A Supplement to 'Comparison of the Effects of Trade Liberalization under Different Market Structures'.

Shiro Takeda*

Department of Economics, Kanto Gakuen University
200 Fujiakucho, Ota city, Gunma, 373–8515, Japan.

e-mail: <zbc08106@park.zero.ad.jp>

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Abstract

This is a supplementary paper to Takeda (2007) (the main paper, hereafter). In this paper, we present results of the simulation omitted in the main paper, and describe the complete model structure, parameterizations, data construction.

Contents

*Email: zbc08106@park.zero.ad.jp. The computer programs for the simulation is available from the author upon request or the author's website: http://park.zero.ad.jp/~zbc08106/

1 Output effects

Section 5 of the main paper only reports a part of output effects. So, in this section, we present all output effects. Output effects (percentage change in sectoral output) under Scenario SG are reported in Table 1.

Table 1: Output effects (percentage change in sectoral outputs)

Region	Sector	PC	CD	LGMC	CH	CF	QCV	BD	IC	IB	AVG	STDEV	M-M
CJK	AFF	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	0	0
CJK	MIN	-9	-10	-10	-9	-9	-10	-10	-11	-10	-10	1	2
CJK	FBT	0	0	0	1	1	0	0	1	0	0	0	0
CJK	TWA	26	35	35	28	24	35	35	37	35	32	5	13
CJK	WPP	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
CJK	CHM	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1
CJK	MET	-1	-2	-2	-1	-1	-2	-2	-2	-2	-2	1	2
CJK	MVT	6	8	7	7	6	8	7	8	7	7	0	1
CJK	ELE	-2	-3	-3	-2	-1	-3	-3	-4	-3	-3	1	3
CJK	OME	-3	-4	-5	-3	-2	-4	-5	-5	-5	-4	1	3
CJK	OMF	0	0	0	1	0	0	0	0	0	0	0	1
CJK	TAT	0	0	0	0	0	0	0	0	0	0	0	0
CJK	OSE	0	0	0	0	0	0	0	0	0	0	0	0
IDN	AFF	1	0	0	1	1	0	0	0	0	0	0	0
IDN	MIN	-14	-18	-18	-15	-14	-18	-18	-17	-18	-16	2	4
IDN	FBT	6	5	5	6	6	5	5	5	5	5	1	2
IDN	TWA	31	61	56	35	31	63	56	55	56	49	13	32
IDN	WPP	0	-3	-3	0	1	-3	-3	-3	-3	-2	2	4
IDN	CHM	0	0	0	0	0	0	0	0	0	0	0	1
IDN	MET	-13	-22	-22	-13	-12	-23	-22	-22	-22	-19	5	11
IDN	MVT	-10	-13	-14	-11	-9	-14	-14	-14	-14	-12	2	5
IDN	ELE	1	-5	-5	1	2	-7	-5	-4	-5	-3	3	9
IDN	OME	0	-7	-7	0	0	-7	-7	-6	-7	-5	3	8
IDN	OMF	-7	-12	-13	-8	-7	-13	-13	-13	-13	-11	3	6
IDN	TAT	1	1	1	1	1	1	1	1	1	1	0	0
IDN	OSE	0	0	0	0	0	0	0	0	0	0	0	1
MYS	AFF	-5	-1	-1	-5	-5	-2	-1	-1	-1	-2	2	5
MYS	MIN	-16	-20	-20	-16	-15	-19	-20	-20	-20	-19	2	5
MYS	FBT	63	122	121	67	61	112	121	120	120	101	28	61
MYS	TWA	57	124	117	59	56	120	118	116	118	99	31	68
MYS	WPP	21	33	35	22	22	33	35	35	34	30	6	13
MYS	CHM	-2	-4	-5	-2	-1	-3	-5	-5	-5	-4	1	3
MYS	MET	-4	-9	-9	-4	-3	-8	-9	-9	-9	-7	3	6
MYS	MVT	-10	-20	-18	-11	-7	-20	-19	-18	-18	-16	5	13
MYS	ELE	-4	-13	-13	-4	-4	-12	-13	-13	-13	-10	4	9
MYS	OME	9	11	11	9	9	14	11	11	11	10	2	5
MYS	OMF	5	7	7	6	6	8	7	8	7	7	1	2
MYS	TAT	0	-3	-3	-2	-2	-3	-3	-3	-3	-3	1	3
MYS	OSE	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	0	1
PHL	AFF	0	0	0	0	0	0	0	0	0	0	0	0
PHL	MIN	-23	-28	-27	-23	-22	-27	-27	-27	-27	-26	2	5
PHL	FBT	-2	-3	-3	-2	-2	-3	-3	-3	-3	-3	0	1
PHL	TWA	31	65	61	34	31	66	61	60	61	52	15	35
PHL	WPP	-3	-9	-8	-4	-3	-8	-8	-9	-8	-7	3	6
PHL	CHM	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1
PHL	MET	-6	-10	-10	-6	-6	-10	-10	-10	-10	-9	2	4
PHL	MVT	17	20	22	17	17	19	22	22	22	20	2	6
PHL	ELE	1	-4	-3	0	0	-4	-3	-3	-3	-2	2	4
PHL	OME	5	2	1	5	5	1	1	2	2	3	2	4
PHL	OMF	-6	-9	-10	-6	-6	-8	-10	-10	-9	-8	2	4
PHL	TAT	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1
PHL	OSE	0	-1	-1	0	0	-1	-1	-1	-1	-1	0	0
SGP	AFF	0	-1	-1	0	-1	-1	-1	-1	-1	-1	0	1
SGP	MIN	-16	-28	-29	-17	-15	-25	-29	-29	-29	-24	6	14
SGP	FBT	29	41	40	30	27	42	40	40	40	37	6	14
SGP	TWA	3	-4	-7	3	2	-3	-7	-8	-7	-3	5	11
SGP	WPP	2	14	14	3	2	14	14	14	14	10	6	13
SGP	CHM	1	-2	-3	2	1	0	-3	-3	-3	-1	2	5
SGP	MET	-4	-13	-14	-4	-4	-12	-14	-14	-14	-10	5	11
SGP	MVT	-19	-31	-33	-19	-19	-30	-33	-33	-33	-28	7	14
SGP	ELE	-2	-11	-11	-2	-2	-11	-11	-11	-11	-8	4	9
SGP	OME	-1	-14	-15	-1	-1	-13	-15	-15	-15	-10	7	14
SGP	OMF	220	573	600	229	219	530	600	606	601	464	183	387
SGP	TAT	-1	-4	-4	-3	-3	-4	-4	-5	-4	-4	1	3
SGP	OSE	-2	-4	-3	-3	-2	-3	-3	-4	-3	-3	0	1

Table 1 (continued)

Region	Sector	PC	CD	LGMC	CH	CF	QCV	BD	IC	IB	AVG	STDEV	M-M
THA	AFF	1	2	2	1	1	2	2	2	2	2	0	1
THA	MIN	-26	-26	-27	-25	-26	-26	-26	-26	-26	-26	0	1
THA	FBT	15	17	17	16	16	17	17	18	17	17	1	3
THA	TWA	7	5	3	8	8	6	3	3	4	5	2	5
THA	WPP	-12	-17	-17	-12	-11	-16	-17	-17	-17	-15	3	6
THA	CHM	-1	-2	-2	-1	-1	-1	-2	-2	-2	-2	0	1
THA	MET	-5	-8	-8	-5	-5	-8	-8	-8	-8	-7	1	3
THA	MVT	-9	-13	-13	-9	-7	-13	-13	-13	-13	-12	3	7
THA	ELE	6	10	9	7	5	11	9	10	10	9	2	5
THA	OME	15	27	26	16	14	28	26	27	26	23	6	14
THA	OMF	-9	-16	-17	-10	-9	-15	-17	-17	-17	-14	4	8
THA	TAT	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	0	1
THA	OSE	-1	-1	-1	-1	-2	-1	-1	-1	-1	-1	0	0
VNM	AFF	-3	-6	-6	-3	-3	-6	-6	-6	-6	-5	1	3
VNM	MIN	-15	-35	-35	-15	-15	-33	-35	-34	-35	-28	10	21
VNM	FBT	-7	-15	-15	-7	-6	-14	-15	-15	-15	-12	4	9
VNM	TWA	126	246	238	130	121	237	239	238	239	202	57	125
VNM	WPP	34	33	37	37	35	36	37	38	36	36	2	5
VNM	CHM	-1	-7	-7	0	0	-6	-7	-7	-7	-5	3	7
VNM	MET	-21	-41	-41	-22	-20	-40	-41	-41	-41	-34	10	21
VNM	MVT	2	-18	-15	1	2	-18	-15	-16	-16	-10	9	21
VNM	ELE	-12	-33	-32	-13	-11	-32	-32	-32	-32	-26	10	22
VNM	OME	4	-21	-20	3	3	-15	-20	-20	-20	-12	12	25
VNM	OMF	0	-2	-3	1	-1	-1	-3	-3	-2	-1	1	3
VNM	TAT	7	5	6	7	5	6	6	6	6	6	1	2
VNM	OSE	-6	-9	-8	-6	-6	-8	-8	-9	-8	-8	1	3
XSE	AFF	2	2	2	2	2	2	2	2	2	2	0	0
XSE	MIN	17	20	21	17	17	20	21	21	21	20	2	5
XSE	FBT	-3	-4	-4	-4	-4	-4	-4	-4	-4	-4	0	1
XSE	TWA	0	-2	-3	0	0	-2	-3	-3	-3	-2	1	3
XSE	WPP	3	4	4	4	3	4	4	5	4	4	1	2
XSE	CHM	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1
XSE	MET	-4	-3	-3	-4	-4	-3	-3	-3	-3	-3	0	1
XSE	MVT	-5	-5	-5	-5	-5	-5	-5	-4	-5	-5	0	1
XSE	ELE	-4	-3	-3	-4	-4	-3	-3	-3	-3	-3	0	1
XSE	OME	-10	-9	-9	-10	-9	-10	-9	-9	-9	-9	0	1
XSE	OMF	-3	-4	-4	-3	-3	-3	-4	-3	-4	-3	0	0
XSE	TAT	0	0	0	0	-1	0	0	0	0	0	0	0
XSE	OSE	0	0	0	0	0	0	0	0	0	0	0	0
XAS	AFF	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
XAS	MIN	-30	-31	-31	-30	-30	-31	-31	-31	-31	-30	1	1
XAS	FBT	-4	-5	-5	-4	-4	-5	-5	-5	-5	-5	1	2
XAS	TWA	15	16	14	16	14	16	14	14	14	15	1	3
XAS	WPP	-1	-1	-2	-1	-1	-2	-2	-2	-2	-2	0	1
XAS	CHM	3	4	4	4	4	4	4	4	4	4	0	1
XAS	MET	-2	-3	-3	-1	0	-3	-3	-2	-3	-2	1	3
XAS	MVT	-5	-7	-8	-5	-3	-7	-8	-7	-8	-6	1	4
XAS	ELE	9	19	17	10	8	19	17	18	17	15	4	11
XAS	OME	7	7	8	7	8	8	8	9	8	8	1	2
XAS	OMF	-8	-12	-13	-7	-6	-13	-13	-13	-13	-11	3	7
XAS	TAT	0	0	0	0	0	0	0	0	0	0	0	0
XAS	OSE	0	0	0	0	0	0	0	0	0	0	0	0
NAF	AFF	6	6	6	6	6	6	6	6	6	6	0	0
NAF	MIN	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1
NAF	FBT	2	2	2	2	2	2	2	2	2	2	0	1
NAF	TWA	-12	-15	-16	-12	-10	-16	-16	-16	-16	-14	2	6
NAF	WPP	0	0	0	0	0	0	0	0	0	0	0	0
NAF	CHM	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
NAF	MET	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
NAF	MVT	-2	-2	-3	-2	-2	-2	-3	-3	-3	-2	0	1
NAF	ELE	0	1	1	0	0	1	1	1	1	1	1	2
NAF	OME	-1	0	-1	-1	-1	-1	-1	-1	-1	-1	0	0
NAF	OMF	14	17	18	15	14	18	18	19	18	17	2	4
NAF	TAT	0	0	0	0	0	0	0	0	0	0	0	0
NAF	OSE	0	0	0	0	0	0	0	0	0	0	0	0

Table 1 (continued)

Region	Sector	PC	CD	LGMC	CH	CF	QCV	BD	IC	IB	AVG	STDEV	M-M
XCS	AFF	9	9	9	9	8	9	9	9	9	9	0	1
XCS	MIN	0	2	2	0	0	2	2	2	2	1	1	2
XCS	FBT	3	3	3	3	3	3	3	3	3	3	0	1
XCS	TWA	-8	-12	-13	-9	-7	-12	-13	-13	-13	-11	2	6
XCS	WPP	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	0	0
XCS	CHM	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	0	1
XCS	MET	-5	-5	-5	-5	-4	-5	-5	-5	-5	-5	0	1
XCS	MVT	2	3	4	3	3	3	4	4	4	3	1	2
XCS	ELE	-5	-5	-5	-5	-4	-5	-5	-5	-5	-5	0	1
XCS	OME	-12	-14	-14	-12	-11	-14	-14	-14	-14	-13	1	3
XCS	OMF	-7	-7	-8	-7	-7	-7	-8	-7	-8	-7	0	1
XCS	TAT	0	0	0	0	0	0	0	0	0	0	0	0
XCS	OSE	0	0	0	0	0	0	0	0	0	0	0	0
EUR	AFF	-1	-2	-2	-1	-1	-2	-2	-2	-2	-1	0	0
EUR	MIN	6	6	6	6	6	6	6	6	6	6	0	0
EUR	FBT	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	0	0
EUR	TWA	-13	-20	-21	-13	-11	-20	-21	-21	-21	-18	4	11
EUR	WPP	1	1	1	1	1	1	1	1	1	1	0	0
EUR	CHM	0	0	0	0	0	0	0	0	0	0	0	0
EUR	MET	1	2	1	1	1	2	1	2	1	1	0	1
EUR	MVT	0	0	0	0	0	0	0	0	0	0	0	0
EUR	ELE	0	2	2	0	0	2	2	2	2	1	1	2
EUR	OME	2	3	3	2	1	3	3	3	3	3	1	2
EUR	OMF	-3	-5	-6	-3	-3	-5	-6	-6	-6	-5	1	3
EUR	TAT	1	0	0	0	0	0	0	0	0	0	0	1
EUR	OSE	0	0	0	0	0	0	0	0	0	0	0	0
ROW	AFF	1	1	1	1	0	1	1	1	1	1	0	0
ROW	MIN	1	2	1	1	1	2	1	1	1	1	0	1
ROW	FBT	6	7	7	7	7	7	7	8	7	7	0	1
ROW	TWA	-11	-19	-20	-12	-9	-19	-20	-20	-20	-17	4	11
ROW	WPP	-4	-4	-5	-4	-4	-4	-5	-5	-5	-5	0	1
ROW	CHM	5	7	6	6	5	7	6	7	6	6	1	2
ROW	MET	2	4	4	3	3	4	4	4	4	4	1	2
ROW	MVT	-5	-7	-7	-5	-4	-7	-7	-7	-7	-6	1	3
ROW	ELE	6	10	10	6	6	11	10	10	10	9	2	5
ROW	OME	-2	-3	-3	-2	-2	-3	-3	-3	-3	-3	0	1
ROW	OMF	-2	-4	-5	-2	-2	-4	-5	-5	-5	-4	1	3
ROW	TAT	0	0	0	0	0	0	0	0	0	0	0	1
ROW	OSE	0	0	0	-1	-1	0	0	0	0	0	0	0

Table 2: List of sectors.

Symbol	Description	The original GTAP sectors
AFF	Agriculture, forestry and fishery	PDR, WHT, GRO, V_F, OSD, C_B, PFB, OCR, CTL, OAP, RMK, WOL, FRS, FSH
MIN	Mining	COA, OIL, GAS, OMN
FBT	Food, Beverages and Tabacco	CMT, OMT, VOL, MIL, PCR, SGR, OFD, B_T
TWA	Textiles, Wearing Apparel, and Leather products	TEX, WAP, LEA
WPP	Wood and Paper products	LUM, PPP
CHM	Chemical products	P_C, CRP, NMM
MET	Metal products	I_S, NFM, FMP
MVT	Motor vehicles and transport equipment	MVH, OTN
ELE	Electronic equipment	ELE
OME	Machinery and equipment nec	OME
OMF	Manufactures nec	OMF
TAT	Trade and transport	TRD, OTP, WTP, ATP
OSE	Other services	ELY, GDT, WTR, CNS, CMN, OFI, ISR, OBS, ROS, OSG, DWE
CGD	Investment goods	CGD

2 Model

In the following, we explain the model used in the main paper. Our analysis is based on a multi-sector multiregion static general equilibrium model. Sectors and regions in the model are listed in Table 2 and 3. We consider not only a perfectly competitive model with CRTS technology but also eight imperfectly competitive models with IRTS technology. First, we explain perfectly competitive model and then explain imperfect competition models. In what follows, notations are defined as follows:

- $i, j \cdots$ Index of sectors and goods.
- $r, s, r' \cdots$ Index of regions.
- $v, v', l \cdots$ Index of firms (varieties).
- $f \cdots$ Index of primary factors.
- $I \cdots$ Set of sectors and goods.
- $C \cdots$ Set of perfectly competitive sectors.
- $K \cdots$ Set of imperfectly competitive sectors.
- $R \cdots$ Set of regions.
- cgd $\cdots$ Index of investment goods.
- $z\ (\in R) \cdots$ Index of a region whose value of consumption is the largest at the benchmark equilibrium.

2.1 Perfectly competitive model

As the perfectly competitive model, we use the simplified version of the GTAP standard model (Hertel, 1997). Our model differs from the GTAP model in four main aspects. First, savings and

Table 3: List of regions.

Symbol	Description	The original GTAP regions
CJK	China, Japan, and Korea	CHN, HKG, JPN, KOR
IDN	Indonesia	IDN
MYS	Malaysia	MYS
PHL	Philippines	PHL
SGP	Singapore	SGP
THA	Thailand	THA
VNM	Vietnam	VNW
XSE	Rest of Southeast Asia	XSE
XAS	Rest of Asia	TWN, XEA, BGD, IND, LKA, XSA
NAF	NAFTA	CAN, USA, MEX
XCS	Central and Southern America	COL, PER, VEN, XAP, ARG, BRA, CHL, URY, XSM, XCA, XFA, XCB
EUR	European countries and the former Soviet Union	AUT, BEL, DNK, FIN, FRA, DEU, GBR, GRC, IRL, ITA, LUX, NLD, PRT, ESP, SWE, CHE, XEF, XER, ALB, BGR, HRV, CYP, CZE, HUN, MLT, POL, ROM, SVK, SVN, EST, LVA, LTU, RUS, XSU
ROW	Rest of the world	AUS, NZL, XOC, XNA, TUR, XME, MAR, TUN, XNF, BWA, ZAF, XSC, MWI, MOZ, TZA, ZMB, ZWE, XSD, MDG, UGA, XSS

investment are determined endogenously in the GTAP model, while they are exogenously constant at the benchmark level in our model.[1] Second, the regional welfare (utility) in the GTAP model is determined through a Cobb-Douglas function of private demand, government expenditure, and savings, while we aggregate private demand and government expenditure into a single final demand and assume that utility is derived only from this final demand. Third, the GTAP model aggregates consumption through a CDE function, while our model aggregates it through a Cobb-Douglas function. Finally, the GTAP model assumes that the aggregation of domestic and imported goods (Armington aggregation) is conducted separately according to their uses, while our model assumes that Armington aggregation is conducted as a whole irrespective of their uses.

In the remainder of this section, we consider the following four activities: production activity, consumption, Armington aggregation, and international transport. It is assumed that all these activities are done under the assumption of CRTS technology, perfect competition, and profit maximization.[2]

2.1.1 Production side

Using intermediate inputs and primary factors, firms produce goods under constant returns to scale (CRS) technology to maximize profits. All markets are assumed to be perfectly competitive and thus all producers are price takers. The production function is a nested CES function represented by Figure 1. The sigmas in the figure represent elasticities of substitution between inputs. Output is produced with fixed coefficient aggregation of intermediate inputs and primary-factor composite. The primary factor composite is a CES aggregation of four primary factors (capital, skilled labor, unskilled labor and land) with an elasticity of σ_i^{PF}.

Let Y_{ir} denote output of sector i, Q_{jir}^{I} denote intermediate input j of sector i, and Q_{ir}^{PF} denote

[1] Strictly speaking, *real investment* is fixed.

[2] Consumption is regarded as an activity that produces utility.

