous)
BOP	Capital flows to foreign countries (exogenous)
e_{ir}^{TOT}	Parameter for changing terms of trade (exogenous). Initial value = 1. e_{ir}^{TOT} > 1 means improvement of terms of trade.

2.4. Model

2.4.1. Unit cost and price index

Price index of output: Output of each sector is allocated to domestic and export supply according to a CET (constant elasticity of transformation) function with elasticity η_i. The price index of output of sector i in region r is expressed as follows:

$$p_{ir}^{Y} = \bar{p}_{ir}^{Y}\left[\theta_{ir}^{X}\left[\frac{p_{ir}^{X}}{\bar{p}_{ir}^{X}}\right]^{1+\eta_i} + (1-\theta_{ir}^{X})\left[\frac{p_{ir}^{D}}{\bar{p}_{ir}^{D}}\right]^{1+\eta_i}\right]^{\frac{1}{1+\eta_i}} \qquad \{p_{ir}^{Y}\}$$

Price index of value added (primary factors): Capital and labor (and land in sector AGR) are aggregated through a CES function. Therefore, price index of value added is expressed as follows:

$$p_{ir}^{\mathrm{VA}} = \bar{p}_{ir}^{\mathrm{VA}}\left[\theta_{ir}^{\mathrm{VAL}}\left[\frac{p_{r}^{L}}{\bar{p}_{r}^{L}}\right]^{1-\sigma_i^{\mathrm{VAL}}} + \theta_{ir}^{\mathrm{VAK}}\left[\frac{p_{r}^{K}}{\bar{p}_{r}^{K}}\right]^{1-\sigma_i^{\mathrm{VA}}} + \theta_{ir}^{\mathrm{VAN}}\left[\frac{p_{ir}^{\mathrm{LND}}}{\bar{p}_{ir}^{\mathrm{LND}}}\right]^{1-\sigma_i^{\mathrm{VA}}}\right]^{\frac{1}{1-\sigma_i^{\mathrm{VA}}}} \qquad \{p_{ir}^{\mathrm{VA}}\}$$

Unit cost of production: Unit cost of production of sector i in region r is

$$c_{ir}^{Y} = \bar{c}_{ir}^{Y}\left[\sum_j \theta_{jir}^{I}\frac{(1+t_{jir}^{I})p_{jr}^{A}}{(1+\bar{t}_{jir}^{I})\bar{p}_{jr}^{A}} + \theta_{ir}^{\mathrm{VA}}\frac{p_{ir}^{\mathrm{VA}}}{\bar{p}_{ir}^{\mathrm{VA}}}\right] \qquad \{c_{ir}^{Y}\}$$

Since intermediate inputs and composite primary factor are used through a Leontief function, unit cost is the linear combination of prices of intermediate inputs and composite primary factor.

Unit cost of Armington aggregation: Since Armington aggregation (aggregation of domestic and imported goods) is done through a CES function, unit cost of Armington aggregation of goods i in region r is

$$c_{ir}^{A} = \bar{c}_{ir}^{A}\left[\theta_{ir}^{\mathrm{AD}}\left[\frac{p_{ir}^{\mathrm{AD}}}{\bar{p}_{ir}^{\mathrm{AD}}}\right]^{1-\sigma_i^{\mathrm{DM}}} + \theta_{ir}^{M}\left[\frac{p_{ir}^{M}}{\bar{p}_{ir}^{M}}\right]^{1-\sigma_i^{\mathrm{DM}}}\right]^{\frac{1}{1-\sigma_i^{\mathrm{DM}}}} \qquad \{c_{ir}^{A}\}$$

Unit cost of aggregation of domestic goods: Domestic goods from different regions are aggregated through a CES function. Thus unit cost of aggregation of domestic goods i in regins r is

$$c_{ir}^{\mathrm{AD}} = \bar{c}_{ir}^{\mathrm{AD}}\left[\sum_s \theta_{isr}^{D}\left[\frac{p_{is}^{D}}{\bar{p}_{is}^{D}}\right]^{1-\sigma_i^{\mathrm{DD}}}\right]^{\frac{1}{1-\sigma_i^{\mathrm{DD}}}} \qquad \{c_{ir}^{\mathrm{AD}}\}$$