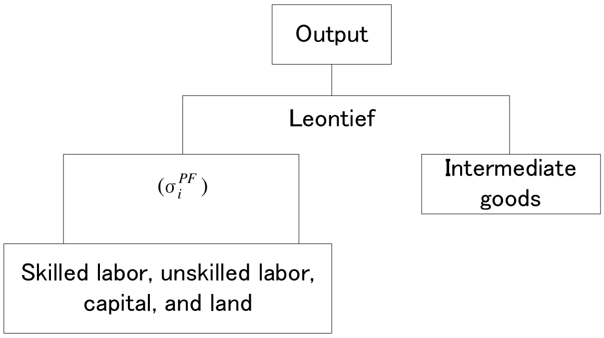

Figure 1: Production function.

primary factor composite of sector i. Then, the production function is represented as follows:

$$Y_{ir} = Y_{ir}(\{Q^I_{jir}\}_{j\in I}, Q^{\mathrm{PF}}_{ir}) = \min\left[\left\{\frac{Q^I_{jir}}{\bar{a}^I_{jir}}\right\}_{j\in I}, \frac{Q^{\mathrm{PF}}_{ir}}{\bar{a}^{\mathrm{PF}}_{ir}}\right]$$

where a^I_{jir} fixed input coefficient of intermediate goods j, and $\bar{a}^{\mathrm{PF}}_{ir}$ is fixed input coefficient of primary factor composite. Since the primary factor composite is a CES aggregation of primary factors, it is represented as follows:

$$Q^{\mathrm{PF}}_{ir} = Q^{\mathrm{PF}}_{ir}(\{Q^F_{fir}\}) = \left[\sum_f \alpha^F_{fir}(Q^F_{fir})^{\frac{\sigma_i^{\mathrm{PF}}-1}{\sigma_i^{\mathrm{PF}}}}\right]^{\frac{\sigma_i^{\mathrm{PF}}}{\sigma_i^{\mathrm{PF}}-1}}$$

where Q^F_{fir} denotes the amount of primary factor f used in sector i. Note that elasticities of substitution among four primary factors (σ_i^{PF}) have different values across sectors.

By the assumption of profit maximizing (cost minimization) and CRTS technology, we can define unit cost function as follows:

$$\begin{aligned} c^Y_{ir} &\equiv \min_{\{Q^I_j\}, Q^{\mathrm{PF}}}\left[\sum_j \tilde{p}^A_{Ijir}Q^I_j + p^{\mathrm{PF}}_{ir}Q^{\mathrm{PF}} \mid Y_{ir}(\{Q^I_j\}, Q^{\mathrm{PF}}) = 1\right] \\ &= \sum_j \tilde{p}^A_{Ijir}\bar{a}^I_{jir} + p^{\mathrm{PF}}_{ir}\bar{a}^{\mathrm{PF}}_{jir} \end{aligned}$$

where $\tilde{p}^A_{Ijir} = (1+t^I_{jir})p^A_{jr}$ is a producer price of intermediate goods j , t^I_{jir} is a tax rate on intermediate inputs, and p^{PF}_{ir} is price index of a primary factor composite of sector i.

Similarly, the combination of primary factors is determined so as to minimize cost. Thus, price index of a primary factor composite is defined as follows:

$$\begin{aligned} p^{\mathrm{PF}}_{ir} &\equiv \min_{\{Q^F_f\}}\left[\sum_f \tilde{p}^F_{fir}Q^F_f \mid Q^{\mathrm{PF}}_{ir}(\{Q^F_f\}) = 1\right] \\ &= \left[\sum_f (\alpha^F_{fir})^{\sigma_i^{\mathrm{PF}}}(\tilde{p}^F_{fir})^{1-\sigma_i^{\mathrm{PF}}}\right]^{\frac{\sigma_i^{\mathrm{PF}}}{\sigma_i^{\mathrm{PF}}-1}} \end{aligned}$$

where $\tilde{p}^F_{fir}$ is the producer price of primary factor f which is equal to $(1+t^F_{fir})p^F_{fr}$.

Next, let us consider output side. In CGE analysis, it is often assumed that goods produced for domestic market and goods produced for export are differentiated. However, we assume that goods of an industry are perfect substitutes regardless of destination. Thus, the price of output is given by a single price p_{ir}^{Y}. Since a tax whose rate is t_{ir}^{Y} is imposed on output, the produced price of output is $(1-t_{ir}^{Y})p_{ir}^{Y}$.

From the results derived above, the profit of sector i is given by

$$\pi_{ir} = \left[(1-t_{ir}^{Y})p_{ir}^{Y} - c_{ir}^{Y}\right] Y_{ir}$$

From this, the zero profit condition (the condition for profit maximization) for sector i is given by

$$\frac{\partial \pi_{ir}}{\partial Y_{ir}} = 0: \;\; c_{ir}^{Y} = (1-t_{ir}^{Y})p_{ir}^{Y}$$

In general, the FOC for profit maximization is given by "marginal revenue = marginal cost". In our model, we have "marginal revenue = price" by the assumption of perfect competition, and have "marginal cost = unit cost" by the assumption of CRTS technology. Thus, "price = unit cost" becomes the FOC for profit maximization.

Finally, we derive demand function for inputs. First, demand for intermediate input is given by $\bar{a}_{jir}^{I} Y_{ir}$. On the other hand, demand for primary factors can be derived by applying Shephard's lemma to the price index of primary factor composites. Let a_{fir}^{F} denote the unit demand for primary factor f. Then, a_{fir}^{F} is represented as follows:

$$a_{fir}^{F} = \frac{\partial c_{ir}^{Y}}{\partial \tilde{p}_{fir}^{F}} = \frac{\partial c_{ir}^{Y}}{\partial p_{ir}^{\text{PF}}} \frac{\partial p_{ir}^{\text{PF}}}{\partial \tilde{p}_{fir}^{F}} = \bar{a}_{ir}^{\text{PF}} \left[\frac{\alpha_{fir}^{F} p_{ir}^{\text{PF}}}{\tilde{p}_{fir}^{F}}\right]^{\sigma_i^{\text{PF}}}$$

Total demand for factor f of sector i is given by $a_{fir}^{F} Y_{ir}$.

2.1.2 Demand side

To represent the demand side, we assume a representative household for each region. Since we do not consider government explicitly, final demand is the sum of private demand and government expenditure. Final demand is derived from the optimizing behavior of this household. The utility function for the household is a Cobb-Douglas function of consumption goods. Thus, utility U_r is represented as follows:

$$U_r = U_r(\{C_{ir}\}) = \prod_i (C_{ir})^{\theta_{ir}^{C}}$$

where C_{ir} is final demand of goods i.

By the homogeneity of the utility function, we can define the unit cost (expenditure) function of utility as follows:

$$c_r^{U} \equiv \min_{\{C_i\}} \left[\sum_i \tilde{p}_{Cir}^{A} C_i \mid U_r(\{C_i\}) = 1\right] = \prod_i \left[\frac{\tilde{p}_{ir}^{A}}{\theta_{ir}^{C}}\right]^{\theta_{ir}^{C}}$$

where $\tilde{p}_{Cir}^{A}$ is demand price of goods i which is equal to $(1+t_{ir}^{C})p_{ir}^{A}$. If we regard consumption as utility producing activity, the FOC for consumption is given by unit cost = price as in production activity:

$$c_r^{U} = p_r^{U}$$

where p_r^{U} is the price of utility.

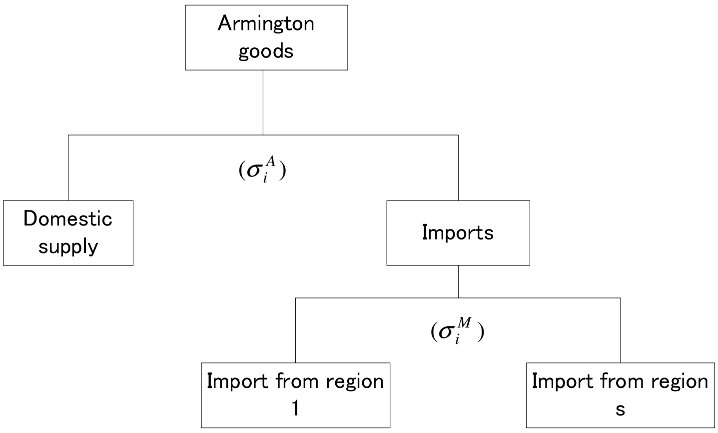

Figure 2: Armington structure in perfectly competitive model.

In addition, let H_r denote the income spent on consumption. Then, the level of utility is represented as follows:

$$U_r = H_r / p_r^U$$

Finally, we derive the demand for consumption goods. From Shephard's lemma, compensated unit demand function is given by

$$a_{ir}^C = \frac{\partial c_r^U}{\partial \tilde{p}_{Cir}^A} = \frac{\theta_{ir}^C c_r^U}{\tilde{p}_{Cir}^A}$$

Total consumption demand for goods i is given by $a_{ir}^C U_r$.

2.1.3 Investment

We assume that investment INV_r is assumed to be constant at the benchmark level. As in the GTAP model, production of investment goods is treated in the same way as ordinary goods.

2.1.4 International trade

Like other CGE analyses, we use the Armington assumption to explain cross-hauling in trade (Armington, 1969). The Armington assumption implies that goods produced in different regions are imperfect substitutes. Goods produced in different regions are aggregated through a two-stage CES function (see Figure 2). First, imports from different regions are aggregated into a import composite and then domestic goods and import composite are aggregated. σ_{ir}^A is the elasticity of substitution between domestic and import goods and σ_{ir}^M is the elasticity of substitution between imports from different regions. In the following, goods that aggregate domestic and imports are called Armington goods, and aggregation of domestic and import goods and aggregation of import goods from different regions are called Armington aggregation and import aggregation respectively. Armington goods are used for both intermediate input and final consumption.

Let us express Armington and import aggregation by mathematical formula. First, the import composite of region r is given by

$$\text{AM}_{ir} = \text{AM}_{ir}(\{M_{isr}\}_s) = \left[\sum_s \alpha_{isr}^M (M_{isr})^{\frac{\sigma_i^M - 1}{\sigma_i^M}} \right]^{\frac{\sigma_i^M}{\sigma_i^M - 1}}$$

where M_{isr} denote import of goods i from region s to region r.

The import composite AM_{ir} and domestic goods AD_{ir} are are aggregated into Armington goods A_{ir} through a CES function.

$$A_{ir} = A_{ir}(\text{AD}_{ir}, \text{AM}_{ir}) = \left[\alpha_{ir}^{\text{AD}} (\text{AD}_{ir})^{\frac{\sigma_i^A - 1}{\sigma_i^A}} + \alpha_{ir}^{\text{AM}} (\text{AM}_{ir})^{\frac{\sigma_i^A - 1}{\sigma_i^A}} \right]^{\frac{\sigma_i^A}{\sigma_i^A - 1}}$$

It is assumed that two aggregation activities are conducted so as to maximize profit. Thus, we can define unit costs for aggregation activities. First, the unit cost of import aggregation is given by

$$c_{ir}^{\text{AM}} \equiv \min \left[\sum_s \tilde{p}_{isr}^M M_{is} \mid \text{AM}_{ir}(\{M_{is}\}_s) = 1 \right] = \left[\sum_s (\alpha_{isr}^M)^{\sigma_i^M} (\tilde{p}_{isr}^M)^{1-\sigma_i^M} \right]^{\frac{1}{1-\sigma_i^M}}$$

where $\tilde{p}_{isr}^M$ is the price of import from region s to region r. This import price includes export tax, import tax and transport cost. Let t_{isr}^X and t_{isr}^M denote export subsidy and import tax imposed on good i from region s to region r. Moreover, let τ_{isr} denote the amount of transport services required to ship one unit of good i from region s to region r and let p^T denote the price of transport services. Then, the CIF price of import is given by

$$\tilde{p}_{isr}^X = (1 - t_{isr}^X) p_{is}^X + p^T \tau_{isr}$$

$\tilde{p}_{isr}^M$ is derived by adding import tariffs to $\tilde{p}_{isr}^X$:

$$\tilde{p}_{isr}^M = (1 + t_{isr}^M) \tilde{p}_{isr}^X$$

Similarly, the unit cost of Armington aggregation is

$$\begin{aligned} c_{ir}^A &\equiv \min \left[p_{ir}^Y \text{AD} + p_{ir}^{\text{AM}} \text{AM} \mid A_{ir}(\text{AD}, \text{AM}) = 1 \right] \\ &= \left[(\alpha_{ir}^{\text{AD}})^{\sigma_i^A} (p_{ir}^Y)^{1-\sigma_i^A} + (\alpha_{ir}^{\text{AM}})^{\sigma_i^A} (p_{ir}^{\text{AM}})^{1-\sigma_i^A} \right]^{\frac{1}{1-\sigma_i^A}} \end{aligned}$$

where p_{ir}^Y is the domestic price in region r and p_{ir}^{AM} is the price index of an import composite.

Since the profit of import aggregation activity is given by $(p_{ir}^{\text{AM}} - c_{ir}^{\text{AM}})\text{AM}_{ir}$, the FOC for profit maximization is

$$c_{ir}^{\text{AM}} = p_{ir}^{\text{AM}}$$

Similarly, if p_{ir}^A denotes the price of Armington goods, the FOC for profit maximization in Armington aggregation is

$$c_{ir}^A = p_{ir}^A$$

For both activities, FOCs for profit maximization are equivalent to zero profit conditions.

From the unit const functions defined above, we can derive demand functions for domestic goods and imports. First, region r's unit demand for import of good i from region s is

$$a_{isr}^M = \frac{\partial c_{ir}^{\text{AM}}}{\partial \tilde{p}_{isr}^M} = \left[\frac{\alpha_{isr}^M c_{ir}^{\text{AM}}}{\tilde{p}_{isr}^M} \right]^{\sigma_i^M}$$

Similarly, unit demand functions for domestic and import composite are given by

$$a_{ir}^{\text{AD}} = \frac{\partial c_{ir}^A}{\partial p_{ir}^Y} = \left[\frac{\alpha_{ir}^{\text{AD}} c_{ir}^A}{p_{ir}^Y} \right]^{\sigma_i^A}$$

$$a_{ir}^{\text{AM}} = \frac{\partial c_{ir}^A}{\partial p_{ir}^{\text{AM}}} = \left[\frac{\alpha_{ir}^{\text{AM}} c_{ir}^A}{p_{ir}^{\text{AM}}} \right]^{\sigma_i^A}$$

2.1.5 International transport sector

As in the GTAP model, we assume that there is a global transport sector which supplies international transport services. Transport service is produced through a Cobb–Douglas production function, using inputs supplied from various regions. Let Q^T_{ir} denote region r's input of goods i to transport service. Then, output of transport service T is represented as follows:

$$Y^T = \prod_{i,r} (Q^T_{ir})^{\theta^T_{ir}}$$

International transport sector aims to maximize his profit. Thus, we can define the unit cost of transport service as

$$c^T = \prod_{i,r} \left[\frac{p^Y_{ir}}{\theta^T_{ir}} \right]^{\theta^T_{ir}}$$

Let p^T denote the price of transport service. Then, the FOC for profit maximization of transport sector is represented by

$$c^T = p^T$$

By applying Shephard's lemma, demand for input is given by

$$a^T_{ir} = \frac{\partial c^T}{\partial p^Y_{ir}} = \frac{\theta^T_{ir} c^T}{p^Y_{ir}}$$

On the other hand, demand for transport service associated with import of goods i from region s to region r is given by

$$\tau_{isr} a^M_{isr} \mathrm{AM}_{ir}$$

where τ_{isr} is unit of transport service required to ship one unit of goods i from region s to region r.

2.2 Imperfectly competitive model (IRTS model)

Next, we explain imperfectly competitive models. This paper consider eight imperfectly competitive models listed in Table 4. Note that even in the imperfectly competitive models, sectors AFF and MIN are assumed to be perfectly competitive sectors with CRTS technology. The assumption that AFF is a perfectly competitive sector is common in many CGE studies. The assumption that MIN is perfectly competitive is for a computational reason.[3]

Model PC in Table 4 is a perfectly competitive model explained in the previous section. Model CD is a benchmark model of all imperfectly competitive models.[4] Alternative imperfectly competitive models are derived from model CD by changing the assumptions. So, we first explain the structure of model CD in detail. In model CD, we make the following assumptions.

A1: Economies of scale arise from the existence of fixed costs.

A2: Varieties of different firms in a sector are assumed to be differentiated and aggregated using a CES function.

A3: Each firm behaves in a Cournot fashion, that is, each firm determines its output, taking the output of all other firms as fixed.

A4: Markets in different regions are segmented.

A5: Free entry and exit are possible.

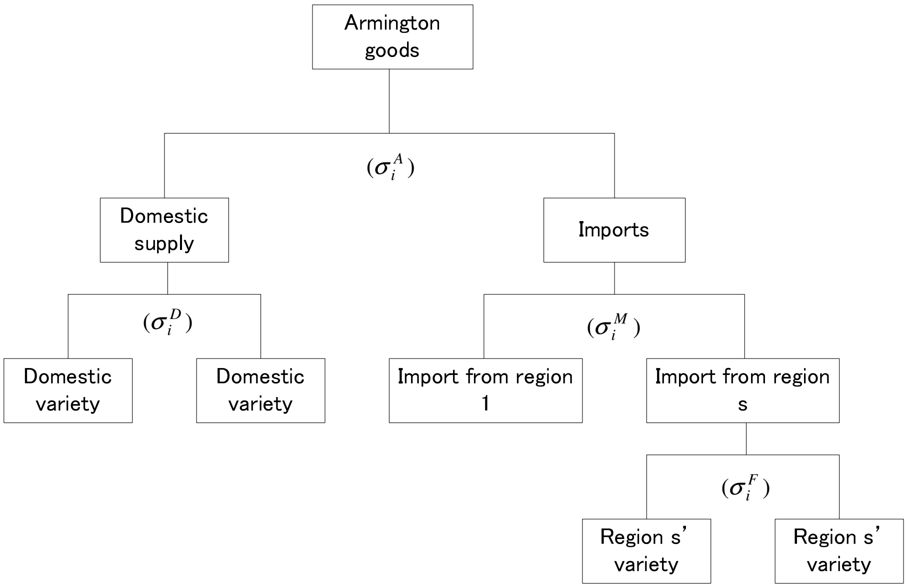

Figure 3: Armington aggregation with IRTS goods.

Table 4: Model list.

Model name	Description
Model PC	Perfectly competitive model.
Model CD	Cournot model.
Model LGMC	Large group monopolistic competition model.
Model CH	Cournot model with homogeneous varieties.
Model CF	Cournot model with fixed number of firms.
Model QCV	Quantity competition model with non-Cournot conjectural variation.
Model BD	Bertrand model.
Model IC	Integrated market Cournot model.
Model IB	Integrated market Bertrand model.

Figure 4: Armington aggregation.

A1 is the assumption on the way for introducing economies of scale. There are two approaches to incorporate economies of scale: (1) to assume a production function with increasing returns to scale, and (2) to assume fixed cost. Like many trade CGE models, model CD uses approach (2). Assumption A1 is applied to all imperfectly competitive models in Table 4, while A2-A5 are modified according to the different models.

Assumption A2 is the assumption on product variety. Like other trade CGE models, model CD assumes that varieties of different firms in a sector are differentiated (imperfect substitutes). In addition, model CD assumes that these differentiated varieties are aggregated through a CES function. By this assumption, model CD possesses love of variety. With respect to variety aggregation, we add it into the model, keeping the Armington assumption (see Figure 2). Although some imperfectly competitive CGE models abandon the Armington aggregation and only consider

[3]When MIN is assumed to be imperfectly competitive, the model becomes significantly unstable and cannot be solved. To make the model stable, MIN is assumed to be perfectly competitive even in the imperfectly competitive model.

[4]Bchir, Decreux, Guérin and Jean (2002) employ a model similar to model CD.

the variety aggregation[5], we adopt the Armington-variety aggregation because the absence of the Armington aggregation can significantly alter the structure of the model and thus makes difficult the comparisons between perfectly competitive and imperfectly competitive models. Since our aim is to compare different assumptions on market structure, we want to keep the same structure in other aspects. So, we decide to keep the Armington aggregation also in imperfectly competitive models. By this assumption, variety aggregation stage is added to Armington structure (see Figure 4). σ_i^D in the figure represents elasticity of substitution between domestic varieties and and σ_i^F represents elasticity of substitution between import varieties.

A3 is the assumption on forms of competition. Although the basic structure of model CD is monopolistic competition, there are various types of forms of competition. Model CD assumes that firms behave with Cournot conjecture. A4 is the assumption on market segmentation. There are two alternative assumptions for it: (1) segmented market and (2) integrated market. Model CD assumes the former type. Under the segmented market assumption, firms can cntrol output and price for different regions independently. Finally, A5 is the assumption on entry and exit. For this, there are two alternative cases: (1) free entry and exit are possible, and (2) entry and exit are restricted. Model CD adopts the former assumption. Free entry and exit mean that the number of firms is endogenously determined so that zero profit condition is satisfied.

2.2.1 Other IRTS models

Here, we briefly explain main characteristics of other IRTS models.

Model LGMC: Model LGMC is the large group monopolistic competition model frequently used in theoretical analysis. In this model, it is assumed that each firm recognizes the number of firms as sufficiently large. As a result, model LGMC has the following two features: (1) markup rate is kept constant (equal to the inverse of the elasticity of substitution), and (2) scale of each firm (total output of each firm) is kept constant. As these features seem to be somewhat unrealistic, the validity of this model may be questionable. However, this model is frequently used not only in theoretical analysis but also in CGE studies, and thus we decided to consider also this model.[6]

Model CH: Model CH changes the assumption of product variety. It assumes that product varieties of different firms are homogeneous (perfect substitutes). Homogeneous variety means that σ_i^D and σ_i^F in Figure 4 are infinite.

Model CF: In model CF, the assumption on entry is modified. It assumes that the number of firms is fixed at the benchmark level. This assumption indicates (1) a situation where there are strong entry barriers to markets, or (2) a situation in the short run. The former situation is of importance because entry barriers are often observed in actual economies; the latter situation is also worth analyzing because it often takes some time for economies to adjust to external shocks. In addition, theoretical analysis such as that in Horstmann and Markusen (1986) and Markusen and Venables (1988) shows that the effects of trade policy can vary drastically, depending on whether free entry and exit are possible or not. Thus, we consider the model of a fixed number of firms as well. Note that in our model, each firm produces one variety and thus the assumption of a fixed number of firms implies the fixed number of varieties.

Model QCV: Model QCV changes the assumption on conjectural variation. Model CD assumes Cournot conjecture, that is, each firm determines its output, taking the output of all other firms as fixed. On the other hand, in model QCV, each firm determines its output, taking the output of all other firms as variable. Although this non-Cournot conjecture model may rarely be used in

[5]For example, the Francois model (Francois, van Meijl and Tongeren, 2005) and the Michigan model (Brown, Deardorff and Stern, 2002; Brown, Kiyota and Stern, 2006) abandon the Armington assumption and only consider the variety aggregation.

[6]For example, the following papers employ model LGMC: Francois, McDonald and Nordström (1996), Francois and Roland-Holst (1997), and Francois et al. (2005).

theoretical analysis due to its complexity, it is often used in CGE analysis.[7] The Cournot competition model is the representative model in the imperfect competition models and is used in both theoretical and empirical analysis. However, this does not necessarily guarantee the actual validity of the Cournot competition model. Moreover, Eaton and Grossman (1986) demonstrate that the welfare effects of trade policy can be strongly influenced by the assumptions on conjectural variation. Thus, it is of great importance to show how the assumptions on conjectural variation affect results.

Model BD: Model BD is a Bertrand competition version of model CD, that is, it assumes that a firm's strategic variable is price and that each firm determines its prices, taking the prices of all other firms as fixed. As with the Cournot model, the Bertrand model is one of the most popular imperfectly competitive models and is used frequently in both theoretical and empirical works. However, because it is difficult to evaluate which model is the more realistic, we decided to consider the Bertrand model as well as the Cournot model.

Model IC and IB: Although all models listed so far assume segmented markets, there is another frequently used model: the integrated market model. In the integrated market model, where arbitrage trade across different regions is possible, firms cannot independently set output for markets in different regions and only control total output. Moreover, they cannot set different prices for different regions. Studies such as Markusen and Venables (1988) show that the effects on trade policy can vary significantly, depending on whether the market is segmented or integrated. Thus, we attempted to consider the integrated market model and examine differences generated by the two alternative assumptions. Model IC is the integrated market version of model CD and model IB is the integrated market version of model BD.

2.2.2 Cost structure

In this section, using model CD as an example, we explain in detail the structure of the imperfectly competitive models. In imperfectly competitive models, economies of scale internal to firms are present in all sectors except in sectors AFF and MIN. In the following, a sector with economies of scale is called the IRTS sector, and a sector without economies of scale is called the CRTS sector. The structure of CRTS sectors is the same as in the perfectly competitive model.

It is assumed that economies of scale arise from the existence of fixed cost. Let q^T_{vir} denote the total output of firm v in sector i. Then, the total cost TC_{vir} is given by

$$\mathrm{TC}_{vir} = \mathrm{MC}_{vir}\left[q^T_{vir} + \mathrm{fc}_{vir}\right] \tag{1}$$

where MC_{vir} is the marginal cost and $\mathrm{MC}_{vir} \times \mathrm{fc}_{vir}$ is the fixed cost.[8] The marginal cost is assumed to be independent from output. Moreover, we assume that the input structure (production function) is the same as that in the perfectly competitive model. The form of MC $\times$ fc means that combination of intermediate goods and primary factors used for fixed and variable costs is the same.

2.2.3 Output side

As to the output side, model CD assumes that product varieties of different firms in a sector are differentiated. Since each variety is aggregated using a CES function, the Armington-Variety aggregation for model QD is modified as in Figure 4. Markets in different regions are assumed to be segmented; each firm determines separately the level of supply to different regions. Given the Armington structure of Figure 4, the firm determines the optimal supply to different regions.

[7] For example, the following studies adopt a non-Cournot conjectural variation models: Burniaux and Waelbroeck (1992), de Melo and Tarr (1992, Chap.7), Harrison, Rutherford and Tarr (1996, 1997), Francois and Roland-Holst (1997), and de Santis (2002a,b).

[8] Although we use the term "fixed cost", it does not mean that MC $\times$ fc is constant. If the marginal cost MC changes, MC $\times$ fc also changes. The term "fixed cost" in this case means that it does not depend on the level of output.

The profit of firm v of IRTS sector i in region r is given by

$$\pi_{vir} = (1 - t^Y_{ir}) \left[p^D_{vir} q^D_{vir} + \sum_s p^X_{virs} q^X_{virs} \right] - \text{MC}_{vir} \left[q^D_{vir} + \sum_s q^X_{virs} - \text{fc}_{vir} \right] \tag{2}$$

where t^Y_{ir} is the output tax rate, q^D_{vir} is the supply to the domestic market, q^X_{virs} is the supply to region s, p^D_{vir} is the price in the domestic market, p^X_{virs} is the export price to region s, MC_{vir} is the marginal cost, and $\text{MC}_{vir} \times \text{fc}_{vir}$ is the fixed cost. Since q^T_{vir} is total output, the following relation holds.

$$q^T_{vir} = q^D_{vir} + \sum_s q^X_{virs}$$

In the perfectly competitive model, domestic supply and export supply have a common price, that is, p^Y_{ir}. On the contrary, in the imperfectly competitive model, prices are distinguished by destination because all markets in different regions are segmented.

Each firm determines supply to the domestic and export markets so as to maximize profit. FOCs of profit maximization of firm v are

$$\frac{\partial \pi_{vir}}{\partial q^D_{vir}} = 0: \quad (1 - t^Y_{ir}) p^D_{vir} \left[1 - \frac{1}{\varepsilon^D_{vir}} \right] = \text{MC}_{vir} \tag{3}$$

$$\frac{\partial \pi_{vir}}{\partial q^X_{virs}} = 0: \quad (1 - t^Y_{ir}) p^X_{virs} \left[1 - \frac{1}{\varepsilon^X_{virs}} \right] = \text{MC}_{vir} \tag{4}$$

where ε^D_{vir} and ε^X_{virs} denote the perceived elasticities of demand in domestic and export markets, respectively, and defined as follows:

$$\varepsilon^D_{vir} \equiv -\frac{\partial \ln q^D_{vir}}{\partial \ln p^D_{vir}} \qquad \varepsilon^X_{virs} \equiv -\frac{\partial \ln q^X_{virs}}{\partial \ln p^X_{virs}}$$

Let us define $\mu^D_{vir} \equiv 1/\varepsilon^D_{vir}$, $\mu^X_{virs} \equiv 1/\varepsilon^X_{virs}$ and $\hat{p}^D_{vir} = (1 - t^Y_{ir}) p^D_{ir}$, $\hat{p}^X_{virs} = (1 - t^Y_{ir}) p^X_{virs}$. Then, (3)–(4) are written as

$$\mu^D_{vir} = \frac{\hat{p}^D_{vir} - \text{MC}_{vir}}{\hat{p}^D_{vir}} \qquad \mu^X_{virs} = \frac{\hat{p}^X_{virs} - \text{MC}_{vir}}{\hat{p}^X_{virs}} \tag{5}$$

From these equations, we can see that μ^D_{vir} and μ^X_{virs} represent markup rates for domestic and export markets, respectively.

2.2.4 Markup rates

To incorporate (3)–(4) (or (5)) into the simulation, it is necessary to derive explicit formula of markup rates. Below, we assume that all firms (varieties) in an IRTS industry are symmetric.

Markup rate for domestic market

First, we derive markup formula for the domestic market (μ^D_{vir}). Since μ^D_{vir} is a reciprocal of the price elasticity of the domestic demand (ε^D_{vir}), we need to derive ε^D_{vir}. It is assumed that each firm determines its output, taking account of the Armington structure depicted in Figure 3. Aggregation in all stages is conducted with CES functions. Thus, the structure of aggregation is represented as follows:

$$A_{ir} = \left[\alpha_{ir}^{\text{AD}} (\text{AD}_{ir})^{\frac{\sigma_i^A - 1}{\sigma_i^A}} + \alpha_{ir}^{\text{AM}} (\text{AM}_{ir})^{\frac{\sigma_i^A - 1}{\sigma_i^A}} \right]^{\frac{\sigma_i^A}{\sigma_i^A - 1}} \tag{6}$$

$$\text{AM}_{ir} = \left[\sum_s \alpha_{isr}^M (M_{isr})^{\frac{\sigma_i^M - 1}{\sigma_i^M}} \right]^{\frac{\sigma_i^M}{\sigma_i^M - 1}} \tag{7}$$

$$\text{AD}_{ir} = \left[\sum_v \beta_{vir}^D (q_{vir}^D)^{\frac{\sigma_i^D - 1}{\sigma_i^D}} \right]^{\frac{\sigma_i^D}{\sigma_i^D - 1}} \tag{8}$$

$$M_{isr} = \left[\sum \beta_{visr}^M (q_{visr}^X)^{\frac{\sigma_i^F - 1}{\sigma_i^F}} \right]^{\frac{\sigma_i^F}{\sigma_i^F - 1}} \tag{9}$$

where A_{ir} is Armington goods i which is created from domestic and imported goods, AM_{ir} is a composite import, AD_{ir} is a composite of domestic varieties, and M_{isr} is a composite of import varieties.

Since all quantity indices are linearly homogeneous CES functions, we can defines price indices as follows:

$$p_{ir}^A = \left[(\alpha_{ir}^{\text{AD}})^{\sigma_i^A} (p_{ir}^{\text{AD}})^{1-\sigma_i^A} + (\alpha_{ir}^{\text{AM}})^{\sigma_i^A} (p_{ir}^{\text{AM}})^{1-\sigma_i^A} \right]^{\frac{1}{1-\sigma_i^A}} \tag{10}$$

$$p_{ir}^{\text{AM}} = \left[\sum_s (\alpha_{isr}^M)^{\sigma_i^M} (\tilde{p}_{isr}^M)^{1-\sigma_i^M} \right]^{\frac{1}{1-\sigma_i^M}} \tag{11}$$

$$p_{ir}^{\text{AD}} = \left[\sum_v (\beta_{vir}^D)^{\sigma_i^D} (p_{vir}^D)^{1-\sigma_i^D} \right]^{\frac{1}{1-\sigma_i^D}} \tag{12}$$

$$p_{isr}^M = \left[\sum_v (\beta_{visr}^M)^{\sigma_i^F} (\tilde{p}_{visr}^X)^{1-\sigma_i^F} \right]^{\frac{1}{1-\sigma_i^F}} \tag{13}$$

where $\tilde{p}_{isr}^M = (1 + t_{isr}^M) p_{isr}^M$ and $\tilde{p}_{visr}^X = (1 - t_{isr}^X) p_{visr}^X + p^T \tau_{isr}$.

From the price indices, we can derive compensated demand functions.

$$\text{AD}_{ir} = \frac{\partial p_{ir}^A}{\partial p_{ir}^{\text{AD}}} A_{ir} = \left[\frac{\alpha_{ir}^{\text{AD}} p_{ir}^A}{p_{ir}^{\text{AD}}} \right]^{\sigma_i^A} A_{ir} \tag{14}$$

$$\text{AM}_{ir} = \frac{\partial p_{ir}^A}{\partial p_{ir}^{\text{AM}}} A_{ir} = \left[\frac{(1 - \alpha_{ir}^{\text{AD}}) p_{ir}^A}{p_{ir}^{\text{AM}}} \right]^{\sigma_i^A} A_{ir} \tag{15}$$

$$M_{isr} = \frac{\partial p_{ir}^{\text{AM}}}{\partial \tilde{p}_{isr}^M} \text{AM}_{ir} = \left[\frac{\alpha_{isr}^M p_{ir}^{\text{AM}}}{\tilde{p}_{isr}^M} \right]^{\sigma_i^M} \text{AM}_{ir} \tag{16}$$

$$q_{vir}^D = \frac{\partial p_{ir}^{\text{AD}}}{\partial p_{vir}^D} \text{AD}_{ir} = \left[\frac{\beta_{vir}^D p_{ir}^{\text{AD}}}{p_{vir}^D} \right]^{\sigma_i^D} \text{AD}_{ir} \tag{17}$$

$$q_{visr}^X = \frac{\partial p_{isr}^M}{\partial \tilde{p}_{visr}^X} M_{isr} = \left[\frac{\beta_{visr}^M p_{isr}^M}{\tilde{p}_{visr}^X} \right]^{\sigma_i^F} M_{isr} \tag{18}$$

From the above results, let us derive markup rates (price elasticity of demand). First, we derive inverse demand function for domestic variety from (17).

$$p_{vir}^{D}=\left[\frac{\mathrm{AD}_{ir}}{q_{vir}^{D}}\right]^{1/\sigma_i^D}\beta_{vir}^{D}p_{ir}^{\mathrm{AD}} \tag{19}$$

Taking logarithm of both sides, we have

$$\ln p_{vir}^{D}=\frac{1}{\sigma_i^D}\ln A_{ir}-\frac{1}{\sigma_i^D}q_{vir}^{D}+\ln p_{ir}^{\mathrm{AD}}+\ln\beta_{vir}^{D}$$

From this, the following relationship holds.

$$\frac{\partial\ln p_{vir}^{D}}{\partial\ln q_{vir}^{D}}=-\frac{1}{\sigma_i^D}+\frac{1}{\sigma_i^D}\frac{q_{vir}^{D}}{\mathrm{AD}_{ir}}\frac{\partial\mathrm{AD}_{ir}}{\partial q_{vir}^{D}}+\frac{q_{vif}^{D}}{p_{ir}^{\mathrm{AD}}}\frac{\partial p_{ir}^{\mathrm{AD}}}{\partial\mathrm{AD}_{ir}}\frac{\partial\mathrm{AD}_{ir}}{\partial q_{vir}^{D}} \tag{20}$$

By (8), $\partial\mathrm{AD}_{ir}/\partial q_{vir}^{D}$ in (20) is written as

$$\frac{\partial\mathrm{AD}_{ir}}{\partial q_{vir}^{D}}=(\mathrm{AD}_{ir})^{1/\sigma_i^D}\left[\beta_{vir}^{D}(q_{vir}^{D})^{-1/\sigma_i^D}+\sum_{v'\neq v}\beta_{v'ir}^{D}(q_{v'ir}^{D})^{-1/\sigma_i^D}\frac{q_{v'ir}^{D}}{q_{vir}^{D}}\phi_{vir}^{D}\right] \tag{21}$$

where ϕ_{vir}^{D} is firm v's conjectural elasticity defined as follows.

$$\phi_{vir}^{D}\equiv\frac{\partial\ln q_{v'ir}^{D}}{\partial\ln q_{vir}^{D}}\qquad v'\neq v$$

Substituting (19) into (21), we get

$$\frac{\partial\mathrm{AD}_{ir}}{\partial q_{vir}^{D}}=\frac{p_{vir}^{D}}{p_{ir}^{\mathrm{AD}}}\left[1+\sum_{v'\neq v}\frac{p_{v'ir}^{D}q_{v'ir}^{D}}{p_{vir}^{D}q_{vir}^{D}}\phi_{vir}^{D}\right]$$

From this, (20) is written as follows:

$$\frac{\partial\ln p_{vir}^{D}}{\partial\ln q_{vir}^{D}}=-\frac{1}{\sigma_i^D}+\left[\frac{1}{\sigma_i^D}\frac{p_{vir}^{D}q_{vir}^{D}}{p_{ir}^{\mathrm{AD}}\mathrm{AD}_{ir}}+\frac{p_{vir}^{D}q_{vir}^{D}}{p_{ir}^{\mathrm{AD}}\mathrm{AD}_{ir}}\frac{\partial p_{ir}^{\mathrm{AD}}}{\partial\mathrm{AD}_{ir}}\frac{\mathrm{AD}_{ir}}{p_{ir}^{\mathrm{AD}}}\right]\left[1+\sum_{v'\neq v}\frac{p_{v'ir}^{D}q_{v'ir}^{D}}{p_{vir}^{D}q_{vir}^{D}}\phi_{vir}^{D}\right]$$

$p_{vir}^{D}q_{vir}^{D}/(p_{ir}^{\mathrm{AD}}\mathrm{AD}_{ir})$ in the above equation indicates share of a firm in the domestic market. Since we assume symmetry of all firms, we have $p_{vir}^{D}q_{vir}^{D}/(p_{ir}^{\mathrm{AD}}\mathrm{AD}_{ir})=1/n_{ir}$. Similarly, symmetry means $p_{vir}^{D}=p_{v'ir}^{D}$ and $q_{vir}^{D}=q_{v'ir}^{D}$. Thus, the following relation holds.

$$\sum_{v'\neq v}\frac{p_{v'ir}^{D}q_{v'ir}^{D}}{p_{vir}^{D}q_{vir}^{D}}\phi_{vir}^{D}=(n_{ir}-1)\phi_{ir}^{D}$$

In addition, we define

$$\varepsilon_{ir}^{\mathrm{AD}}\equiv-\frac{\partial\mathrm{AD}_{ir}}{\partial p_{ir}^{\mathrm{AD}}}\frac{p_{ir}^{\mathrm{AD}}}{\mathrm{AD}_{ir}}$$

From above results, markup rate μ_{ir}^{D} is given by

$$\mu_{ir}^{D}=\frac{1}{\varepsilon_{ir}^{D}}=-\frac{\partial\ln p_{vir}^{D}}{\partial\ln q_{vir}^{D}}=\frac{1}{\sigma_i^D}+\left[\frac{1}{\varepsilon_{ir}^{\mathrm{AD}}}-\frac{1}{\sigma_i^D}\right]\frac{1+(n_{ir}-1)\phi_{ir}^{D}}{n_{ir}} \tag{22}$$

This indicates markup rate of each firm of IRTS sector i in region r. Since all firms in an industry are assumed to be symmetric, index v is omitted.

Following the similar procedure, $1/\varepsilon_{ir}^{\mathrm{AD}}$ is derived as follows:

$$\frac{1}{\varepsilon_{ir}^{\mathrm{AD}}} = \frac{1}{\sigma_i^A} + \left[\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A}\right]\left[S_{ir}^{\mathrm{AD}} + (1 - S_{ir}^{\mathrm{AD}})\phi_{ir}^{\mathrm{DM}}\right] \tag{23}$$

where S_{ir}^{AD} is share of domestic supply, ϕ_{ir}^{DM} is conjectural elasticity, and ε_{ir}^A is price elasticity of Armington demand defined as follows:

$$S_{ir}^{\mathrm{AD}} \equiv \frac{p_{ir}^{\mathrm{AD}}\mathrm{AD}_{ir}}{p_{ir}^A A_{ir}} \qquad \phi_{ir}^{\mathrm{DM}} \equiv \frac{\partial \ln AM_{ir}}{\partial \ln AD_{ir}} \qquad \varepsilon_{ir}^A \equiv -\frac{\partial \ln A_{ir}}{\partial \ln p_{ir}^A}$$

Combining (22) and (23), markup rate can be written as follows:

$$\mu_{ir}^D = \frac{1}{\sigma_i^D} + \left\{\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^D} + \left[\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A}\right]\left[S_{ir}^{\mathrm{AD}} + (1 - S_{ir}^{\mathrm{AD}})\phi_{ir}^{\mathrm{DM}}\right]\right\}\frac{1 + (n_{ir} - 1)\phi_{ir}^D}{n_{ir}} \tag{24}$$

Since model CD assumes Cournot conjecture, conjectural elasticity parameters ϕ_{ir}^D and ϕ_{ir}^{DM} are zero. Thus, (24) reduces to

$$\mu_{ir}^D = \frac{1}{\sigma_i^D} + \left\{\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^D} + \left[\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A}\right] S_{ir}^{\mathrm{AD}}\right\}\frac{1}{n_{ir}}$$

This is the markup rate for domestic supply.