Aggregation of consumption goods: Consumption is the Cobb-Douglas aggregation of individual goods. Thus, its unit cost is

$$c_r^C = \bar{c}_r^C \prod_i \left[\frac{p_{ir}^A}{\bar{p}_{ir}^A} \right]^{\theta_{ir}^C} \qquad \{c_r^C\}$$

Unit cost of investment: Regional investment is the Leontief aggregation of individual goods. Thus, unit cost of investment in region r is

$$c_r^{\mathrm{INV}} = \bar{c}_r^{\mathrm{INV}} \sum_i \theta_{ir}^{\mathrm{INV}} \frac{p_{ir}^A}{\bar{p}_{ir}^A} \qquad \{c_r^{\mathrm{INV}}\}$$

Unit cost of government expenditure: Region government expenditure is the Cobb-Douglas aggregation of individual goods. Thus, unit cost of government expenditure in region r is

$$c_r^G = \bar{c}_r^G \prod_i \left[\frac{p_{ir}^A}{\bar{p}_{ir}^A} \right]^{\theta_{ir}^G} \qquad \{c_r^G\}$$

Unit cost of utility: Utility function is the Cobb-Douglas function of consumption and saving. Thus unit cost of utility is

$$c_r^U = \bar{c}_r^U \left[\frac{p_r^C}{\bar{p}_r^C} \right]^{\theta_{C,r}^U} \left[\frac{p_r^{\mathrm{INV}}}{\bar{p}_r^{\mathrm{INV}}} \right]^{\theta_{S,r}^U} \qquad \{c_r^U\}$$

2.4.2. Unit demand and supply functions

Export supply: Unit export supply of sector i in region r is

$$a_{ir}^{\mathrm{YX}} = \bar{a}_{ir}^{\mathrm{YX}} \left[\frac{p_{ir}^X / \bar{p}_{ir}^X}{p_{ir}^Y / \bar{p}_{ir}^Y} \right]^{\eta_i} \qquad \{a_{ir}^X\}$$

This is derived by differentiating the price index of output (p_{ir}^Y) with respect to p_{ir}^X.

Domestic supply: Similarly, unit domestic supply of sector i in region r is

$$a_{ir}^{\mathrm{YD}} = \bar{a}_{ir}^{\mathrm{YD}} \left[\frac{p_{ir}^D / \bar{p}_{ir}^D}{p_{ir}^Y / \bar{p}_{ir}^Y} \right]^{\eta_i} \qquad \{a_{ir}^D\}$$

Unit demand for capital: Unit demand for capital of sector i in region r is

$$a_{ir}^{K} = \bar{a}_{ir}^{K} \left[\frac{p_{ir}^{\mathrm{VA}} / \bar{p}_{ir}^{\mathrm{VA}}}{p_{r}^{K} / \bar{p}_{r}^{K}} \right]^{\sigma_i^{\mathrm{VA}}} \qquad \{a_{ir}^{K}\}$$

This is derived by differentiating the price index of VA (p_{ir}^{VA}) with respect to p_r^K (Shephard's lemma).

Unit demand for composite labor: Similarly, unit demand for composite labor of sector i in region r is

$$a_{ir}^{L} = \bar{a}_{ir}^{L} \left[\frac{p_{ir}^{\mathrm{VA}} / \bar{p}_{ir}^{\mathrm{VA}}}{p_{r}^{L} / \bar{p}_{r}^{L}} \right]^{\sigma_i^{\mathrm{VA}}} \qquad \{a_{ir}^{L}\}$$

Unit demand for land ($i \in \mathrm{AGR}$): Unit demand for land of sector i in region r is

$$a_{ir}^{\mathrm{LND}} = \bar{a}_{ir}^{\mathrm{LND}} \left[\frac{p_{ir}^{\mathrm{VA}} / \bar{p}_{ir}^{\mathrm{VA}}}{p_{ir}^{\mathrm{LND}} / \bar{p}_{ir}^{\mathrm{LND}}} \right]^{\sigma_i^{\mathrm{VA}}} \qquad \{a_{ir}^{\mathrm{LND}}\}_{i \in \mathrm{AGR}}$$