Markup rate for export markets

Next, let us consider markup rate for export market μ_{visr}^X. The procedure is the same as in the derivation of μ_{vir}^D. First, from (18), inverse demand for import from region s to region r is given by

$$\tilde{p}_{visr}^X = \left[\frac{M_{isr}}{q_{visr}^X}\right]^{1/\sigma_i^F} \beta_{visr}^M p_{isr}^M \tag{25}$$

Taking logarithm of both sides, we have

$$\ln \tilde{p}_{visr}^X = \frac{1}{\sigma_i^F}\ln M_{isr} - \frac{1}{\sigma_i^F}\ln q_{visr}^X + \ln p_{isr}^M + \ln \beta_{visr}^M$$

This leads to

$$\frac{\partial \ln \tilde{p}_{visr}^X}{\partial \ln q_{visr}^X} = -\frac{1}{\sigma_i^F} + \frac{1}{\sigma_i^F}\frac{q_{visr}^X}{M_{isr}}\frac{\partial M_{isr}}{\partial q_{visr}^X} + \frac{q_{visr}^X}{p_{isr}^M}\frac{\partial p_{isr}^M}{\partial M_{isr}}\frac{\partial M_{isr}}{\partial q_{visr}^X} \tag{26}$$

From (9), $\partial M_{isr}/\partial q_{visr}^X$ is written as

$$\frac{\partial M_{isr}}{\partial q_{visr}^X} = (M_{isr})^{1/\sigma_i^F}\left[\beta_{visr}^M (q_{vir}^D)^{-1/\sigma_i^F} + \sum_{v' \neq v}\beta_{v'isr}^M (q_{v'isr}^X)^{-1/\sigma_i^F}\frac{q_{v'isr}^X}{q_{visr}^X}\phi_{visr}^X\right] \tag{27}$$

where ϕ_{visr}^X is conjectural elasticity defined as follows:

$$\phi_{visr}^X \equiv \frac{\partial \ln q_{v'isr}^X}{\partial \ln q_{visr}^X} \qquad v' \neq v \tag{28}$$

Applying (25) to this, we have

$$\frac{\partial M_{isr}}{\partial q^X_{visr}} = \frac{\tilde{p}^X_{visr}}{p^M_{isr}} \left[1 + \sum_{v' \neq v} \frac{\tilde{p}^X_{v'isr} q^X_{v'isr}}{\tilde{p}^X_{visr} q^X_{visr}} \phi^X_{visr} \right]$$

From this, (26) is written as

$$\frac{\partial \ln \tilde{p}^X_{visr}}{\partial \ln q^X_{visr}} = -\frac{1}{\sigma^F_i} + \left[\frac{1}{\sigma^F_i} \frac{q^X_{visr}}{M_{isr}} \frac{\tilde{p}^X_{visr}}{p^M_{isr}} + \frac{q^X_{visr}}{M_{isr}} \frac{\tilde{p}^X_{visr}}{p^M_{isr}} \frac{\partial p^M_{isr}}{\partial M_{isr}} \frac{M_{isr}}{p^M_{isr}} \right] \left[1 + \sum_{v' \neq v} \frac{\tilde{p}^X_{v'isr} q^X_{v'isr}}{\tilde{p}^X_{visr} q^X_{visr}} \phi^X_{visr} \right]$$

By the symmetry assumption, $q^X_{visr} / M_{isr} = 1/n_{is}$ holds. Similarly, symmetry means

$$\sum_{v' \neq v} \frac{\tilde{p}^X_{v'isr} q^X_{v'isr}}{\tilde{p}^X_{visr} q^X_{visr}} \phi^X_{visr} = (n_{is} - 1) \phi^X_{isr}$$

In addition, we define elasticity as follows:

$$\varepsilon^M_{isr} \equiv -\frac{\partial M_{isr}}{\partial p^M_{isr}} \frac{p^M_{isr}}{M_{isr}} \tag{29}$$

Then, markup rates are derived as follows:

$$\tilde{\mu}^X_{isr} = \frac{1}{\tilde{\varepsilon}^X_{isr}} = -\frac{\partial \ln \tilde{p}^X_{isr}}{\partial \ln q^X_{isr}} = \frac{1}{\sigma^F_i} + \left[\frac{1}{\varepsilon^M_{isr}} - \frac{1}{\sigma^F_i} \right] \frac{1 + (n_{is} - 1)\phi^X_{isr}}{n_{is}} \tag{30}$$

Index v is omitted here because of symmetry assumption.

Following the similar procedure, elasticities are derived as follows:

$$\frac{1}{\varepsilon^M_{isr}} = \frac{1}{\sigma^M_i} + \left[\frac{1}{\varepsilon^{\mathrm{AM}}_{ir}} - \frac{1}{\sigma^M_i} \right] [S^M_{isr} + (1 - S^M_{isr}) \phi^{\mathrm{XM}}_{isr}] \tag{31}$$

$$\frac{1}{\varepsilon^{\mathrm{AM}}_{ir}} = \frac{1}{\sigma^A_i} + \left[\frac{1}{\varepsilon^A_{ir}} - \frac{1}{\sigma^A_i} \right] [S^{\mathrm{AM}}_{ir} + (1 - S^{\mathrm{AM}}_{ir}) \phi^{\mathrm{XMD}}_{isr}] \tag{32}$$

where

$$S^M_{isr} \equiv \frac{\tilde{p}^M_{isr} M_{isr}}{p^{\mathrm{AM}}_{ir} \mathrm{AM}_{ir}} \qquad S^{\mathrm{AM}}_{ir} \equiv \frac{p^{\mathrm{AM}}_{ir} \mathrm{AM}_{ir}}{p^A_{ir} A_{ir}} = 1 - S^{\mathrm{AD}}_{ir}$$

$$\phi^{\mathrm{XM}}_{isr} \equiv \frac{\partial \ln M_{is'r}}{\partial \ln M_{isr}} \qquad \phi^{\mathrm{XMD}}_{isr} \equiv \frac{\partial \ln \mathrm{AD}_{ir}}{\partial \ln \mathrm{AM}_{ir}}$$

From (30)–(32), markup rate for export of region s to region r is represented by

$$\tilde{\mu}^X_{isr} = \frac{1}{\sigma^F_i} + \left\{ \frac{1}{\sigma^M_i} - \frac{1}{\sigma^F_i} + \left[\frac{1}{\sigma^A_i} - \frac{1}{\sigma^M_i} + \left(\frac{1}{\varepsilon^A_{ir}} - \frac{1}{\sigma^A_i} \right) \right.\right.$$
$$\left.\left. \times [S^{\mathrm{AM}}_{ir} + (1 - S^{\mathrm{AM}}_{ir}) \phi^{\mathrm{XMD}}_{isr}] \right] [S^M_{isr} + (1 - S^M_{isr}) \phi^{\mathrm{XM}}_{isr}] \right\} \frac{1 + (n_{is} - 1)\phi^X_{isr}}{n_{is}} \tag{33}$$

Since model CD assumes Cournot conjecture, conjectural elasticity parameters ϕ^X_{isr}, $\phi^{\mathrm{XMD}}_{isr}$, and ϕ^{XM}_{isr} are zero. Thus, (33) reduces to

$$\tilde{\mu}^X_{isr} = \frac{1}{\sigma^F_i} + \left\{ \frac{1}{\sigma^M_i} - \frac{1}{\sigma^F_i} + \left[\frac{1}{\sigma^A_i} - \frac{1}{\sigma^M_i} + \left(\frac{1}{\varepsilon^A_{ir}} - \frac{1}{\sigma^A_i} \right) S^{\mathrm{AM}}_{ir} \right] S^M_{isr} \right\} \frac{1}{n_{is}} \tag{34}$$

In our model, it is necessary to make further adjustment to (34) because $\tilde{\mu}^X_{isr}$ deviates from markup rate for firms μ^X_{isr}. The difference between two markup rates is due to transport cost.

$\tilde{\varepsilon}^X_{isr}$ is defined as follows:

$$\tilde{\varepsilon}^X_{isr} = -\frac{\partial \ln q^X_{isr}}{\partial \ln \tilde{p}^X_{isr}}$$

Since $\tilde{p}^X_{isr} = (1 - t^X_{isr})p^X_{isr} + p^T \tau_{isr}$, we have

$$\tilde{\varepsilon}^X_{isr} = -\frac{\partial q^X_{isr}}{\partial p^X_{isr}} \frac{\partial p^X_{isr}}{\partial \tilde{p}^X_{isr}} \frac{\tilde{p}^X_{isr}}{q^X_{isr}} = -\frac{\partial q^X_{isr}}{\partial p^X_{isr}} \frac{p^X_{isr}}{q^X_{isr}} \frac{\tilde{p}^X_{isr}}{(1 - t^X_{isr})p^X_{isr}}$$

From this, markup rate for each firm μ^X_{isr} is given by

$$\mu^X_{isr} = \tilde{\mu}^X_{isr} \frac{\tilde{p}^X_{isr}}{(1 - t^X_{isr})p^X_{isr}}$$

2.2.5 Profit maximization

In this section, we summarize results derived above. Below, we assume that all firms in an IRTS sector are symmetric. This means that all firms in an the same sector set the same prices, outputs, and markup rates.

The FOCs of profit maximization of a firm of IRTS sector s in region r are given by (3)–(4). Since input structure of IRTS model is the same as that of CRTS model, marginal cost in IRTS model (i.e. MC_{vir}) is equal to unit cost in CRTS model (c^Y_{ir}). In addition, by the symmetry assumption, index v can be omitted. So, FOCs for profit maximization are written as

$$(1 - t^Y_{ir})p^D_{ir} \left[1 - \mu^D_{ir}\right] = c^Y_{ir} \tag{35}$$

$$(1 - t^Y_{ir})p^X_{irs} \left[1 - \mu^X_{irs}\right] = c^Y_{ir} \tag{36}$$

(35) is the FOC for domestic supply q^D_{ir} and (36) is the FOC for export supply q^X_{irs}. By these conditions, each firm determines q^D_{ir} and q^X_{irs}.

Markup rates are given by

$$\mu^D_{ir} = \frac{1}{\sigma^D_i} + \left\{ \frac{1}{\sigma^A_i} - \frac{1}{\sigma^D_i} + \left[\frac{1}{\varepsilon^A_{ir}} - \frac{1}{\sigma^A_i} \right] S^{\text{AD}}_{ir} \right\} \frac{1}{n_{ir}} \tag{37}$$

$$\tilde{\mu}^X_{irs} = \frac{1}{\sigma^F_i} + \left\{ \frac{1}{\sigma^M_i} - \frac{1}{\sigma^F_i} + \left[\frac{1}{\sigma^A_i} - \frac{1}{\sigma^M_i} + \left(\frac{1}{\varepsilon^A_{is}} - \frac{1}{\sigma^A_i} \right) S^{\text{AM}}_{is} \right] S^M_{irs} \right\} \frac{1}{n_{ir}} \tag{38}$$

$$\mu^X_{irs} = \tilde{\mu}^X_{irs} \frac{\tilde{p}^X_{irs}}{(1 - t^X_{irs})p^X_{irs}} \tag{39}$$

$$\tilde{p}^X_{irs} = (1 - t^X_{irs})p^X_{irs} + p^T \tau_{irs} \tag{40}$$

Share variables are defined as follows:

$$S^{\text{AD}}_{ir} = \frac{p^{\text{AD}}_{ir} \text{AD}_{ir}}{p^A_{ir} A_{ir}} \qquad S^{\text{AM}}_{is} = \frac{p^{\text{AM}}_{is} \text{AM}_{is}}{p^A_{is} A_{is}} = 1 - S^{\text{AD}}_{is}$$

$$S^M_{irs} = \frac{\tilde{p}^M_{irs} M_{irs}}{p^{\text{AM}}_{is} \text{AM}_{is}} \qquad \sum_r S^M_{irs} = 1$$

2.2.6 Zero profit conditions

Model CD assumes free entry–exit. So, zero profit condition is satisfied in the equilibrium.

$$\pi_{ir} = (1 - t_{ir}^{Y}) \left[p_{ir}^{D} q_{ir}^{D} + \sum_{s} p_{irs}^{X} q_{irs}^{X} \right] - c_{ir}^{Y} \left[q_{ir}^{D} + \sum_{s} q_{irs}^{X} - \mathrm{fc}_{ir} \right] = 0 \tag{41}$$

The number of firms in IRTS sector i is determined so that this zero profit condition is satisfied.

2.2.7 Average cost

Here, we see average cost of each firm. Let q_{ir}^{T} denote total output of each firm in IRTS sector i. That is, $q_{ir}^{T} = q_{ir}^{D} + \sum_{s} q_{irs}^{X}$. Then, total cost is represented as $c_{ir}^{Y}(q_{ir}^{T} + \mathrm{fc}_{ir})$. $c_{ir}^{Y} q_{ir}^{T}$ indicates variable cost and $c_{ir}^{Y} \mathrm{fc}_{ir}$ indicates fixed cost. Since average cost is defined as total cost divided by total output, it is represented as follows:

$$\mathrm{AC}_{ir} = \frac{c_{ir}^{Y}(q_{ir}^{T} + \mathrm{fc}_{ir})}{q_{ir}^{T}} = c_{ir}^{Y} \left[1 + \frac{\mathrm{fc}_{ir}}{q_{ir}^{T}} \right]$$

From this, we can confirm that average cost of each firm declines as total output increases. In view of this, increase in scale of each firm generates positive impacts on the economy.

2.2.8 Price index

By (12) and (13), we define price indices for aggregated varieties. Here, we derive price indices in a symmetric model. First, by symmetry, β_{vir}^{D} and p_{vir}^{D} are equal for any v. Thus, summation with respect to v means multiplication by n_{ir}. It follows that (12) is written as

$$p_{ir}^{\mathrm{AD}} = \left[n_{ir} (\beta_{ir}^{D})^{\sigma_i^D} (p_{ir}^{D})^{1-\sigma_i^D} \right]^{\frac{1}{1-\sigma_i^D}} = (n_{ir})^{\frac{1}{1-\sigma_i^D}} (\beta_{ir}^{D})^{\frac{\sigma_i^D}{1-\sigma_i^D}} p_{ir}^{D}$$

From this, we can see effects of n_{ir} on price index. Assume that $\sigma_i^D > 1$ (this is indeed assumed in the simulation). Then, we have $\partial p_{ir}^{\mathrm{AD}} / \partial n_{ir} < 0$. That is, the increase in n_{ir} decreases p_{ir}^{AD}.

Similarly, p_{isr}^{M} is written as

$$p_{isr}^{M} = \left[n_{is} (\beta_{isr}^{M})^{\sigma_i^F} (\tilde{p}_{isr}^{X})^{1-\sigma_i^F} \right]^{\frac{1}{1-\sigma_i^F}} = (n_{is})^{\frac{1}{1-\sigma_i^F}} (\beta_{isr}^{M})^{\frac{\sigma_i^F}{1-\sigma_i^F}} \tilde{p}_{isr}^{X}$$

Also in this case, increase in varieties lowers the price index.

2.2.9 Demand function

In this section, we consider demand functions of IRTS sectors. These demand functions are derived from (14)–(18). First, unit domestic demand for outputs of a firm in region r is given by

$$a_{ir}^{\mathrm{DD}} = \frac{\partial p_{ir}^{\mathrm{AD}}}{\partial p_{vir}^{D}} = \left[\frac{\beta_{vir}^{D} p_{ir}^{\mathrm{AD}}}{p_{vir}^{D}} \right]^{\sigma_i^D}$$

Similarly, unit demand of region r for outputs of a firm in region s is

$$a_{isr}^{\mathrm{MM}} = \frac{\partial \tilde{p}_{isr}^{M}}{\partial \tilde{p}_{isr}^{X}} = \left[\frac{\beta_{isr}^{M} p_{isr}^{M}}{\tilde{p}_{isr}^{X}} \right]^{\sigma^F}$$

2.2.10 Supply of IRTS sector to international transport sector

There are IRTS sectors that supply their output to international transport sector. We assume that products of IRTS sectors supplied to international transport is produced under CRTS and perfect competition even if those sectors are classified as IRTS sectors. Supply to international transport sector by IRTS sectors is denoted by Y_{ir}^{XT}.

2.3 Market clearing conditions

Below, we present market clearing conditions. These conditions are different across CRTS sectors and IRTS sectors. Market clearing conditions for the perfectly competitive model is represented by the case where $C = I$ (i.e. $K = \emptyset$). In equations below, the LHS represents supply and the RHS represents demand.

2.3.1 Output of CRTS sectors ($i \in C$)

First, let us consider output of CRTS sectors. Supply is given by Y_{ir}. Demand is the sum of domestic demand ($a_{ir}^{\mathrm{AD}} A_{ir}$), import demand of region s ($a_{irs}^{M} \mathrm{AM}_{irs}$), and demand from international transport sector ($a_{ir}^{T} Y^{T}$).

$$Y_{ir} \geq a_{ir}^{\mathrm{AD}} A_{ir} + \sum_{s} a_{irs}^{M} \mathrm{AM}_{irs} + a_{ir}^{T} Y^{T} \qquad i \in C, i \neq \mathrm{cgd}$$

With respect to investment goods (i.e. $i = \mathrm{cgd}$), demand consists of only investment demand INV_r. Thus, market clearing condition is given by

$$Y_{ir} \geq \mathrm{INV}_r \qquad i = \mathrm{cgd}$$

Note that investment demand INV_r is exogenously given constant.

2.3.2 Markets for goods of IRTS sectors ($i \in K$)

As to IRTS goods, we must consider market clearing conditions for an individual firm. First, domestic supply of a IRTS firm in region r is q_{ir}^{D}. On the other hand, domestic demand for a IRTS firm in region r is $a_{ir}^{\mathrm{DD}} \mathrm{AD}_{ir}$.

$$q_{ir}^{D} \geq a_{ir}^{\mathrm{DD}} \mathrm{AD}_{ir}$$

Similarly, export supply of IRTS sector i in region r to region s is q_{irs}^{X} and demand for it is $a_{irs}^{\mathrm{MM}} M_{irs}$. Thus, we have

$$q_{irs}^{X} \geq a_{irs}^{\mathrm{MM}} M_{irs}$$

2.3.3 Markets for Armington goods

Supply of Armington goods is given by A_{ir} and demand is sum of intermediate demand and final demand. Final demand is represented by $a_{ir}^{C} U_r$. Intermediate demand of CRTS sector j is $\bar{a}_{ijr}^{I} Y_{jr}$. On the other hand, intermediate demand of a single firm in IRTS sector j is $\bar{a}_{ijr}^{I} q_{jr}^{T} + \bar{a}_{ijr}^{I} \mathrm{fc}_{jr}$. Thus, market clearing condition for Armington goods i is

$$A_{ir} \geq \sum_{j \in C} \bar{a}_{ijr}^{I} Y_{jr} + \sum_{j \in K} n_{jr} \bar{a}_{ijr}^{I} q_{jr}^{\mathrm{TT}} + a_{ir}^{C} U_r \tag{42}$$

where q_{jr}^{TT} is defined as follows:

$$q_{jr}^{\mathrm{TT}} \equiv q_{jr}^{T} + \mathrm{fc}_{jr} \qquad j \in K$$

2.3.4 Market clearing condition for international transport service

Supply of transport service is Y^T and demand is the sum of $\tau_{irs} a^M_{irs} \mathrm{AM}_{irs}$:

$$Y^T \geq \sum_{i,r,s} \tau_{irs} a^M_{irs} \mathrm{AM}_{irs}$$

2.3.5 Markets of primary factors

Supply of primary factors is ($\bar{F}_{fr}$) which is assumed to be constant. On the other hand, demand of primary factors is demand from production.

$$\bar{F}_{fr} \geq \sum_{i \in C} a^F_{fir} Y_{ir} + \sum_{i \in K} n_{ir} a^F_{fir} q^{\mathrm{TT}}_{ir}$$

2.4 Income of the household

In this section, we derive household income that is spent on consumption (H_r). Income is derived from factor income, tax revenue, and net capital inflow, plus profit. Taxes include production tax, intermediate tax, factor tax, consumption tax, export tax, and tariff. Net capital inflow is represented by $p^U_Z \mathrm{BOP}_r$.[9] Finally, we subtract investment expenditure from the above income.[10]

$$\begin{aligned}
H_r = & \sum_f p^F_{fr} \bar{F}_{fr} + \sum_{i \in C} t^Y_{ir} p^Y_{ir} Y_{ir} + \sum_{i \in K} t^Y_{ir} n_{ir} \left[p^D_{ir} q^D_{ir} + \sum_s p^X_{irs} q^X_{irs} \right] \\
& + \sum_{j,i \in C} t^I_{jir} p^A_{jr} \bar{a}^I_{jir} Y_{ir} + \sum_{j,i \in K} t^I_{jir} p^A_{jr} \bar{a}^I_{jir} n_{ir} q^{\mathrm{TT}}_{ir} + \sum_{f,i \in C} t^F_{fir} p^F_{fr} a^F_{fir} Y_{ir} + \sum_{f,i \in K} t^F_{fir} p^F_{fr} a^F_{fir} n_{ir} q^{\mathrm{TT}}_{ir} \\
& - \sum_{i \in C,s} t^X_{irs} p^Y_{ir} a^M_{irs} \mathrm{AM}_{is} - \sum_{i \in K,s} t^X_{irs} p^X_{irs} n_{ir} \tilde{q}^X_{irs} + \sum_{i \in C,s} t^M_{isr} \tilde{p}^X_{isr} a^M_{isr} \mathrm{AM}_{ir} + \sum_{i \in K,s} t^M_{isr} \tilde{p}^X_{isr} n_{is} \tilde{q}^X_{isr} \\
& + \sum_i t^C_{ir} p^A_{ir} a^C_{ir} U_r + \sum_{i \in K} \pi_{ir} + \; p^U_z \mathrm{BOP}_r - p^{\mathrm{INV}}_r \mathrm{INV}_r
\end{aligned}$$

2.5 Other imperfectly competitive models

So far, we have used model CD to represent the imperfectly competitive model. Below, we explain other variants of imperfectly competitive models. Note that cost structure is the same as model CD.