Unit demand for import goods: Unit demand for import goods i in region r is

$$a_{ir}^{M} = \bar{a}_{ir}^{M} \left[\frac{c_{ir}^{A} / \bar{c}_{ir}^{A}}{p_{ir}^{M} / \bar{p}_{ir}^{M}} \right]^{\sigma_i^{\mathrm{DM}}} \qquad \{a_{ir}^{M}\}$$

Unit demand for composite domestic goods: Unit demand for composite domestic goods i in region r is

$$a_{ir}^{\mathrm{AD}} = \bar{a}_{ir}^{\mathrm{AD}} \left[\frac{c_{ir}^{A} / \bar{c}_{ir}^{A}}{p_{ir}^{\mathrm{AD}} / \bar{p}_{ir}^{\mathrm{AD}}} \right]^{\sigma_i^{\mathrm{DM}}} \qquad \{a_{ir}^{\mathrm{AD}}\}$$

Unit demand for domestic goods: Unit demand of region r for domestic goods i supplied from region s is

$$a_{isr}^{D} = \bar{a}_{isr}^{D} \left[\frac{c_{ir}^{\mathrm{AD}} / \bar{c}_{ir}^{\mathrm{AD}}}{p_{is}^{D} / \bar{p}_{is}^{D}} \right]^{\sigma_i^{\mathrm{DD}}} \qquad \{a_{isr}^{D}\}$$

Unit consumption demand: Unit consumption demand for goods i in region r is

$$a_{ir}^{\mathrm{CC}} = \bar{a}_{ir}^{\mathrm{CC}} \frac{c_{r}^{C} / \bar{c}_{r}^{C}}{p_{ir}^{A} / \bar{p}_{ir}^{A}} \qquad \{a_{ir}^{\mathrm{CC}}\}$$

Unit demand for composite consumption: Unit demand for composite consumption in region r is

$$a_r^C = \bar{a}_r^C \frac{c_r^U / \bar{c}_r^U}{p_r^C / \bar{p}_r^C} \qquad \{a_r^C\}$$

Unit demand for saving: Unit demand for saving in region r is

$$a_r^S = \bar{a}_r^S \frac{c_r^U / \bar{c}_r^U}{p_r^{\mathrm{INV}} / \bar{p}_r^{\mathrm{INV}}} \qquad \{a_r^S\}$$

Unit government demand: Unit demand of government expenditure in region r for goods i is

$$a_{ir}^{\mathrm{GOV}} = \bar{a}_{ir}^{\mathrm{GOV}} \frac{c_r^G / \bar{c}_r^G}{p_{ir}^A / \bar{p}_{ir}^A} \qquad \{a_{ir}^{\mathrm{GOV}}\}$$

Unit investment demand: Unit demand of investment in region r for goods i is

$$a_{ir}^{\mathrm{INV}} = \bar{a}_{ir}^{\mathrm{INV}} \qquad \{a_{ir}^{\mathrm{INV}}\}$$

2.4.3. Zero profit conditions

In the following, the left-hand side represents unit cost and the right-hand side represents unit revenue (price).

Production: Production of sector i in region r. Unit cost is c_{ir}^Y and producer price is $(1-t_{ir}^Y)p_{ir}^Y$. Thus the zero profit condition is given by

$$c_{ir}^Y \geq (1-t_{ir}^Y)p_{ir}^Y \qquad \{Y_{ir}\}$$

Armington aggregation: Zero profit condition for Armington aggregation of goods i in region r is

$$c_{ir}^A \geq p_{ir}^A \qquad \{A_{ir}\}$$

Aggregation of domestic goods: Zero profit condition for aggregation of domestic goods i in region r is

$$c_{ir}^{\mathrm{AD}} \geq p_{ir}^{\mathrm{AD}} \qquad \{\mathrm{AD}_{ir}\}$$

Import: One unit of foreign exchange is converted to one unit of goods i. Cost to obtain one unit of foreign exchange is given by $(1+t_{ir}^{M})p^{\mathrm{FX}}$ while the value of one unit of goods i is p_{ir}^{M}. Thus zero profit condition for import activity of goods i in region r is