2.5.1 Model LGMC

Model LGMC presents the large group monopolistic competitive model frequently used in theoretical analyses (for example Krugman, 1980). In this model, each firm recognizes that the number of firms in the industry is sufficiently large. By this assumption, markup rates are modified as follows:

$$\mu^D_{ir} = 1/\sigma^D_i \qquad \tilde{\mu}^X_{irs} = 1/\sigma^F_i \tag{43}$$

These markup rates are derived by setting $n_{ir} \to \infty$ in (37)–(38). [11]

As the above equations show, markup rates in model LGMC are equal to the reciprocals of elasticities of substitution and are constant. Other components in the model are the same as model CD.

[9] We use the price of utility in region z for the price index of international capital flow.

[10] Saving in region r is given by $p^{\mathrm{INV}}_r \mathrm{INV}_r - p^U_Z \mathrm{BOP}_r$.

[11] Although we assume $n_r \to \infty$ in deriving markup rates, it is merely a firm's conjecture and it does not mean that the actual number of firms is infinite. The actual number of firms is endogenously determined so that the zero profit condition is satisfied.

2.5.2 Model CH

Model CD assumes that all varieties in an industry are differentiated. On the other hand, model CH assumes that all varieties in an industry are homogeneous. The assumption of homogeneous varieties means that elasticities of substitution among varieties are infinite. Thus, we can get markup rates for model CH by setting $\sigma_i^D \to \infty$ and $\sigma_i^F \to \infty$ in (37) and (38).

$$\mu_{ir}^D = \left[\frac{1}{\sigma_i^A} + \left(\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A} \right) S_{ir}^{\mathrm{AD}} \right] \frac{1}{n_{ir}}$$

$$\tilde{\mu}_{irs}^X = \left\{ \frac{1}{\sigma_i^M} + \left[\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^M} + \left(\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A} \right) S_{is}^{\mathrm{AM}} \right] S_{irs}^M \right\} \frac{1}{n_{ir}}$$

2.5.3 Model CF

In model CD, free entry–exit is assumed. On the other hand, model CF assumes that the number of firms (varieties) in an industry is fixed. This change in the assumption means that zero profit condition is not satisfied. We assume that profit is transferred to the household in lump-sum fashion.

2.5.4 Model QCV

Model CD assumes that each firm competes under Cournot conjecture. That is, each firm determines his outputs, viewing the outputs of all other firms as fixed. Model QCV also assumes quantity competition as in model CD, but assumes non-zero conjectural variation. Markup rates with conjectural variation parameters are given by (24) and (34). In addition, we assume that conjectural elasticity parameters of a firm against all rival firms in a market are equalized. Let ϕ_{ir}^D denote conjectural elasticity of rival's supply with respect to a change in own supply, and ϕ_{irs}^X denote conjectural elasticity of rival's supply to region s. Then, the above assumption implies the following relations.

$$\phi_{ir}^D \equiv \frac{\partial \ln q_{v'ir}^D}{\partial \ln q_{vir}^D} = \frac{\partial \ln q_{lisr}^X}{\partial \ln q_{vir}^D} \qquad v' \neq v$$

$$\phi_{irs}^X \equiv \frac{\partial \ln q_{v'irs}^X}{\partial \ln q_{virs}^X} = \frac{\partial \ln q_{lir's}^X}{\partial \ln q_{virs}^X} = \frac{\partial \ln q_{lis}^D}{\partial \ln q_{virs}^X} \qquad v' \neq v,\ r' \neq r$$

In the following, using ϕ_{ir}^D and ϕ_{irs}^X, we rewrite ϕ_{ir}^{DM}, ϕ_{irs}^{XM}, and $\phi_{irs}^{\mathrm{XMD}}$ in (24) and (34). First, let us consider ϕ_{ir}^{DM}. It is defined as

$$\phi_{ir}^{\mathrm{DM}} \equiv \frac{\partial \ln \mathrm{AM}_{ir}}{\partial \ln \mathrm{AD}_{ir}}$$

From (8) and (19), we have

$$d \ln \mathrm{AD}_{ir} = \sum_{v' \neq v} S_{v'ir}^D d \ln q_{v'ir}^D + S_{vir}^D d \ln q_{vir}^D$$

where

$$S_{v'ir}^D \equiv p_{v'ir}^D q_{v'ir}^D / (p_{ir}^{\mathrm{AD}} \mathrm{AD}_{ir})$$

By the definition of ϕ_{vir}^D, this reduces to

$$d \ln \mathrm{AD}_{ir} = \left[\sum_{v' \neq v} S_{v'ir}^D \phi_{vir}^D + S_{vir}^D \right] d \ln q_{vir}^D$$

In addition, from the symmetry assumption, $S^D_{v'ir} = S^D_{vir} = 1/n_{ir}$ holds. So, we have

$$d \ln \mathrm{AD}_{ir} = \frac{1 + (n_{ir} - 1)\phi^D_{ir}}{n_{ir}} d \ln q^D_{ir} \tag{44}$$

Similarly, from (7), (9), (16), and (18), we have

$$d \ln \mathrm{AM}_{ir} = \sum_s S^M_{isr} d \ln M_{isr} \qquad d \ln M_{isr} = \sum_z S^X_{zisr} d \ln q^X_{zisr}$$

where

$$S^X_{zisr} \equiv \tilde{p}^X_{zisr} q^X_{zisr} / (p^M_{isr} M_{isr})$$

From the definition of ϕ^D_{ir}, we have $d \ln q^X_{zisr} = \phi^D_{ir} d \ln q^D_{ir}$. Thus, $d \ln M_{isr}$ is represented as

$$d \ln M_{isr} = \sum_z S^X_{zisr} \phi^D_{ir} d \ln q^D_{ir} = \phi^D_{ir} d \ln q^D_{ir}$$

Using this relation, we can express $d \ln \mathrm{AM}_{ir}$ as follows:

$$d \ln \mathrm{AM}_{ir} = \sum_s S^M_{isr} \phi^D_{ir} d \ln q^D_{ir} = \phi^D_{ir} d \ln q^D_{ir} \tag{45}$$

From (44) and (45), we have

$$\phi^{\mathrm{DM}}_{ir} = \frac{\phi^D_{ir}}{[1 + (n_{ir} - 1)\phi^D_{ir}]/n_{ir}} \tag{46}$$

Using Eq. (46), we can rewrite (24) as follows:

$$\mu^D_{ir} = \frac{1}{\sigma^D_i} + \left[\frac{1}{\sigma^A_i} - \frac{1}{\sigma^D_i}\right] \frac{1 + (n_{ir} - 1)\phi^D_{ir}}{n_{ir}} + \left[\frac{1}{\varepsilon^A_{ir}} - \frac{1}{\sigma^A_{ir}}\right] \frac{S^{\mathrm{AD}}_{ir} + (n_{ir} - S^{\mathrm{AD}}_{ir})\phi^D_{ir}}{n_{ir}} \tag{47}$$

This is the markup rate for domestic supply in model QCV.

Following the same procedure, we next rewrite (34). ϕ^{XM}_{isr} and $\phi^{\mathrm{XMD}}_{isr}$ in (34) are defined as follows:

$$\phi^{\mathrm{XM}}_{isr} \equiv \frac{\partial \ln M_{is'r}}{\partial \ln M_{isr}} \qquad \phi^{\mathrm{XMD}}_{isr} \equiv \frac{\partial \ln \mathrm{AD}_{ir}}{\partial \ln \mathrm{AM}_{ir}}$$

From (9) and (18), we have

$$d \ln M_{isr} = \sum_{v' \neq v} S^X_{v'isr} d \ln q^X_{v'isr} + S^X_{visr} d \ln q^X_{visr}$$

By the definition of ϕ^X_{visr} and the symmetry assumption, we have

$$d \ln M_{isr} = \frac{1 + (n_{is} - 1)\phi^X_{isr}}{n_{is}} d \ln q^X_{visr} \tag{48}$$

Following the same procedure, $d \ln M_{is'r}$ is expressed as

$$d \ln M_{is'r} = \sum_{v'} S^X_{v'is'r} d \ln q^X_{v'is'r}$$

By the definition of ϕ^X_{visr}, we have $d \ln q^X_{v'is'r} = \phi^X_{isr} d \ln q^X_{visr}$ for $\forall v'$ and s'. Thus,

$$d \ln M_{is'r} = \phi^X_{isr} d \ln q^X_{isr} \tag{49}$$

From (48) and (49), ϕ_{isr}^{XM} reduces to

$$\phi_{isr}^{\text{XM}} = \frac{\phi_{isr}^{X}}{[1 + (n_{is} - 1)\phi_{isr}^{X}]/n_{is}} \tag{50}$$

Next, we consider ϕ_{isr}^{XMD}. First, from (8) and (17), we have

$$d \ln \text{AD}_{ir} = \sum_{v'} S_{v'ir}^{D} d \ln q_{v'ir}^{D} = \sum_{v'} S_{v'ir}^{D} \phi_{visr}^{X} d \ln q_{visr}^{X} = \phi_{visr}^{X} d \ln q_{visr}^{X} \tag{51}$$

Similarly, from (7), (18), (48), and (49), we have

$$\begin{aligned} d \ln \text{AM}_{ir} &= \sum_{s' \neq r} S_{is'r}^{M} d \ln M_{is'r} + S_{isr}^{M} d \ln M_{isr} \\ &= \sum_{s' \neq r} S_{is'r}^{M} \phi_{isr}^{X} d \ln q_{isr}^{X} + S_{isr}^{M} \frac{1 + (n_{is} - 1)\phi_{isr}^{X}}{n_{is}} d \ln q_{isr}^{X} \end{aligned} \tag{52}$$

From (51) and (52), ϕ_{isr}^{XMD} is expressed as follows:

$$\phi_{isr}^{\text{XMD}} = \frac{\phi_{isr}^{X}}{\phi_{isr}^{X} + (1 - \phi_{isr}^{X}) S_{isr}^{M}/n_{is}} \tag{53}$$

Substituting (50) and (53) into (34), markup rate for export supply in model QCV is given by

$$\begin{aligned} \tilde{\mu}_{irs}^{X} = \frac{1}{\sigma_i^F} &+ \left[\frac{1}{\sigma_i^M} - \frac{1}{\sigma_i^F}\right] \frac{1 + (n_{ir} - 1)\phi_{irs}^{X}}{n_{ir}} + \left[\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^M}\right] \frac{S_{irs}^{M} + (n_{ir} - S_{irs}^{M})\phi_{isr}^{X}}{n_{ir}} \\ &+ \left[\frac{1}{\varepsilon_{is}^A} - \frac{1}{\sigma_i^A}\right] \frac{S_{irs}^{M} S_{is}^{\text{AM}} + (n_{ir} - S_{irs}^{M} S_{is}^{\text{AM}})\phi_{isr}^{X}}{n_{ir}} \end{aligned}$$

2.5.5 Model BD

Model BD is a Bertrand competition version of model CD, that is, it assumes that firm's strategic variable is price and that each firm determines his prices, viewing the prices of all other firms as fixed.

First, we derive markup rate for domestic supply. From (17), we have

$$\frac{\partial \ln q_{vir}^{D}}{\partial \ln p_{vir}^{D}} = -\sigma_i^D + \sigma_i^D \frac{p_{vir}^{D}}{p^{\text{AD}_{ir}}} \frac{\partial p_{ir}^{\text{AD}}}{\partial p_{vir}^{D}} + \frac{p_{vir}^{D}}{\text{AD}_{ir}} \frac{\partial \text{AD}_{ir}}{\partial p_{ir}^{\text{AD}}} \frac{\partial p_{ir}^{\text{AD}}}{\partial p_{vir}^{D}} \tag{54}$$

From (12), we have

$$\frac{\partial p_{ir}^{\text{AD}}}{\partial p_{vir}^{D}} = (p_{ir}^{\text{AD}})^{\sigma_{ir}^{D}} \left[(\beta_{vir}^{D})^{\sigma_{ir}^{D}} (p_{vir}^{D})^{-\sigma_{ir}^{D}} + \sum_{v' \neq v} (\beta_{v'ir}^{D})^{\sigma_{ir}^{D}} (p_{v'ir}^{D})^{-\sigma_{ir}^{D}} \frac{p_{v'ir}^{D}}{p_{vir}^{D}} \frac{\partial \ln p_{v'ir}^{D}}{\partial \ln p_{vir}^{D}} \right] \tag{55}$$

Since model BD assumes Bertrand conjecture, we have $\partial \ln p_{v'ir}^{D} / \partial \ln p_{vir}^{D} = 0$. Thus, (55) reduces to

$$\frac{\partial p_{ir}^{\text{AD}}}{\partial p_{vir}^{D}} = \left[\frac{p_{ir}^{\text{AD}} \beta_{vir}^{D}}{p_{vir}^{D}}\right]^{\sigma_{ir}^{D}}$$

From (12), this equation is rewritten as

$$\frac{\partial p_{ir}^{\text{AD}}}{\partial p_{vir}^{D}} = \frac{q_{vir}^{D}}{\text{AD}_{ir}}$$

Thus, (54) is

$$\frac{\partial \ln q^D_{vir}}{\partial \ln p^D_{vir}} = -\sigma^D_i - (\varepsilon^{\text{AD}}_{ir} - \sigma^D_i)\frac{1}{n_{ir}} \tag{56}$$

Similarly, $\varepsilon^{\text{AD}}_{ir}$ is

$$\varepsilon^{\text{AD}}_{ir} = -\frac{\partial \ln \text{AD}_{ir}}{\partial \ln p^{\text{AD}}_{ir}} = \sigma^A_i + (\varepsilon^A_{ir} - \sigma^A_i)S^{\text{AD}}_{ir} \tag{57}$$

Combining (56) and (57), markup rate for domestic supply is given by

$$1/\mu^D_{ir} = \varepsilon^D_{ir} = \sigma^D_i + [\sigma^A_i - \sigma^D_i + (\varepsilon^A_{ir} - \sigma^A_i)S^{\text{AD}}_{ir}]\frac{1}{n_{ir}}$$

Following the same procedure, markup rates for export supply are derived as follows:

$$1/\tilde{\mu}^X_{isr} = \sigma^F_i + \{\sigma^M_i - \sigma^F_i + [\sigma^A_i - \sigma^M_i + (\varepsilon^A_{ir} - \sigma^A_i)S^{\text{AM}}_{ir}]S^M_{isr}\}\frac{1}{n_{is}}$$

2.5.6 Model IC

Model IC is the integrated market version of model CD. In contrast to the case of the segmented market model, each firm in the integrated market model sets a common price for all markets. Moreover, each firm can control only total outputs. Thus, the profit of a firm is represented as follows:

$$\pi_{ir} = \left[(1 - t^Y_{ir})p_{ir} - c^Y_{ir}\right] q^T_{ir}$$

From this, the first order condition for profit maximization of each firm reduces to a single equation.

$$(1 - t^Y_{ir})p_{ir}(1 - \mu_{ir}) = c^Y_{ir} \tag{58}$$

In integrated market Cournot model, it is assumed that each firm determines his outputs, viewing domestic and foreign firms do not change their total outputs. From this assumption, we can derive the expression of markup rates. However, it is quite difficult to derive explicit expression. So, we derive markup rates implicitly in this case.

First, the overall markup rate is expressed as follows:

$$\mu_{ir} = -\hat{p}_{vir}/\hat{q}^T_{vir} \tag{59}$$

where hat variable means the rate of change. This relation means that it is necessary to obtain $\hat{p}_{vir}$ and $\hat{q}^T_{vir}$ to derive markup rate.

First, we consider change in own quantity of firm v in region r. Since $q^T_{vir} \equiv q^D_{vir} + \sum_{s \neq r} q^X_{virs}$, $\hat{q}^T_{vir}$ is expressed as follows:

$$\hat{q}^T_{vir} = \delta^D_{ir}\hat{q}^D_{vir} + \sum_s \delta^X_{irs}\hat{q}^X_{virs} \tag{60}$$

where

$$\delta^D_{ir} \equiv q^D_{ir}/q^T_{ir} \qquad \delta^X_{irs} \equiv q^X_{irs}/q^T_{ir}$$

From (17) and (18), $\hat{q}^D_{vir}$ and $\hat{q}^X_{virs}$ in (60) are given by

$$\hat{q}^D_{vir} = -\sigma^D_i \hat{p}_{vir} + (\sigma^D_i - \sigma^A_i)\hat{p}^{\text{AD},r}_{ir} + (\sigma^A_i - \varepsilon^A_{ir})\hat{p}^{A,r}_{ir} \tag{61}$$

$$\hat{q}^X_{virs} = -\sigma^F_i \hat{p}^X_{virs} + (\sigma^F_i - \sigma^M_i)\hat{p}^{M,r}_{irs} + (\sigma^M_i - \sigma^A_i)\hat{p}^{\text{AM},r}_{is} + (\sigma^A_i - \varepsilon^A_{is})\hat{p}^{A,r}_{is} \tag{62}$$

Note that superscript r indicates that changes in variables are conjectured by firm v in region r.

Next, we consider changes in quantity of rival firms implied by Cournot conjecture. Rate of change in outputs of rival firms is given by

$$\delta^D_{it}\hat{q}^{D,r}_{it} + \sum_s \delta^X_{its}\hat{q}^{X,r}_{its} = 0 \tag{63}$$

$$\hat{q}^{D,r}_{it} = -\sigma^D_i \hat{p}^r_{it} + (\sigma^D_i - \sigma^A_i)\hat{p}^{\text{AD},r}_{it} + (\sigma^A_i - \varepsilon^A_{it})\hat{p}^{A,r}_{it} \tag{64}$$

$$\hat{q}^{X,r}_{its} = -\sigma^F_i \hat{p}^{X,r}_{its} + (\sigma^F_i - \sigma^M_i)\hat{p}^{M,r}_{its} + (\sigma^M_i - \sigma^A_i)\hat{p}^{\text{AM},r}_{is} + (\sigma^A_i - \varepsilon^A_{is})\hat{p}^{A,r}_{is} \tag{65}$$

where $t = r$ means the domestic rival firm and $t \neq r$ means the foreign rival firm. Note that conjectured change in total outputs of rival firms is set to zero due to Cournot conjecture.

(61)–(65) include conjectured rate of change in price. Next, let us derive these price changes. First, from (12), $\hat{p}^{\text{AD},r}_{ir}$ is given by

$$\hat{p}^{\text{AD},r}_{ir} = \sum_l S^D_{lir}\hat{p}^r_{lir} = \frac{\hat{p}_{vir} + (n_{ir} - 1)\hat{p}^r_{ir}}{n_{ir}} \tag{66}$$

Similarly, $\hat{p}^{\text{AD},r}_{is}$ ($s \neq r$) is

$$\hat{p}^{\text{AD},r}_{is} = \sum_l S^D_{lis}\hat{p}^r_{lis} = \hat{p}^r_{is} \tag{67}$$

From (13), $\hat{p}^{M,r}_{its}$ is

$$\hat{p}^{M,r}_{irs} = \sum_l S^X_{lirs}\hat{p}^{X,r}_{lir} = \frac{\hat{p}^X_{virs} + (n_{ir} - 1)\hat{p}^{X,r}_{irs}}{n_{ir}} \tag{68}$$

$$\hat{p}^{M,r}_{its} = \sum_l S^X_{lits}\hat{p}^{X,r}_{lit} = \hat{p}^{X,r}_{its} \qquad t \neq r \tag{69}$$

Following the same procedure, other price variables are derived as follows.

$$\hat{p}^{\text{AM},r}_{is} = \sum_t S^M_{its}\hat{p}^{M,r}_{its} \tag{70}$$

$$\hat{p}^{A,r}_{is} = S^{\text{AD}}_{is}\hat{p}^{\text{AD},r}_{is} + S^{\text{AM}}_{is}\hat{p}^{\text{AM},r}_{is} \tag{71}$$

$$\hat{p}^X_{virs} = \beta_{irs}\hat{p}_{vir} \tag{72}$$

$$\hat{p}^{X,r}_{its} = \beta_{its}\hat{p}^r_{it} \tag{73}$$

Finally, we normalize $\hat{p}_{vir}$ to unity.

$$\hat{p}_{vir} = 1 \tag{74}$$

In the simulation, we incorporate a system of (59)–(74) and then derive the value of μ_{ir} implicitly.

2.5.7 Model IB

As in model IC, each firm sets a common price for all markets and can control only total outputs. Thus, the first order condition for profit maximization of each firm reduces to a single equation.

$$(1 - t^Y_{ir})p_{ir}(1 - \mu_{ir}) = c^Y_{ir}$$

Total demand for a firm is the sum of domestic demand and import demand. So, we have

$$q^T_{ir} = q^D_{ir} + \sum_s q^X_{irs}$$

From this, price elasticity of total demand (ε_{ir}) is

$$\varepsilon_{ir} = -\frac{\partial \ln q^T_{ir}}{\partial \ln p_{ir}} = -\left[\frac{\partial q^D_{ir}}{\partial p_{ir}} + \sum_s \frac{\partial q^X_{irs}}{\partial p_{ir}}\right] \frac{p_{ir}}{q^T_{ir}} = \delta^D_{ir}\varepsilon^D_{ir} + \sum_s \delta^X_{irs}\varepsilon^X_{irs}$$

This means that overall elasticity of demand is equal to the weighted average of elasticity of demand in each market. Since markup rates are reciprocals of demand elasticity, the following relation holds for markup rates.

$$1/\mu_{ir} = \delta^D_{ir}/\mu^D_{ir} + \sum_s \delta^X_{irs}/\mu^X_{irs}$$

Table 5: Values of elasticity of substitution (σ_i^A and σ_i^{PF}).

Sectors & goods	σ_i^A	σ_i^{PF}
AFF	2.42	0.23
MIN	5.75	0.20
FBT	1.99	1.12
TWA	3.02	1.26
WPP	3.10	1.26
CHM	2.92	1.26
MET	3.56	1.26
MVT	3.15	1.26
ELE	3.52	1.26
OME	4.05	1.26
OMF	3.00	1.26
TAT	1.90	1.68
OSE	1.95	1.28

3 Data

3.1 Source of data

As the benchmark data, we use GTAP version 6 whose benchmark year is 2001.[12] The original GTAP 6 data contain 87 regions and 57 sectors. We first aggregate the original data according to region and sector classification in Table 2 and 3.

3.2 Services trade barriers

We assume that 30% of tariff imposed on services trade and modified the GTAP data. By introducing services tariffs, value of imports including tariffs increase. This reduces value of total final demand. To restore final demand balance, we adjust value of final consumption as well. Let V_i^M denote the value of import of service i (at the world price), and t_i^0 denote the original tariff rate on service i, and t_i^1 denote the new tariff rate. Then, the rise in tariff rate increases tariff revenue by $\Delta_i^T = (t_i^1 - t_i^0)V_i^M$. To restore final demand balance, we increase the value of final consumption by $Delta_i^T$ and adjust consumption tax rates. For the details of this adjustment, see GAMS program for the simulation.

4 Parameters and calibration

4.1 Elasticity of substitution

Values of elasticity parameters are determined exogenously. We use GTAP 6 values for elasticity of substitution among primary factors (σ_i^{PF}). As to Armington elasticity (σ_i^A), we basically use GTAP 6 values. However, as to sector FBT, TWA, OMF, and ELE, we use values derived by multiplying the original GTAP values by 0.8 for computational reason.[13] As to elasticity of substitution among imports from different regions (σ_i^M), we assume $\sigma_i^M = 2 \times \sigma_i^A$, following the GTAP model. In addition to two elasticities above, imperfectly competitive models include elasticity of substitution of varieties (σ_i^D and σ_i^F). For these two parameters, we assume $\sigma_i^D = \sigma_i^F = 2 \times \sigma_i^M$, following Harrison et al. (1996). With regards to these elasticity parameters, we conduct sensitivity analysis.

[12]For the details of GTAP data, see the GTAP web site `http://www.gtap.agecon.purdue.edu/`.

[13]We use smaller values because when we use the original values, we encounter computational difficulty in solving the model.

4.2 Calibration

Imperfectly competitive models include parameters and variables which do not appear in the perfectly competitive model such as fixed cost, the number of firms, markup rates, and elasticity of substitution of varieties. In addition to these parameters and variables, model QCV includes conjectural variation parameters. Among these parameters, elasticity parameters are determined exogenously.[14] To conduct the simulation, it is necessary to determine values of other parameters and variables by some approach. Since results of the simulation are likely to be influenced by the approach for determining parameters and variables, it is desirable to choose the proper approach. However, there exists no standard method for it and different researches use different methods under the present situation.[15] Here we choose the approach we think the most appropriate for comparing various imperfect competition models in a unified framework. In the following, taking model CD as an example, we explain the approach for calibration.

4.2.1 Calibration method for model CD, CH, CF, BD, IC, and IB

Substituting (35) and (36) (FOCs for profit maximization) into zero profit condition (41), we get

$$(1 - t_{ir}^{Y}) \left[p_{ir}^{D} q_{ir}^{D} \mu_{ir}^{D} + \sum_{s} p_{irs}^{X} q_{irs}^{X} \mu_{irs}^{X} \right] = c_{ir}^{Y} \mathrm{fc}_{ir} \tag{75}$$

On the other hand, markup rates in model CD are

$$\mu_{ir}^{D} = \frac{1}{\sigma_i^{D}} + \left\{ \frac{1}{\sigma_i^{A}} - \frac{1}{\sigma_i^{D}} + \left[\frac{1}{\varepsilon_{ir}^{A}} - \frac{1}{\sigma_i^{A}} \right] S_{ir}^{\mathrm{AD}} \right\} \frac{1}{n_{ir}} \tag{76}$$

$$\tilde{\mu}_{irs}^{X} = \frac{1}{\sigma_i^{F}} + \left\{ \frac{1}{\sigma_i^{M}} - \frac{1}{\sigma_i^{F}} + \left[\frac{1}{\sigma_i^{A}} - \frac{1}{\sigma_i^{M}} + \left(\frac{1}{\varepsilon_{is}^{A}} - \frac{1}{\sigma_i^{A}} \right) S_{is}^{\mathrm{AM}} \right] S_{irs}^{M} \right\} \frac{1}{n_{ir}} \qquad s = 1, \cdots, S \tag{77}$$

These equations must be satisfied at the benchmark equilibrium. The benchmark values of p_{ir}^{D}, p_{irs}^{X}, t_{ir}^{Y}, c_{ir}^{Y}, S_{ir}^{AD}, S_{ir}^{AM}, and S_{irs}^{M} are determined by the benchmark data and, as explained in the previous section, the values of elasticity parameters are determined exogenously. Thus, (75)–(77) include $3 + S$ unknown parameters and variables (i.e., n_{ir}, fc_{ir}, μ_{ir}^{D}, and $\tilde{\mu}_{irs}^{X}$). Since (75)–(77) include $2 + S$ equations, if we determine the value of one parameter or variable, we can determine $2 + S$ parameters and variables by solving the system of (75)–(77).

Here, we assume that the benchmark number of firms is determined exogenously. Thus, we can calibrate the fixed cost and markup rates (fc_{ir}, μ_{ir}^{D}, and $\tilde{\mu}_{irs}^{X}$) by solving (75)–(77).[16] For the benchmark number of firms, we assume 20 for all IRTS sectors in all regions.

4.2.2 Other imperfectly competitive models

In this section, we explain calibration used for other imperfectly competitive models. First, model CF use the same approach as model CD. Model CH, BD, IC, IB also use the same approach as model CD except that markup rate formula is changed.

Model LGMC

In model LGMC, markup rates are independent of the number of firms and depend only on elasticities of substitution (see (43)) and therefore we cannot apply the approach of model CD to model LGMC. Thus, we use the following approach.

[14] Some studies employ the approach where elasticity parameters are calibrated given other parameters and variables (e.g. Smith and Venables 1988). In this approach, elasticity parameters can take quite different values according to models. This feature is undesirable when we compare different models. Thus, this paper does not employ such an approach.

[15] For example, Smith and Venables (1988), Harrison et al. (1996), Francois and Roland-Holst (1997), Grether and Müller (2000), Bchir et al. (2002), and Santis (2002b) adopt different methods for determining parameters and variables.

[16] Strictly speaking, we also use (39) and (40).

Step 1: First, we calibrate markup rates (43) from elasticities of substitution.

Step 2: Second, we determine the number of firms exogenously.

Step 3: Using markup rates and the number of firms determined above, we calibrate fixed cost so that zero profit condition is satisfied.

The number of firms determined exogenously in Step 2 does not affect results of the simulation (rates of change in variables against shocks).[17] Thus, we normalize the benchmark number of firms as unity.

Model QCV

(75) holds in model QCV. However, markup rates for model QCV are changed to

$$\mu_{ir}^{D} = \frac{1}{\sigma_i^D} + \left[\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^D}\right] \frac{1 + (n_{ir} - 1)\phi_{ir}^D}{n_{ir}} + \left[\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_{ir}^A}\right] \frac{S_{ir}^{\mathrm{AD}} + (n_{ir} - S_{ir}^{\mathrm{AD}})\phi_{ir}^D}{n_{ir}} \tag{78}$$

$$\begin{aligned} \mu_{irs}^{X} = {} & \frac{1}{\sigma_i^F} + \left[\frac{1}{\sigma_i^M} - \frac{1}{\sigma_i^F}\right] \frac{1 + (n_{ir} - 1)\phi_{irs}^X}{n_{ir}} + \left[\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^M}\right] \frac{S_{irs}^{M} + (n_{ir} - S_{irs}^{M})\phi_{isr}^X}{n_{ir}} \\ & + \left[\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A}\right] \frac{S_{irs}^{M} S_{is}^{\mathrm{AM}} + (n_{ir} - S_{irs}^{M} S_{is}^{\mathrm{AM}})\phi_{isr}^X}{n_{ir}} \end{aligned} \tag{79}$$

Since conjectural variation parameters ϕ_{ir}^D and ϕ_{irs}^X are added, we cannot apply the approach of model CD to this case. Thus, we adopt the approach of Harrison et al. (1996).

1. First, fixed cost is calibrated, given exogenous CDR (cost disadvantage ratio).[18]
2. Second, the number of firms n_{ir} is determined exogenously.
3. Third, we calibrate μ_{ir}^D, μ_{irs}^X, ϕ_{ir}^D, ϕ_{irs}^X by solving an optimization problem with constraints (75), (78)–(79).

In Step 1, fixed cost is calibrated from CDR in the following way. The cost function is given by (1). So, CDR is represented as follows:

$$\mathrm{CDR} = \frac{\mathrm{AC} - \mathrm{MC}}{\mathrm{AC}} = \frac{\mathrm{FC}}{\mathrm{TC}} \tag{80}$$

Since the benchmark value of total cost TC is given by the benchmark data, if we determine the value of CDR, we can determine the value of fixed cost from (80). For the benchmark value of CDR we assume 0.15 (15%) for all IRTS sectors in all regions.

In step 2, we assume that the number of firms is 20 for all IRTS sectors in all regions. As the objective function in step 3, we assume the following function.

$$\mathrm{Loss}_i = \sum_{s,r,r'} \xi_{ir'r} X_{isr} (\phi_{isr} - \phi_{ir'r})^2 \tag{81}$$

where

$$\begin{aligned} \phi_{isr} &= \begin{cases} \phi_{ir}^D & s = r \\ \phi_{isr}^X & s \neq r \end{cases} \\ X_{isr} &= \begin{cases} p_{ir}^{\mathrm{AD}} \mathrm{AD}_{ir} & s = r \\ \tilde{p}_{isr}^M M_{isr} & s \neq r \end{cases} \\ \theta_{isr} &= X_{isr} / \sum_{r'} X_{ir'r} \\ \xi_{isr} &= \begin{cases} 1 & \text{if } \theta_{isr} = \arg\max_{s'} \{\theta_{is'r}\} \\ 0 & \text{Otherwise.} \end{cases} \end{aligned}$$

[17] Of course, the absolute value of variables depend on choice of the number of firms. However, in the simulation, we only see rates of change in variables.

[18] CDR is defined as $\mathrm{CDR} \equiv (\mathrm{AC} - \mathrm{MC})/\mathrm{AC}$.

ξ_{irs} is a variable which takes unity if supply from region r has the largest share in total supply to regions s and takes zero otherwise.[19] In the calibration, we determine μ^D_{ir}, μ^X_{irs}, and ϕ_{irs} so that the loss function (81) is minimized.

4.2.3 Alternative calibration method

In the sensitivity analysis of the main paper, we try the alternative calibration method for model CD, CH, CF, BD, IC, and IB. The approach in Section 4.2.1 calibrates fixed cost taking the number of firms as given. On the other hand, the alternative approach calibrates the number of firms , taking fixed cost (CDR) as given.

4.3 Elasticity of demand for Armington goods

In models except for model LGMC, elasticity of demand for Armington goods (ε^A_{ir}) are included in markup formula. Using notations defined so far, ε^A_{ir} is represented as

$$\varepsilon^A_{ir} = \frac{C_{ir}}{A_{ir}} \tag{82}$$

We can prove (82) as follows. First, ε^A_{ir} is defined as

$$\varepsilon^A_{ir} \equiv -\frac{\partial A_{ir}}{\partial p^A_{ir}} \frac{p^A_{ir}}{A_{ir}}$$

Demand for Armington goods are the sum of final demand C^D_{ir} and intermediate demand I^D_{ir}. Thus, ε^A_{ir} becomes

$$\varepsilon^A_{ir} = -\left[\frac{\partial C^D_{ir}}{\partial p^A_{ir}} + \frac{\partial I^D_{ir}}{\partial p^A_{ir}}\right] \frac{p^A_{ir}}{A_{ir}}$$

Since intermediate demand are derived from Leontief production technology, $\partial I^D_{ir}/\partial p^A_{ir} = 0$ holds. Thus, we have

$$\varepsilon^A_{ir} = -\frac{\partial C^D_{ir}}{\partial p^A_{ir}} \frac{p^A_{ir}}{A_{ir}} = -\frac{\partial C^D_{ir}}{\partial p^A_{ir}} \frac{p^A_{ir}}{C_{ir}} \frac{C^A_{ir}}{A_{ir}} \tag{83}$$

On the other hand, final demand are derived from Cobb-Douglas utility function. Thus, uncompensated demand is given by

$$C^D_{ir} = \frac{\theta^C_{ir} H_r}{(1+t^C_{ir}) p^A_{ir}}$$

This leads to

$$\frac{\partial C^D_{ir}}{\partial p^A_{ir}} \frac{p^A_{ir}}{C^D_{ir}} = 1 \tag{84}$$

From (83) and (84), we can confirm that (82) holds. (82) shows that ε^A_{ir} is the variable which takes values from 0 to 1.

It is desirable to incorporate (82) into the simulation. However, there is one problem for it. That is, if we determine ε^A_{ir} by (82), ε^A_{ir} for sectors which has no final demand becomes zero. Since ε^A_{ir} enters into markup formula as the denominator, markup formula cannot be defined if ε^A_{ir} is zero. Moreover, even if ε^A_{ir} is not zero, small values of ε^A_{ir} make the model quite unstable. To avoid this problem, we assume $\varepsilon^A_{ir} = 0.5$ for $\forall i$ in the simulation.

[19] In total supply to region r, own supply usually has the largest share. Thus, we usually have $\xi_{irr} = 1$ and $\xi_{isr} = 0$ $(s \neq r)$. However, $\xi_{irr} = 0$ sometimes holds. For example, since the domestic supply of MIN is quite small in Japan, we have $\xi_{\text{MIN,JPN,JPN}} = 0$.

5 Model for the simulation

I have already explained model structure in Section 2. However, in programs for the simulation, the model is described by different notations. Here, I present the model in accordance with simulation programs.

1. In the program, all functions are basically written in calibrated share form (Rutherford, 1998). Thus, we describe the model by calibrated share form below.
2. A variable given in parentheses on the right end indicates the variable determined by the equation or the slack variable in the case that the equation is represented by the inequality. For example, suppose that the equation is represented as follows:

$$f(x,y,z) \geq 0 \qquad \{z\}$$

   Strictly speaking, this equation means the following conditions:

$$f(x,y,z) \geq 0 \qquad f(x,y,z)z = 0 \qquad z \geq 0$$

   This means that if $z > 0$, we must have $f(x,y,z) = 0$, and if $f(x,y,z) > 0$, we must have $z = 0$.
3. Variables with hat indicate value at the benchmark equilibrium.
4. In the GAMS program, values of endogenous variables at the benchmark equilibrium are normalized to unity (except for some variables). For example, Y_{ir} (output of CRTS sector i) is expressed as $Y_{ir} = \bar{Y}_{ir} y_{ir}$ where $\bar{Y}_{ir}$ is the benchmark value of Y_{ir} and we treat y_{ir} as a variable. `y(i,r)` in the program denote this normalized variable y_{ir}. The same normalization is applied to other endogenous variables.

5.1 Notations

First, let us define necessary notations. The last column shows variable name used in GAMS program.

Activity level

Notation	Description	Program
Y_{ir}	Output of CRTS sector ($i \in C$)	`y(i,r)`
Y_{ir}^{XT}	IRTS sector supply to transport sector ($i \in K$)	`y_xt(i,r)`
A_{ir}	Armington activity of goods i	`a(i,r)`
AD_{ir}	Aggregation of domestic varieties ($i \in K$)	`ad(i,r)`
AM_{ir}	Aggregation of imports from different region	`am(i,r)`
M_{isr}	Aggregation of import varieties ($i \in K$)	`m(i,s,r)`
U_r	Utility	`u(r)`
Y^T	International transport service	`yt`

Variables related to IRTS sectors

Notation	Description	Program
π_{ir}	Profit of a firm in IRTS sector ($i \in K$)	`proft(i,r)`
q^{D}_{ir}	Domestic supply of a firm in IRTS sector ($i \in K$)	`q_d(i,r)`
q^{X}_{irs}	Export supply to region s of a firm in IRTS sector in region r ($i \in K$)	`q_x(i,r,s)`
μ^{D}_{ir}	Markup rate for domestic supply ($i \in K$)	`mu_d(i,r)`
μ^{X}_{irs}	Markup rate for export supply ($i \in K$)	`mu_x(i,r,s)`
$\tilde{\mu}^{X}_{irs}$	Markup rate for export supply (adjusted by transport cost) ($i \in K$)	`mu_xx(i,r,s)`
β_{irs}	($i \in K$)	`beta(i,r,s)`
n_{ir}	The number of firms in IRTS sector ($i \in K$)	`n(i,r)`
q^{T}_{ir}	Total output of a firm in IRTS sector ($i \in K$)	`q_t(i,r)`
q^{TT}_{ir}	Total output + fixed input of a firm in IRTS sector ($i \in K$)	`q_tt(i,r)`
S^{AD}_{ir}	Share of domestic supply in Armington aggregation ($i \in K$)	`s_ad(i,r)`
S^{AM}_{ir}	Share of import supply in Armington aggregation ($i \in K$)	`s_am(i,r)`
S^{M}_{isr}	Share of import from region s in total import of region r ($i \in K$)	`s_m(i,s,r)`
AC_{ir}	Average cost ($i \in K$)	`ac(i,r)`

Unit cost and price index

Notation	Description	Program
c^{Y}_{ir}	Unit cost of sector i	`c_y(i,r)`
c^{A}_{ir}	Unit cost of Armington aggregation	`c_a(i,r)`
c^{AD}_{ir}	Unit cost of aggregation of domestic varieties	`c_ad(i,r)`
c^{U}_{r}	Unit cost of utility	`c_u(r)`
c^{AM}_{ir}	Unit cost of import aggregation	`c_am(i,r)`
c^{M}_{isr}	Unit cost of aggregation of import varieties	`c_m(i,s,r)`
c^{T}	Unit cost of international transport service	`c_t`
p^{PF}_{ir}	Price index of the primary factor composite	`p_pf(i,r)`
p^{INV}_{r}	Price index of investment	`p_inv(r)`
p^{Y}_{ir}	Price of output of CRTS sector	`p_y(i,r)`
p^{D}_{ir}	Price of a domestic variety ($i \in K$)	`p_d(i,r)`
p^{X}_{irs}	Price of an export variety ($i \in K$)	`p_x(i,r,s)`
$\tilde{p}^{X}_{irs}$	CIF price of an export variety	`p_x_(i,r,s)`
p^{AD}_{ir}	Price index of aggregated domestic variety ($i \in K$)	`p_ad(i,r)`
p^{AM}_{ir}	Price index of aggregated import	`p_am(i,r)`
p^{M}_{isr}	CIF price of import from region s	`p_m(i,s,r)`
p^{T}	Price of international transport service	`p_t`
p^{A}_{ir}	Price of Armington goods	`p_a(i,r)`
p^{F}_{fr}	Price of primary factor f	`p_f(f,r)`
p^{U}_{r}	Price index of utility	`p_u(r)`

Demand functions

Notation	Description	Program
a^F_{fir}	Unit demand for primary factor f in sector i	a_f(f,i,r)
a^C_{ir}	Unit final demand for goods i	a_c(i,r)
a^{AD}_{ir}	Unit demand for domestic goods in Armington aggregation	a_ad(i,r)
a^{AM}_{ir}	Unit demand for aggregated import in Armington aggregation	a_am(i,r)
a^M_{isr}	Region r's unit demand for import from region s	a_m(i,s,r)
a^{DD}_{ir}	Unit demand for a domestic variety	a_dd(i,r)
a^{MM}_{isr}	Region r's unit demand for a variety from region s	a_mm(i,s,r)
a^T_{ir}	Unit demand for input of transport sector	a_t(i,r)

Share parameters

Share parameters are constant at the benchmark level.