$$(1+t_{ir}^{M})p^{\mathrm{FX}} \geq p_{ir}^{M} \qquad \{M_{ir}\}$$

Export: Export of one unit of goods i generates e_{ir}^{TOT} units of foreign exchange (the benchmark value of e_{ir}^{TOT} is unity). Cost for export of one unit of goods i is p_{ir}^{X} and one unit of export generates the value of $e_{ir}^{\mathrm{TOT}} p^{\mathrm{FX}}$. Thus zero profit condition for export activity of goods i in region r is

$$p_{ir}^{X} \geq e_{ir}^{\mathrm{TOT}} p^{\mathrm{FX}} \qquad \{X_{ir}\}$$

The change in e_{ir}^{TOT} expresses the change in terms of trade (increase in e_{ir}^{TOT} represents improvement of terms of trade).

Consumption:

$$c_{r}^{C} \geq p_{r}^{C} \qquad \{C_{r}\}$$

Utility:

$$c_{r}^{U} \geq p_{r}^{U} \qquad \{U_{r}\}$$

Production of investment goods:

$$c_{r}^{\mathrm{INV}} \geq p_{r}^{\mathrm{INV}} \qquad \{\mathrm{INV}_{r}\}$$

Government expenditure:

$$c_{r}^{G} \geq p_{r}^{G} \qquad \{G_{r}\}$$

2.4.4. Market clearing conditions

This subsection presents market clearing conditions. In the following, the left-hand side represents supply and the right-hand side represents demand.

Domestic goods: Market for domestic goods i produced in region r.

$$a_{ir}^{\mathrm{YD}} Y_{ir} \geq \sum_{s} a_{irs}^{D} \mathrm{AD}_{is} \qquad \{p_{ir}^{D}\}$$

Export goods: Market export goods i produced in region r.

$$a_{ir}^{\mathrm{YX}} Y_{ir} \geq X_{ir} \qquad \{p_{ir}^{X}\}$$

Import goods: Market for import goods in region r.

$$e_{ir}^{T} M_{ir} \geq a_{ir}^{M} A_{ir} \qquad \{p_{ir}^{M}\}$$

Composite domestic goods: Market for composite domestic goods i in regin r.

$$\mathrm{AD}_{ir} \geq a_{ir}^{\mathrm{AD}} A_{ir} \qquad \{p_{ir}^{\mathrm{AD}}\}$$

Armington goods: Market for Armington goods i in region r. Demand consists of consumption demand, investment demand, government demand and intermediate demand.

$$A_{ir} \geq a_{ir}^{\mathrm{CC}} C_{ir} + a_{ir}^{\mathrm{INV}} \mathrm{INV}_r + a_{ir}^{\mathrm{GOV}} G_r + \sum_j \bar{a}_{ijr}^{I} Y_{jr} \qquad \{p_{ir}^{A}\}$$

Investment goods: Market for investment goods in region r.

$$\mathrm{INV}_r \geq a_r^{S} U_r \qquad \{p_r^{\mathrm{INV}}\}$$

Composite consumption:

$$C_r \geq a_r^{C} U_r \qquad \{p_r^{C}\}$$

Government expenditure: Market for government expenditure. Demand is fixed at $\bar{G}_r$.

$$G_r \geq \bar{G}_r \qquad \{p_r^{G}\}$$

Labor: Market for labor in region r.

$$\bar{E}_r^{L} \geq \sum_i a_{ir}^{L} Y_{ir} \qquad \{p_r^{L}\}$$

Capital: Market for capital in region r.

$$\bar{E}_r^{K} \geq \sum_i a_{ir}^{K} Y_{ir} \qquad \{p_r^{K}\}$$

Land ($i \in \mathrm{AGR}$): Market for land in region r. Land is used only in sector AGR.

$$\bar{E}_{ir}^{\mathrm{LND}} \geq a_{ir}^{\mathrm{LND}} Y_{ir} \qquad \{p_{ir}^{\mathrm{LND}}\}_{i \in \mathrm{AGR}}$$