Notation	Description	Program
θ^C_{ir}	Share of goods i in final demand	sh_c(i,r)
θ^F_{fir}	Share of primary factor f in production	sh_f(f,i,r)
θ^I_{jir}	Share of intermediate goods j in production	sh_i(j,i,r)
θ^{PF}_{ir}	Share of the primary factor composite in production	sh_pf(i,r)
θ^{AD}_{ir}	Share of domestic goods in Armington aggregation	sh_ad(i,r)
θ^{AM}_{ir}	Share of import goods in Armington aggregation	sh_am(i,r)
θ^M_{isr}	Share of import from region s in total import of region r	sh_m(i,s,r)
θ^T_{ir}	Share of each input in transport sector	sh_t(i,r)
θ^{IMP}_{ir}	Share of value of import in total value of import including transport cost	sh_imp(i,r,s)

Elasticity of substitution (EOS)

Notation	Description	Program
σ^A_i	EOS between domestic and import goods in Armington aggregation	sig_a(i,r)
σ^M_i	EOS among imports from different region	sig_m(i,r)
σ^{PF}_i	EOS among primary factors	sig_pf(i,r)
σ^D_i	EOS among domestic varieties	sig_dd(i,r)
σ^F_i	EOS among import varieties	sig_ff(i,r)

Tax rates

Notation	Description	Program
t^Y_{ir}	Production tax rate for goods i	rto(i,r)
t^I_{jir}	Tax rate for intermediate input j in sector i	rti(j,i,r)
t^F_{fir}	Tax rate for primary factor f in production	ttf(f,i,r)
t^X_{irs}	Subsidy rate for export	rtxs(i,r,s)
t^M_{irs}	Tariff rate	rtms(i,r,s)
t^C_{ir}	Tax rate for final demand	rtc(i,r)

Variables for integrated market models

Notation	Description	Program
μ_{ir}	Overall markup rate ($i \in K$)	`mu(i,r)`
p^{COM}_{ir}	Price in the integrated market model ($i \in K$)	`p_com(i,r)`
δ^{D}_{ir}	Share of domestic supply in total supply ($i \in K$)	`delta_d(i,r)`
δ^{X}_{irs}	Share of export supply in total supply ($i \in K$)	`delta_x(i,r,s)`

Parameters for model QCV

Notation	Description	Program
ϕ^{D}_{ir}	Conjectural variation parameter for domestic supply ($i \in K$) (exogenous)	`phi0(i,r,r)`
ϕ^{X}_{irs}	Conjectural variation parameter for supply to region s ($i \in K$) (exogenous)	`phi0(i,r,s)`

Variables for model IC

Notation	Description	Program
$\hat{q}^{T}_{vir}$	Change in own total supply ($i \in K$)	`h_qtv(i,r)`
$\hat{q}^{D}_{vir}$	Change in own domestic supply ($i \in K$)	`h_qdv(i,r)`
$\hat{q}^{X}_{virs}$	Change in own export supply ($i \in K$)	`h_qxv(i,r,s)`
$\hat{p}^{r}_{it}$	Conjectured change in rival firm's price ($i \in K$)	`h_p(r,i,t)`
$\hat{q}^{D,r}_{it}$	Conjectured change in rival firm's domestic supply ($i \in K$)	`h_qd(r,i,t)`
$\hat{q}^{X,r}_{its}$	Conjectured change in rival firm's export supply ($i \in K$)	`h_qx(r,i,t,s)`
$\hat{p}^{\text{AD},r}_{is}$	Conjectured change in p^{AD}_{is} ($i \in K$)	`h_pad(r,i,s)`
$\hat{p}^{M,r}_{its}$	Conjectured change in p^{M}_{its} ($i \in K$)	`h_pmu(i,r)`
$\hat{p}^{\text{AM},r}_{is}$	Conjectured change in p^{AM}_{is} ($i \in K$)	`h_pam(r,i,s)`
$\hat{p}^{A,r}_{is}$	Conjectured change in p^{A}_{is} ($i \in K$)	`h_pa(r,i,s)`
$\hat{p}^{X}_{virs}$	Change in own export price ($i \in K$)	
$\hat{p}^{X,r}_{its}$	Conjectured change in p^{X}_{its} ($i \in K$)	

Other variables and parameters

Notation	Description	Program
H_r	Income of the representative household	`inc_ra(r)`
$\bar{a}^{I}_{jir}$	Input coefficient for intermediate goods j in sector i	`v_fm(j,i,r)`
$\bar{F}_{fr}$	Endowment of primary factor f (exogenous)	`evom(f,r)`
INV_r	Investment (exogenous)	`inv(r)`
BOP_r	Capital inflow (exogenous)	`vb(r)`
τ_{irs}	The amount of transport service required to ship one unit of goods i from region r to region s (exogenous)	`tau(i,r,s)`
fc_{ir}	Fixed input in IRTS sector ($i \in K$) (exogenous)	`fc0(i,r)`
ε^{A}_{ir}	Demand elasticity of Armington goods (exogenous)	`eod(i,r)`

5.2 Imperfectly competitive model (model CD)

In this section, equilibrium conditions related to IRTS sectors are presented. First, we explain conditions for model CD. As to other imperfectly competitive models, we present explanation in the last place.

5.2.1 Profit maximization

Profit of a firm in IRTS sector i in region r: Profit of a firm in IRTS sector i in region r is defined as follows:

$$\pi_{ir} = (1 - t^Y_{ir}) \left[p^D_{ir} q^D_{ir} + \sum_s p^X_{irs} q^X_{irs} \right] - c^Y_{ir} q^{\text{TT}}_{ir} \qquad \{\pi_{ir}\}_{i \in K}$$

where

$$q^T_{ir} = q^D_{ir} + \sum_s q^X_{irs} \qquad \{q^T_{ir}\}$$

$$q^{\text{TT}}_{ir} = q^T_{ir} + \text{fc}_{ir} \qquad \{q^{\text{TT}}_{ir}\}$$

FOCs for profit maximization: FOCs for profit maximization is given by

$$c^Y_{ir} \geq (1 - t^Y_{ir}) p^D_{ir} \left[1 - \mu^D_{ir} \right] \qquad \{q^D_{ir}\}_{i \in K}$$

$$c^Y_{ir} \geq (1 - t^Y_{ir}) p^X_{irs} \left[1 - \mu^X_{irs} \right] \qquad \{q^X_{irs}\}_{i \in K}$$

Since all markets are segmented, conditions for profit maximization are distinguished according to destination. The LHS represents marginal revenue and the RHS represents marginal cost.

Zero profit condition: Model CD assumes free entry–exit. Thus, the number of firms (varieties) is determined so that zero profit condition is satisfied.

$$0 \geq \pi_{ir} \qquad \{n_{ir}\}_{i \in K}$$

Average cost:

$$\text{AC}_{ir} = \frac{c^Y_{ir} q^{\text{TT}}_{ir}}{q^T_{ir}} \qquad \{\text{AC}_{ir}\}$$

5.2.2 Markup rates

Markup rates Markup rates are represented as follows:

$$\mu^D_{ir} = \frac{1}{\sigma^D_i} + \left\{ \frac{1}{\sigma^A_i} - \frac{1}{\sigma^D_i} + \left[\frac{1}{\varepsilon^A_{ir}} - \frac{1}{\sigma^A_i} \right] S^{\text{AD}}_{ir} \right\} \frac{1}{n_{ir}} \qquad \{\mu^D_{ir}\}_{i \in K}$$

$$\tilde{\mu}^X_{isr} = \frac{1}{\sigma^F_i} + \left\{ \frac{1}{\sigma^M_i} - \frac{1}{\sigma^F_i} + \left[\frac{1}{\sigma^A_i} - \frac{1}{\sigma^M_i} + \left(\frac{1}{\varepsilon^A_{ir}} - \frac{1}{\sigma^A_i} \right) S^{\text{AM}}_{ir} \right] S^M_{isr} \right\} \frac{1}{n_{is}} \qquad \{\tilde{\mu}^X_{irs}\}_{i \in K}$$

$$\mu^X_{irs} = \tilde{\mu}^X_{irs} / \beta_{irs} \qquad \{\mu^X_{irs}\}_{i \in K}$$

$$\beta_{irs} = \frac{(1 - t^X_{irs}) p^X_{irs}}{(1 - t^X_{irs}) p^X_{irs} + p^T \tau_{irs}} \qquad \{\beta_{irs}\}_{i \in K}$$

As FOCs for profit maximization, markup rates are distinguished according to destination.

Share variables: Share variables in markup rates are defined as follows:

$$S^{\text{AD}}_{ir} = \frac{p^{\text{AD}}_{ir} \text{AD}_{ir}}{p^A_{ir} A_{ir}} \qquad \{S^{\text{AD}}_{ir}\}_{i \in K}$$

$$S^{\text{AM}}_{ir} = \frac{p^{\text{AM}}_{ir} \text{AM}_{ir}}{p^A_{ir} A_{ir}} \qquad \{S^{\text{AM}}_{ir}\}_{i \in K}$$

$$S^M_{irs} = \frac{\tilde{p}^M_{irs} M_{irs}}{p^{\text{AM}}_{is} \text{AM}_{is}} \qquad \{S^M_{irs}\}_{i \in K}$$

5.3 Unit cost and price index

In this section, unit cost and price index are defined.

Price index of the primary factor composite: Price index of the primary factor composite in sector i is given by

$$p_{ir}^{\mathrm{PF}} = \bar{p}_{ir}^{\mathrm{PF}} \left[\sum_f \theta_{fir}^F \left[\frac{(1+t_{fir}^F) p_{fr}^F}{(1+\bar{t}_{fir}^F) \bar{p}_{fr}^F} \right]^{1-\sigma_i^{\mathrm{PF}}} \right]^{\frac{1}{1-\sigma_i^{\mathrm{PF}}}} \qquad \{p_{ir}^{\mathrm{PF}}\}$$

Unit cost of production: Unit cost of sector i is given by

$$c_{ir}^Y = \bar{c}_{ir}^Y \left[\sum_j \theta_{jir}^I \frac{(1+t_{jir}^I) p_{jr}^A}{(1+\bar{t}_{jir}^I) \bar{p}_{jr}^A} + \theta_{ir}^{\mathrm{PF}} \frac{p_{ir}^{\mathrm{PF}}}{\bar{p}_{ir}^{\mathrm{PF}}} \right] \qquad \{c_{ir}^Y\}$$

Since production technology is Leontief type, unit cost is the linear combination of prices of intermediate inputs and the primary factor composite.

Unit cost of Armington aggregation: Unit cost of Armington aggregation is

$$c_{ir}^A = \bar{c}_{ir}^A \left[\theta_{ir}^{\mathrm{AD}} \left(\frac{p_{ir}^Y}{\bar{p}_{ir}^Y} \right)^{1-\sigma_i^A} + \theta_{ir}^{\mathrm{AM}} \left(\frac{p_{ir}^{\mathrm{AM}}}{\bar{p}_{ir}^{\mathrm{AM}}} \right)^{1-\sigma_i^A} \right]^{\frac{1}{1-\sigma_i^A}} \qquad \{c_{ir}^A\}_{i\in C}$$

$$c_{ir}^A = \bar{c}_{ir}^A \left[\theta_{ir}^{\mathrm{AD}} \left(\frac{p_{ir}^{\mathrm{AD}}}{\bar{p}_{ir}^{\mathrm{AD}}} \right)^{1-\sigma_i^A} + \theta_{ir}^{\mathrm{AM}} \left(\frac{p_{ir}^{\mathrm{AM}}}{\bar{p}_{ir}^{\mathrm{AM}}} \right)^{1-\sigma_i^A} \right]^{\frac{1}{1-\sigma_i^A}} \qquad \{c_{ir}^A\}_{i\in K}$$

Unit cost of Armington aggregation differs across CRTS goods and IRTS goods.

Unit cost of aggregation of domestic varieties: In IRTS goods, different varieties are aggregated through CES function. Thus, unit cost of aggregation of domestic varieties is given by

$$c_{ir}^{\mathrm{AD}} = \bar{c}_{ir}^{\mathrm{AD}} \left[\frac{n_{ir}}{\bar{n}_{ir}} \right]^{\frac{1}{1-\sigma_i^D}} \frac{p_{ir}^D}{\bar{p}_{ir}^D} \qquad \{c_{ir}^{\mathrm{AD}}\}_{i\in K}$$

Unit cost of utility: Since utility function is Cobb-Douglas type, unit cost of utility (unit expenditure function) is given by

$$c_r^U = \bar{c}_r^U \prod_i \left[\frac{(1+t_{ir}^C) p_{ir}^A}{(1+\bar{t}_{ir}^C) \bar{p}_{ir}^A} \right]^{\theta_{ir}^C} \qquad \{c_r^U\}$$

Unit cost of import aggregation: Imports from different regions are aggregated into the import composite through a CES function. Thus, unit cost of import aggregation is given by

$$c_{ir}^{\mathrm{AM}} = \bar{c}_{ir}^{\mathrm{AM}} \left[\sum_s \theta_{isr}^M \left[\frac{(1+t_{isr}^M) p_{isr}^M}{(1+\bar{t}_{isr}^M) \bar{p}_{isr}^M} \right]^{1-\sigma_i^M} \right]^{\frac{1}{1-\sigma_i^M}} \qquad \{c_{ir}^{\mathrm{AM}}\}$$

Unit cost of aggregation of import variety: Import varieties are aggregated through CES function. c_{irs}^{M} represents unit cost of aggregation of varieties imported from region r to region s.

$$c_{irs}^{M} = \bar{c}_{irs}^{M} \left[\frac{n_{ir}}{\bar{n}_{ir}} \right]^{\frac{1}{1-\sigma_i^F}} \frac{\tilde{p}_{irs}^{X}}{\bar{\tilde{p}}_{irs}^{X}} \qquad \{c_{irs}^{M}\}_{i\in K}$$

CIF price of import goods: CIF price of import goods is the price which includes export tax and transport cost.

$$\tilde{p}_{irs}^{X} = \bar{\tilde{p}}_{irs}^{X} \left[\theta_{irs}^{\text{IMP}} \frac{(1-t_{irs}^{X}) p_{irs}^{X}}{(1-\bar{t}_{irs}^{X}) \bar{p}_{irs}^{X}} + (1-\theta_{irs}^{\text{IMP}}) \frac{p^T \tau_{irs}}{\bar{p}^T \bar{\tau}_{irs}} \right] \qquad \{\tilde{p}_{irs}^{X}\}_{i\in K}$$

$$\tilde{p}_{irs}^{X} = \bar{\tilde{p}}_{irs}^{X} \left[\theta_{irs}^{\text{IMP}} \frac{(1-t_{irs}^{X}) p_{ir}^{Y}}{(1-\bar{t}_{irs}^{X}) \bar{p}_{ir}^{Y}} + (1-\theta_{irs}^{\text{IMP}}) \frac{p^T \tau_{irs}}{\bar{p}^T \bar{\tau}_{irs}} \right] \qquad \{\tilde{p}_{irs}^{X}\}_{i\in C}$$

Unit cost of international transport sector: International transport service is created through Cobb-Douglas function. Thus, its unit cost is given by

$$c^T = \bar{c}^T \prod_{i,r} \left[\frac{p_{ir}^{Y}}{\bar{p}_{ir}^{Y}} \right]^{\theta_{ir}^{T}} \qquad \{c^T\}$$

Price of investment goods: Price of investment goods (p_r^{INV}) is equal to the price of goods cgd.

$$p_r^{\text{INV}} = p_{\text{cgd},r}^{Y} \qquad \{p_r^{\text{INV}}\}$$

5.3.1 Unit compensated demand

Demand for primary factor:

$$a_{fir}^{F} = \bar{a}_{fir}^{F} \left[\frac{p_{ir}^{\text{PF}} / \bar{p}_{ir}^{\text{PF}}}{(1+t_{fir}^{F}) p_{fr}^{F} / [(1+\bar{t}_{fir}^{F}) \bar{p}_{fr}^{F}]} \right]^{\sigma_i^{\text{PF}}} \qquad \{a_{fir}^{F}\}$$

Final demand :

$$a_{ir}^{C} = \bar{a}_{ir}^{C} \frac{c_r^{U} / \bar{c}_r^{U}}{(1+t_{ir}^{C}) p_{ir}^{A} / [(1+\bar{t}_{ir}^{C}) \bar{p}_{ir}^{A}]} \qquad \{a_{ir}^{C}\}$$

Demand for domestic goods from Armington aggregation:

$$a_{ir}^{\text{AD}} = \bar{a}_{ir}^{\text{AD}} \left[\frac{c_{ir}^{A} / \bar{c}_{ir}^{A}}{p_{ir}^{Y} / \bar{p}_{ir}^{Y}} \right]^{\sigma_i^A} \qquad \{a_{ir}^{\text{AD}}\}_{i\in C}$$

$$a_{ir}^{\text{AD}} = \bar{a}_{ir}^{\text{AD}} \left[\frac{c_{ir}^{A} / \bar{c}_{ir}^{A}}{p_{ir}^{\text{AD}} / \bar{p}_{ir}^{\text{AD}}} \right]^{\sigma_i^A} \qquad \{a_{ir}^{\text{AD}}\}_{i\in K}$$

Demand for import composite from Armington aggregation:

$$a_{ir}^{\text{AM}} = \bar{a}_{ir}^{\text{AM}} \left[\frac{c_{ir}^{A} / \bar{c}_{ir}^{A}}{p_{ir}^{\text{AM}} / \bar{p}_{ir}^{\text{AM}}} \right]^{\sigma_i^A} \qquad \{a_{ir}^{\text{AM}}\}$$

Demand for import:

$$a_{irs}^{M} = \frac{\bar{a}_{irs}^{M}}{\iota_{irs}^{\text{AMS}}} \left[\frac{c_{is}^{\text{AM}} \iota_{irs}^{\text{AMS}} / \bar{c}_{is}^{\text{AM}}}{(1 + t_{irs}^{M}) p_{irs}^{M} / [(1 + \bar{t}_{irs}^{M}) \bar{p}_{irs}^{M}]} \right]^{\sigma_i^M} \qquad \{a_{irs}^{M}\}$$

Demand for the domestic variety:

$$a_{ir}^{\text{DD}} = \bar{a}_{ir}^{\text{DD}} \left[\frac{n_{ir}}{\bar{n}_{ir}} \right]^{\frac{\sigma_i^D}{1-\sigma_i^D}} \qquad \{a_{ir}^{\text{DD}}\}$$

Demand for the import variety:

$$a_{irs}^{\text{MM}} = \bar{a}_{irs}^{\text{MM}} \left[\frac{n_{ir}}{\bar{n}_{ir}} \right]^{\frac{\sigma_i^F}{1-\sigma_i^F}} \qquad \{a_{irs}^{\text{MM}}\}$$

Demand for input of transport sector:

$$a_{ir}^{T} = \bar{a}_{ir}^{T} \frac{c^{T} / \bar{c}^{T}}{p_{ir}^{Y} / \bar{p}_{ir}^{Y}} \qquad \{a_{ir}^{T}\}$$

5.3.2 Zero profit condition

Production activity:

$$c_{ir}^{Y} \geq (1 - t_{ir}^{Y}) p_{ir}^{Y} \qquad \{Y_{ir}\}_{i \in C}$$

Unit cost of production of inputs to transport sector:

$$c_{ir}^{Y} \geq (1 - t_{ir}^{Y}) p_{ir}^{Y} \qquad \{Y_{ir}^{\text{XT}}\}_{i \in K}$$

Armington aggregation:

$$c_{ir}^{A} \geq p_{ir}^{A} \qquad \{A_{ir}\}$$

Aggregation of domestic varieties:

$$c_{ir}^{\text{AD}} \geq p_{ir}^{\text{AD}} \qquad \{\text{AD}_{ir}\}_{i \in K}$$

Aggregation of imports from different regions:

$$c_{ir}^{\text{AM}} \geq p_{ir}^{\text{AM}} \qquad \{\text{AM}_{ir}\}$$

Aggregation of import varieties:

$$c_{irs}^{M} \geq p_{irs}^{M} \qquad \{M_{irs}\}_{i \in K}$$
$$M_{irs} = a_{irs}^{M} \text{AM}_{irs} \qquad \{M_{irs}\}_{i \in C}$$

Utility:

$$c_{r}^{U} \geq p_{r}^{U} \qquad \{U_{r}\}$$

International transport sector:

$$c^T \geq p^T \qquad \{Y^T\}$$

5.3.3 Market clearing conditions

Below, we present market clearing conditions. Basically, the LHS represents supply and the RHS represents demand.