Foreign exchange:

$$\sum_{i,r} e_{ir}^{\mathrm{TOT}} e_{ir}^{T} X_{ir} \geq \sum_{i,r} M_{ir} \qquad \{p^{\mathrm{FX}}\}$$

Utility:

$$U_r \geq M_r^H / p_r^U \qquad \{p_r^U\}$$

2.4.5. Income

Net revenue of the central government: Net revenue of the central government is tax revenue minus capital outflow.

$$\begin{aligned} M^G = & \sum_{i,r} t_{ir}^Y p_{ir}^Y Y_{ir} + \sum_r t_r^K p_r^L \bar{E}_r^L + \sum_r t_r^L p_r^K \bar{E}_r^K \\ & + \sum_{i,r} t_{ir}^M p^{\mathrm{FX}} e_{ir}^T M_{ir} - p^{\mathrm{FX}} \mathrm{BOP} \end{aligned} \qquad \{M^G\}$$

Income of the regional household: Income of the regional household that can be used for consumption and saving. It consists of factor income and transfer from the central government minus regional government expenditure.

$$M_r^H = (1 - t_r^L) p_r^L \bar{E}_r^L + (1 - t_r^K) p_r^K \bar{E}_r^K + \sum_{i \in \mathrm{AGR}} p_{ir}^{\mathrm{LND}} \bar{E}_{ir}^{\mathrm{LND}} + \theta_r^T M^G - p_r^G G_r \qquad \{M_r^H\}$$

where $\theta_r^T M^G$ is transfer from the central government and θ_r^T (constant) indicates the share of transfer to region r in total transfer.

2.4.6. Other variables

GDP: (Real) GDP is defined as follows:

$$\mathrm{GDP}_r = C_r + \mathrm{INV}_r + G_r + \sum_i X_{ir} - \sum_i M_{ir} + \sum_{i,s} a_{irs}^D \mathrm{AD}_{is} - \sum_{i,s} a_{isr}^D \mathrm{AD}_{ir} \qquad \{\mathrm{GDP}_r\}$$

2.4.7. Decomposition of change in per capita GDP and EV

Change in per capita GDP:

$$\Delta \text{pcGDP}_r = \Delta\left(\frac{\text{GDP}_r}{\text{pop}_r}\right)$$

$$= \frac{1}{\text{pop}_r}\left[\Delta C_r + \Delta \text{INV}_r + \Delta G_r + \sum_i \Delta X_{ir} - \sum_i \Delta M_{ir} + \sum_{i,s} \Delta\left(a^D_{irs}\text{AD}_{is}\right) - \sum_{i,s} \Delta\left(a^D_{isr}\text{AD}_{ir}\right)\right]$$

where pop_r is the population of region r, which is constant.

Per capita EV:

$$\text{pcEV}_r = \frac{\overline{p}_r^U}{\text{pop}_r}(U_r - \overline{U}_r) = \frac{\overline{p}_r^U}{\text{pop}_r}\left[\frac{M_r^H}{p_r^U} - \frac{M_r^H}{\overline{p}_r^U}\right]$$

$$= \frac{\overline{p}_r^U}{\text{pop}_r}\left[\frac{FI_r}{p_r^U} - \frac{\overline{FI}_r}{\overline{p}_r^U} + \frac{TR_r}{p_r^U} - \frac{\overline{TR}_r}{\overline{p}_r^U} - \frac{GE_r}{p_r^U} + \frac{\overline{GE}_r}{\overline{p}_r^U}\right]$$

where FI, TR, GE are factor income, transfer income and government expenditure respectively.

References

- Rutherford, Thomas F. (1998) "CES Preferences and Technology: A Practical Introduction." in "Economic Equilibrium Modeling with GAMS: An Introduction to GAMS/MCP and GAMS/MPSGE (GAMS/MPSGE Solver Manual)", pp. 89-115, (available at: http://www.gams.com/docs/solver/mpsge.pdf).
- Takeda, Shiro and Kanemi Ban (2008) "Regional Effects of Trade Liberalization in Japan: A CGE Analysis Based on Inter-Regional Iunput-Output Table," November. Available at http://shirotakeda.org/.