Market of output of CRTS sector : Supply is Y_{ir} and demand is the sum of domestic demand, export demand, and demand from transport sector.

$$Y_{ir} \geq a^{\mathrm{AD}}_{ir} A_{ir} + \sum_s a^M_{irs} \mathrm{AM}_{is} + a^T_{ir} Y^T \qquad \{p^Y_{ir}\}_{i \in C, i \neq \mathrm{cgd}}$$

Supply of IRTS sectors to international transport sector:

$$Y^{\mathrm{XT}}_{ir} \geq a^T_{ir} Y^T \qquad \{p^Y_{ir}\}_{i \in K}$$

Market of investment goods:

$$Y_{ir} \geq \mathrm{INV}_r \qquad \{p^Y_{ir}\}_{i = \mathrm{cgd}}$$

Market of domestic variety ($i \in K$): Supply of a variety is q^D_{ir} and demand for it is $a^{\mathrm{DD}}_{ir} \mathrm{AD}_{ir}$.

$$q^D_{ir} \geq a^{\mathrm{DD}}_{ir} \mathrm{AD}_{ir} \qquad \{p^D_{ir}\}$$

Market of export variety ($i \in K$): Supply of a variety is q^X_{irs} and demand for it is $a^{\mathrm{MM}}_{irs} M_{irs}$.

$$q^X_{irs} \geq a^{\mathrm{MM}}_{irs} M_{irs} \qquad \{p^X_{irs}\}$$

Market for inputs to transport sector ($i \in K$):

$$Y^{\mathrm{XT}}_{ir} \geq a^T_{ir} Y^T \qquad \{p^Y_{ir}\}$$

Market of domestic goods: Market of aggregated domestic variety. Supply is AD_{ir} and demand for it is $a^{\mathrm{AD}}_{ir} A_{ir}$.

$$\mathrm{AD}_{ir} \geq a^{\mathrm{AD}}_{ir} A_{ir} \qquad \{p^{\mathrm{AD}}_{ir}\}$$

Market of import composite: Supply of import composite is AM_{ir} and demand is $a^{\mathrm{AM}}_{ir} A_{ir}$.

$$\mathrm{AM}_{ir} \geq a^{\mathrm{AM}}_{ir} A_{ir} \qquad \{p^{\mathrm{AM}}_{ir}\}$$

Market of import goods: Supply is M_{irs} and demand is $a^M_{irs} \mathrm{AM}_{is}$. In the case of CRTS goods ($i \in C$), the next relation defines CIF price of import goods.

$$M_{irs} \geq a^M_{irs} \mathrm{AM}_{is} \qquad \{p^M_{irs}\}_{i \in K}$$
$$p^M_{irs} = \tilde{p}^X_{irs} \qquad \{p^M_{irs}\}_{i \in C}$$

Market of transport service: Supply is Y^T and demand is the sum of demand from transport of CRTS goods ($\tau_{isr} a^M_{isr} \mathrm{AM}_{ir}$) and demand from transport of IRTS goods ($\tau_{isr} n_{is} a^{\mathrm{MM}}_{isr} M_{isr}$).

$$Y^T \geq \sum_{i \in C, s, r} \tau_{isr} a^M_{isr} \mathrm{AM}_{ir} + \sum_{i \in K, s, r} \tau_{isr} n_{is} a^{\mathrm{MM}}_{isr} M_{isr} \qquad \{p^T\}$$

Market of Armington goods: Supply is A_{ir} and demand consists of intermediate demand and final demand.

$$A_{ir} \geq \sum_{j \in C} \bar{a}^I_{ijr} Y_{jr} + \sum_{j \in K} \bar{a}^I_{ijr} (Y^{\mathrm{XT}}_{jr} + n_{jr} q^{\mathrm{TT}}_{jr}) + a^C_{ir} U_r \qquad \{p^A_{ir}\}$$

Market of primary factors:

$$\bar{E}^F_f \geq \sum_{i \in C} a^F_{fir} Y_{ir} + \sum_{i \in K} a^F_{fir} (Y^{\mathrm{XT}}_{ir} + n_{ir} q^{\mathrm{TT}}_{ir}) \qquad \{p^F_{fr}\}$$

Utility: This is the condition that income is equal to expenditure.

$$H_r \geq p^U_r U_r \qquad \{p^U_r\}$$

5.3.4 Income of the representative household

Income: Income spent on consumption is the sum of factor income, tax revenue, profit, and net capital inflow minus investment expenditure.

$$\begin{aligned} H_r = & \sum_f p^F_{fr} E^F_{fr} \\ & + \sum_{i \in C} t^Y_{ir} p^Y_{ir} Y_{ir} + \sum_{i \in K} t^Y_{ir} p^Y_{ir} Y^{\mathrm{XT}}_{ir} + \sum_{i \in K} t^Y_{ir} n_{ir} \left[p^D_{ir} q^D_{ir} + \sum_s p^X_{irs} q^X_{irs} \right] \\ & + \sum_j \left[\sum_{i \in C} t^I_{jir} p^A_{jr} \bar{a}^I_{jir} Y_{ir} + \sum_{i \in K} t^I_{jir} p^A_{jr} \bar{a}^I_{jir} (Y^{\mathrm{XT}}_{ir} + n_{ir} q^{\mathrm{TT}}_{ir}) \right] \\ & + \sum_f \left[\sum_{i \in C} t^F_{fir} p^F_{fr} a^F_{fir} Y_{ir} + \sum_{i \in K} t^F_{fir} p^F_{fr} a^F_{fir} (Y^{\mathrm{XT}}_{ir} + n_{ir} q^{\mathrm{TT}}_{ir}) \right] \\ & - \sum_{i \in C, s \in R} t^X_{irs} p^Y_{ir} a^M_{irs} \mathrm{AM}_{is} - \sum_{i \in K, s \in R} t^X_{irs} p^X_{irs} n_{ir} q^X_{irs} \\ & + \sum_{i \in C, s \in R} t^M_{isr} p^M_{isr} a^M_{isr} \mathrm{AM}_{ir} + \sum_{i \in K, s \in R} t^M_{isr} [(1 - t^X_{isr}) p^X_{isr} + \tau_{isr} p^T] n_{is} q^X_{isr} \\ & + \sum_i t^C_{ir} p^A_{ir} a^C_{ir} U_r + \sum_{i \in K} \pi_i + p^U_z \mathrm{BOP}_r - p^{\mathrm{INV}}_r \mathrm{INV}_r \end{aligned}$$

5.4 Other imperfectly competitive models

So far, we assume model CD as the imperfectly competitive model. In this section, we explain how equilibrium conditions are modified under different models.

5.4.1 Model CH

First, we consider model CH. In model CH, it is assumed that varieties in an industry are homogeneous. Since aggregation of varieties is not included in model CH, variables c^{AD}_{ir}, c^M_{irs}, a^{DD}_{ir}, and a^{MM}_{irs} do not disappear in model CH.

Markup rates: Markup rates are modified as follows:

$$\mu_{ir}^{D} = \left[\frac{1}{\sigma_i^A} + \left(\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A}\right) S_{ir}^{\mathrm{AD}}\right] \frac{1}{n_{ir}} \qquad \{\mu_{ir}^D\}_{i \in K}$$

$$\tilde{\mu}_{irs}^{X} = \left\{\frac{1}{\sigma_i^M} + \left[\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^M} + \left(\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_i^A}\right) S_{is}^{\mathrm{AM}}\right] S_{irs}^{M}\right\} \frac{1}{n_{ir}} \qquad \{\tilde{\mu}_{irs}^X\}_{i \in K}$$

Demand for domestic goods:

$$\mathrm{AD}_{ir} = a_{ir}^{\mathrm{AD}} A_{ir} \qquad \{\mathrm{AD}_{ir}\}_{i \in K}$$

Demand for import goods:

$$M_{irs} = a_{irs}^{M} \mathrm{AM}_{is} \qquad \{M_{irs}\}_{i \in K}$$

Domestic variety: Total domestic supply is sum of supply of each variety.

$$n_{ir} q_{ir}^{D} \geq \mathrm{AD}_{ir} \qquad \{p_{ir}^D\}_{i \in K}$$

Price of export goods:

$$p_{irs}^{M} = \tilde{p}_{irs}^{X} \qquad \{p_{irs}^X\}_{i \in K}$$

Price of domestic goods:

$$p_{ir}^{\mathrm{AD}} = p_{ir}^{D} \qquad \{p_{ir}^D\}_{i \in K}$$

Export variety: Total export supply is sum of supply of each variety.

$$n_{ir} q_{irs}^{X} \geq M_{irs} \qquad \{p_{irs}^M\}_{i \in K}$$

5.4.2 Model CF

Model CF assumes that the number of firms (n_{ir}) is exogenously constant.

The number of firms: The number of firms ($\bar{n}$) is constant at the benchmark value.

$$n_{ir} = \bar{n}_{ir} \qquad \{n_{ir}\}_{i \in K}$$

5.4.3 Model LGMC

Model LGMC is the large group monopolistic competition model.

Markup rates: Since each firm conjectures $n_{ir} \to \infty$, markup rates are modified as follows:

$$\mu_{ir}^{D} = 1/\sigma_i^{D} \qquad \{\mu_{ir}^D\}_{i \in K}$$
$$\tilde{\mu}_{irs}^{X} = 1/\sigma_i^{F} \qquad \{\tilde{\mu}_{irs}^X\}_{i \in K}$$

5.4.4 Model QCV

Model QC assumes non-zero conjectural variation. Thus, form of markup rates are modified.

Markup rates:

$$\mu_{ir}^{D} = \frac{1}{\sigma_i^D} + \left[\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^D}\right] \frac{1+(n_{ir}-1)\phi_{ir}^D}{n_{ir}} + \left[\frac{1}{\varepsilon_{ir}^A} - \frac{1}{\sigma_{ir}^A}\right] \frac{S_{ir}^{\mathrm{AD}} + (n_{ir} - S_{ir}^{\mathrm{AD}})\phi_{ir}^D}{n_{ir}} \qquad \{\mu_{ir}^D\}_{i\in K}$$

$$\tilde{\mu}_{irs}^{X} = \frac{1}{\sigma_i^F} + \left[\frac{1}{\sigma_i^M} - \frac{1}{\sigma_i^F}\right] \frac{1+(n_{ir}-1)\phi_{irs}^X}{n_{ir}} + \left[\frac{1}{\sigma_i^A} - \frac{1}{\sigma_i^M}\right] \frac{S_{irs}^{M} + (n_{ir} - S_{irs}^{M})\phi_{isr}^X}{n_{ir}} + \left[\frac{1}{\varepsilon_{is}^A} - \frac{1}{\sigma_i^A}\right] \frac{S_{irs}^{M}S_{is}^{\mathrm{AM}} + (n_{ir} - S_{irs}^{M}S_{is}^{\mathrm{AM}})\phi_{isr}^X}{n_{ir}} \qquad \{\tilde{\mu}_{irs}^X\}_{i\in K}$$

5.4.5 Model BD

Model BD assumes Bertrand competition.

Markup rate:

$$1/\mu_{ir}^D = \varepsilon_{ir}^D = \sigma_i^D + [\sigma_i^A - \sigma_i^D + (\varepsilon_{ir}^A - \sigma_i^A)S_{ir}^{\mathrm{AD}}]\frac{1}{n_{ir}} \qquad \{\mu_{ir}^D\}_{i\in K}$$

$$1/\tilde{\mu}_{irs}^X = \sigma_i^F + \{\sigma_i^M - \sigma_i^F + [\sigma_i^A - \sigma_i^M + (\varepsilon_{is}^A - \sigma_i^A)S_{is}^{\mathrm{AM}}]S_{irs}^M\}\frac{1}{n_{ir}} \qquad \{\tilde{\mu}_{irs}^X\}_{i\in K}$$

5.4.6 Model IC

Model IC assumes Cournot competition and integrated market.

FOC for profit maximization:

$$c_{ir}^Y \geq (1 - t_{ir}^Y)p_{ir}^{\mathrm{COM}}(1 - \mu_{ir}) \qquad \{q_{ir}^T\}_{i\in K}$$

Overall markup rate:

$$\mu_{ir} = -\hat{p}_{vir}/\hat{q}_{vir}^T \qquad \{\mu_{ir}\}_{i\in K}$$

Change in own quantity of firm v in region r: Superscript r indicates that changes in variables are conjectured by a firm in region r.

$$\hat{q}_{vir}^T = \delta_{ir}^D\hat{q}_{vir}^D + \sum_s \delta_{irs}^X\hat{q}_{virs}^X \qquad \{\hat{q}_{vir}^T\}_{i\in K}$$

$$\hat{q}_{vir}^D = -\sigma_r^D\hat{p}_{vir} + (\sigma_r^D - \sigma_r^A)\hat{p}_{ir}^{\mathrm{AD},r} + (\sigma_r^A - \varepsilon_{ir}^A)\hat{p}_{ir}^{A,r} \qquad \{\hat{q}_{vir}^D\}_{i\in K}$$

$$\hat{q}_{virs}^X = -\sigma_s^F\hat{p}_{virs}^X + (\sigma_s^F - \sigma_s^M)\hat{p}_{irs}^{M,r} + (\sigma_s^M - \sigma_s^A)\hat{p}_{is}^{\mathrm{AM},r} + (\sigma_s^A - \varepsilon_{is}^A)\hat{p}_{is}^{A,r} \qquad \{\hat{q}_{virs}^X\}_{i\in K}$$

Changes in quantity of rival firms implied by Cournot conjecture: $t = r$ means the domestic rival firm and $t \neq r$ means the foreign rival firm.

$$\delta_{it}^D\hat{q}_{it}^{D,r} + \sum_s \delta_{its}^X\hat{q}_{its}^{X,r} = 0 \qquad \{\hat{p}_{it}^r\}_{i\in K}$$

$$\hat{q}_{it}^{D,r} = -\sigma_t^D\hat{p}_{it}^r + (\sigma_t^D - \sigma_t^A)\hat{p}_{it}^{\mathrm{AD},r} + (\sigma_t^A - \varepsilon_{it}^A)\hat{p}_{it}^{A,r} \qquad \{\hat{q}_{it}^{D,r}\}_{i\in K}$$

$$\hat{q}_{its}^{X,r} = -\sigma_s^F\hat{p}_{its}^{X,r} + (\sigma_s^F - \sigma_s^M)\hat{p}_{its}^{M,r} + (\sigma_s^M - \sigma_s^A)\hat{p}_{is}^{\mathrm{AM},r} + (\sigma_s^A - \varepsilon_{is}^A)\hat{p}_{is}^{A,r} \qquad \{\hat{q}_{its}^{X,r}\}_{i\in K}$$

Conjectured changes in prices:

$$\hat{p}_{is}^{\text{AD},r} = \begin{cases} \dfrac{1}{n_{is}}[\hat{p}_{vis} + (n_{is}-1)\hat{p}_{is}^{r}] & s = r \\ \hat{p}_{is}^{r} & s \neq r \end{cases} \qquad \{\hat{p}_{is}^{\text{AD},r}\}_{i\in K}$$

$$\hat{p}_{its}^{M,r} = \begin{cases} \dfrac{1}{n_{it}}[\hat{p}_{vits}^{X} + (n_{it}-1)\hat{p}_{its}^{X,r}] & t = r \\ \hat{p}_{its}^{X,r} & t \neq r \end{cases} \qquad \{\hat{p}_{its}^{M,r}\}_{i\in K}$$

$$\hat{p}_{is}^{\text{AM},r} = \sum_{t} \delta_{its}^{M} \hat{p}_{its}^{M,r} \qquad \{\hat{p}_{is}^{\text{AM},r}\}_{i\in K}$$

$$\hat{p}_{is}^{A,r} = \delta_{is}^{\text{AD}} \hat{p}_{is}^{\text{AD},r} + \delta_{is}^{\text{AM}} \hat{p}_{is}^{\text{AM},r} \qquad \{\hat{p}_{is}^{A,r}\}_{i\in K}$$

$$\hat{p}_{virs}^{X} = \beta_{irs} \hat{p}_{vir} \qquad \{\hat{p}_{virs}^{X}\}_{i\in K}$$

$$\hat{p}_{its}^{X,r} = \beta_{its} \hat{p}_{it}^{r} \qquad \{\hat{p}_{its}^{X,r}\}_{i\in K}$$

Normalization:

$$\hat{p}_{vir} = 1 \qquad \{\hat{p}_{vir}\}_{i\in K}$$

Markets for outputs:

$$q_{ir}^{T} \geq a_{ir}^{\text{DD}} \text{AD}_{ir} + \sum_{s} a_{irs}^{\text{MM}} M_{irs} \qquad \{p_{ir}^{\text{COM}}\}_{i\in K}$$

Price of outputs:

$$p_{ir}^{D} = p_{ir}^{\text{COM}} \qquad \{p_{ir}^{D}\}_{i\in K}$$

$$p_{irs}^{X} = p_{ir}^{\text{COM}} \qquad \{p_{irs}^{X}\}_{i\in K}$$

Supply for each market:

$$q_{ir}^{D} = a_{ir}^{\text{DD}} \text{AD}_{ir} \qquad \{q_{ir}^{D}\}_{i\in K}$$

$$q_{irs}^{X} = a_{irs}^{\text{MM}} M_{irs} \qquad \{q_{irs}^{X}\}_{i\in K}$$

5.4.7 Model IB

Model IB assumes Bertrand competition and integrated market.

FOC for profit maximization: Each firm determines total output so as to maximize profit.

$$c_{ir}^{Y} \geq (1 - t_{ir}^{Y}) p_{ir}^{\text{COM}} (1 - \mu_{ir}) \qquad \{q_{ir}^{T}\}_{i\in K}$$

Overall markup rate:

$$1/\mu_{ir} = \sum_{s} \delta_{irs}^{X} / \mu_{irs}^{X} + \delta_{ir}^{D} / \mu_{ir}^{D} \qquad \{\mu_{ir}\}_{i\in K}$$

where δ_{ir}^{D} and δ_{irs}^{X} are supply shares defined as follows

$$\delta_{irs}^{X} = q_{irs}^{X} / q_{ir}^{T} \qquad \{\delta_{irs}^{X}\}_{i\in K}$$

$$\delta_{ir}^{D} = q_{ir}^{D} / q_{ir}^{T} \qquad \{\delta_{ir}^{D}\}_{i\in K}$$

Markup rates for each market:

$$1/\mu_{ir}^{D} = \varepsilon_{ir}^{D} = \sigma_i^{D} + [\sigma_i^{A} - \sigma_i^{D} + (\varepsilon_{ir}^{A} - \sigma_i^{A})S_{ir}^{\mathrm{AD}}]\frac{1}{n_{ir}} \qquad \{\mu_{ir}^{D}\}_{i\in K}$$

$$1/\tilde{\mu}_{irs}^{X} = \sigma_i^{F} + \{\sigma_i^{M} - \sigma_i^{F} + [\sigma_i^{A} - \sigma_i^{M} + (\varepsilon_{is}^{A} - \sigma_i^{A})S_{is}^{\mathrm{AM}}]S_{irs}^{M}\}\frac{1}{n_{ir}} \qquad \{\tilde{\mu}_{irs}^{X}\}_{i\in K}$$

Markets for outputs:

$$q_{ir}^{T} \geq a_{ir}^{\mathrm{DD}}\mathrm{AD}_{ir} + \sum_{s} a_{irs}^{\mathrm{MM}} M_{irs} \qquad \{p_{ir}^{\mathrm{COM}}\}_{i\in K}$$

Price of outputs:

$$p_{ir}^{D} = p_{ir}^{\mathrm{COM}} \qquad \{p_{ir}^{D}\}_{i\in K}$$
$$p_{irs}^{X} = p_{ir}^{\mathrm{COM}} \qquad \{p_{irs}^{X}\}_{i\in K}$$

Supply for each market:

$$q_{ir}^{D} = a_{ir}^{\mathrm{DD}}\mathrm{AD}_{ir} \qquad \{q_{ir}^{D}\}_{i\in K}$$
$$q_{irs}^{X} = a_{irs}^{\mathrm{MM}} M_{irs} \qquad \{q_{irs}^{X}\}_{i\in K}$$

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